PH-291 Physics Lab
Professor Corn-Agostini
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Lab # 2: Index of Refraction

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Read and sign Academic Integrity Statement:

I hereby attest that I have not given or received any unauthorized assistance on this assignment.

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Grading Rubric

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1 Purpose

In this lab, Pfund's Method and Snell's Law will be used to determine the index of refraction of an unknown liquid. Two methods will be used to find this index of refraction, the first utilizing Pfund's Method, while the second uses Snell's Law. The index of refraction of the glass petri dish is determined via Pfund's Method. This then allows us to proceed with both methods. As light is refracted through a medium, a small circular region is observed, surrounded by a darker region with a radius that can be used to calculate the unknown index of refraction via Pfund's Law. In the second method, the incident and reflected angles of the light are measured and plugged into Snell's Law. We can compare the two calculated values to determine the unknown substance.

2 Data

Measurement	**Thickness (mm)**
1	1.695
2	1.995
3	1.990
4	2.095
5	1.930
6	1.995
Instrumental Error: 0.005 mm	
Random Error: 0.055 mm	
Thickness: 1.950 ± 0.055 mm	

Table 1: Petri Dish Thickness

Measurement	**Thickness (mm)**
1	7.98
2	8.86
3	8.24
4	7.60
5	8.16
6	7.66
Instrumental Error: 0.02 mm	
Random Error: 0.19 mm	
Thickness: 8.08 ± 0.19 mm	

Table 2: Ring Diameter Without Liquid

Measurement	**Diameter (mm)**
1	19.68
2	19.60
3	19.58
4	19.58
5	19.56
6	19.62
Instrumental Error: 0.02 mm	
Random Error: 0.017 mm	
Thickness: 19.60 ± 0.02 mm	

Table 3: Ring Diameter With Liquid

Measurement	**Incident Angle**	**Refracted Angle**
1	50.0°	28.0°
2	28.0°	38.0°
3	36.0°	20.0°
4	17.0°	40.0°
5	43.0°	27.0°
6	28.0°	39.0°
Instrumental Error: 0.5°		

Table 4: Snell's Law

3 Calculations

Part A: Pfund's Method

1. Index of Refraction (n_{glass})

$$n_{\text{glass}} = \frac{\sqrt{d^2 + 16t^2}}{d}$$

Before the index of refraction of the liquid (n_{liquid}) could be found, we must first find the index of refraction of our petri dishes (n_{glass}). Here, d represents the diameter of the ring of light reflected by the glass and t represents the thickness of the petri dish. When solving for n_{glass} mean values of d and t were used. The error of each measurement was propagated through this calculation using the following equations.

Error Propagation:

1.1 Partial Derivative of Eq. 1 w.r.t *d*

$$\frac{\partial n_{\text{glass}}}{\partial d} = -\frac{16t^2}{d^2\sqrt{d^2 + 16t^2}}$$

1.2 Partial Derivative of Eq. 1 w.r.t *t*

$$\frac{\partial n_{\text{glass}}}{\partial t} = \frac{16t}{d\sqrt{16t^2 + d^2}}$$

Each of these partial derivatives are used in the final calculation of the total error associated with n_{glass}. Because d and t are both independent variables we use the following equation to find the total error.

1.3 Total Error Associated with the Index of Refraction (n_{glass})

$$\delta n_{\text{glass}} = \sqrt{\left(\frac{\partial n_{\text{glass}}}{\partial t}\delta t\right)^2 + \left(\frac{\partial n_{\text{glass}}}{\partial d}\delta d\right)^2}$$

The error associated with each set of measurements was calculated (shown in the results section). The mean of those values provides our final error associated with (n_{glass}).

2. Index of Refraction (n_{liquid})

$$n_{\text{liquid}} = \frac{n_{\text{glass}} d}{\sqrt{d^2 + 16t^2}}$$

We proceed to calculate (n_{liquid}), which relies on (n_{glass}) (calculated above), the diameter d of the first ring reflected by the liquid, and the thickness of the petri dish t. The total error of (n_{liquid}) is found using the following equations.

Error Propagation:

2.1 Partial Derivative of Eq. 2 w.r.t *d*

$$\frac{\partial n_{\text{liquid}}}{\partial d} = \frac{16 n_{\text{glass}} t^2}{(d^2 + 16t^2)^{\frac{3}{2}}}$$

2.2 Partial Derivative of Eq. 2 w.r.t *t*

$$\frac{\partial n_{\text{liquid}}}{\partial t} = -\frac{16 d n_{\text{glass}} t}{(16t^2 + d^2)^{\frac{3}{2}}}$$

2.3 Partial Derivative of Eq. 2 w.r.t $\boldsymbol{n}_{\textbf{glass}}$

$$\frac{\partial n_{\text{liquid}}}{\partial n_{\text{glass}}} = \frac{d}{\sqrt{16t^2 + d^2}}$$

Each of these partial derivatives are used in calculating the total error in the following equation. All of the variables $((n_{\text{glass}}), d,$ and $t)$ are independent, which allows us to use the same process as above (Equation 1.3).
2.4 Total Error Associated with the Index of Refraction ($\boldsymbol{n}_{\textbf{liquid}}$)

$$\delta n_{\text{liquid}} = \sqrt{\left(\frac{\partial n_{\text{liquid}}}{\partial t}\delta t\right)^2 + \left(\frac{\partial n_{\text{liquid}}}{\partial d}\delta d\right)^2 + \left(\frac{\partial n_{\text{liquid}}}{\partial n_{\text{glass}}}\delta n_{\text{glass}}\right)^2}$$

As in Equation 1.3, this equation gives us the total error associated with (n_{liquid}). The mean of these error values provided us with our final error associated with our mean (n_{liquid}) value, shown in the results section.

Part B: Snell's Law

3. The Law of Refraction (Snell's Law)

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \Rightarrow n_2 = \frac{n_1 \sin\theta_1}{\sin\theta_2}$$

Part B of the experiment requires us to employ the Law of Refraction, also known as Snell's Law, to find the index of refraction of the liquid, which we'll refer to as n_2. In this equation, θ_1 refers to the incident angle (the angle at which the light enters the medium) and θ_2 refers to the refracted angle (the angle at which the light exits the medium). The final error associated with n_2 is given by the following process.

Error Propagation

3.1 Partial Derivative of Eq. 3 w.r.t $\boldsymbol{\theta}_1$

$$\frac{\partial n_2}{\partial \theta_1} = \frac{n_1 \cos(\theta_1)}{\sin(\theta_2)}$$

3.2 Partial Derivative of Eq. 3 w.r.t $\boldsymbol{\theta}_2$

$$\frac{\partial n_2}{\partial \theta_2} = -\frac{n_1 \sin(\theta_1)\cos(\theta_2)}{\sin^2(\theta_2)}$$

Each of these partial derivatives are used in the following equation to find the total error associated with n_2. Because n_1 was a given value of the index of refraction of air, there is no error associated with that measurement, so it will not factor into the total error. θ_1 and θ_2 are dependent variables, which allows us to use the following equation to find the total error.

3.3 Total Error Associated with the Index of Refraction ($\boldsymbol{n_2}$)

$$\delta n_2 = \left|\left(\frac{\partial n_2}{\partial \theta_1}\delta\theta_1\right)\right| + \left|\left(\frac{\partial n_2}{\partial \theta_2}\delta\theta_2\right)\right|$$

The total error was calculated using this equation for each set of measured angles. The mean of the calculated n_2 values alongside the mean error value (δn_2) provided our final value of the index of refraction of the liquid.

4 Results

5 Conclusion