PH-291 Physics Lab
Professor Corn-Agostini
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Lab # 5: Diffraction & Interference

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Read and sign Academic Integrity Statement:

I hereby attest that I have not given or received any unauthorized assistance on this assignment.

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Grading Rubric

CATEGORY	**POINTS**	**GRADE**
Purpose	2	
Data	6	
Theory & Calculations (includes Q1 and Q2)	6	
Results & Analysis	4	
Conclusion	2	
Total	20	

Contents

1 Purpose

In this experiment, the wave nature of light will be analyzed by observing how a laser beam interacts with certain obstacles and openings. First, the wavelength of the laser beam will be determined through the N-slit experiment. The light will pass through a grating with 1,000 lines/mm onto a screen, and the location of the intensity maxima will then be used to calculate the wavelength. After determining this wavelength, a single slit is created using a Vernier caliper, and the wavelength is determined using the minima from the diffraction pattern generated by the slit. The diameter of a human hair can then be determined using Babinet's principle which states that the diffraction pattern of a shape is identical to the diffraction pattern of a screen with the shape cut out of it. Here, the diffraction pattern of the hair is identical to the diffraction pattern of an opaque screen with a slit that has the same width as the hair.

2 Data

Measurement	$\boldsymbol{D}$ **(cm)**	$\boldsymbol{x_{+1}}$**(cm)**	$\boldsymbol{x_{-1}}$**(cm)**
1	14.95	13.900	13.360
2	14.80	13.430	13.100
3	14.75	13.300	12.942
4	14.75	13.250	13.236
$\boldsymbol{\mu}$	14.81	13.470	13.160
Random Error	0.04	0.13	0.08
Instrumental Error	±0.05	±0.002	

Table 1: Interference due to a grating

Here, D is the distance between the grating and the screen. This was measured using a meter stick. The spacing between each slit is the reciprocal of the number of lines/mm for the grating. Our grating had 1000 lines/mm, which gave us $d = 1 * 10^{-4}$ cm. Using D and the distances between the center and the principal maxima (x_{+1} and x_{-1}), we can calculate θ, which allows us to calculate the wavelength of the laser. When measuring the centers of each peak with vernier calipers, we found that there was an uncertainty of ±0.1 cm in addition to the instrumental error associated with the calipers. This affected our measurements for x_{+1} and x_{-1} and affected the calculated wavelength values.
We then calculated the wavelength by using a single slit. These set using a pair of vernier calipers, which have an instrumental error of 0.002 cm. Measurements were taken for slit widths of 0.2mm, 0.3mm, and 0.4mm.

Measurement	$\boldsymbol{D}$ **(cm)**	$\boldsymbol{x_{+1}}$**(cm)**	$\boldsymbol{x_{-1}}$**(cm)**	$\boldsymbol{x_{+2}}$**(cm)**	$\boldsymbol{x_{-2}}$**(cm)**
1	49.40	0.130	0.156	0.320	0.330
2	49.30	0.136	0.158	0.298	0.320
3	49.40	0.158	0.156	0.320	0.346
4	49.30	0.138	0.160	0.318	0.354
$\boldsymbol{\mu}$	49.35	0.141	0.158	0.314	0.338
Random Error	0.03	0.005	0.0008	0.005	0.007
Instrumental Error	±0.05	±0.002			

Table 2: 0.2mm Single Slit Diffraction

Measurement	$\boldsymbol{D}$ **(cm)**	$\boldsymbol{x_{+1}}$**(cm)**	$\boldsymbol{x_{-1}}$**(cm)**	$\boldsymbol{x_{+2}}$**(cm)**	$\boldsymbol{x_{-2}}$**(cm)**
1	69.20	0.110	0.118	0.246	0.244
2	69.20	0.100	0.120	0.260	0.254
3	69.20	0.108	0.138	0.262	0.268
4	69.20	0.102	0.136	0.258	0.240
$\boldsymbol{\mu}$	69.20	0.105	0.128	0.257	0.252
Random Error	0.00	0.002	0.005	0.003	0.005
Instrumental Error	±0.05	±0.002			

Table 3: 0.3mm Single Slit Diffraction

Measurement	$\boldsymbol{D}$ **(cm)**	$\boldsymbol{x_{+1}}$**(cm)**	$\boldsymbol{x_{-1}}$**(cm)**	$\boldsymbol{x_{+2}}$**(cm)**	$\boldsymbol{x_{-2}}$**(cm)**
1	114.80	0.198	0.180	0.410	0.430
2	114.70	0.260	0.190	0.420	0.432
3	114.75	0.265	0.198	0.420	0.454
4	114.80	0.196	0.194	0.410	0.420
$\boldsymbol{\mu}$	114.76	0.230	0.191	0.415	0.434
Random Error	0.02	0.019	0.004	0.003	0.007
Instrumental Error	±0.05	±0.002			

Table 4: 0.4mm Single Slit Diffraction

Using each of the distance between the center and the peaks (measured with vernier calipers), we were able to calculate a value for the wavelength.

Measurement	$\boldsymbol{D}$	$\boldsymbol{x_{+1}}$	$\boldsymbol{x_{-1}}$	$\boldsymbol{x_{+2}}$	$\boldsymbol{x_{-2}}$	$\boldsymbol{x_{+3}}$	$\boldsymbol{x_{-3}}$
1	56.70	0.340	0.410	0.746	0.770	1.040	1.070
2	56.60	0.310	0.412	0.744	0.760	1.044	1.065
3	56.65	0.330	0.402	0.752	0.782	1.043	1.080
4	56.69	0.326	0.420	0.756	0.720	1.041	1.059
$\boldsymbol{\mu}$	56.66	0.327	0.411	0.750	0.758	1.042	1.069
Random Error	0.02	0.005	0.003	0.002	0.012	0.001	0.004
Instrumental Error	± 0.05	± 0.002					

Table 5: Hair Diffraction (cm)

Using Babinet's principle we are able to calculate the thickness of a hair. The hair was estimated to be rectangular, making it have an equivalent diffraction pattern to a rectangular slit. Using the mean wavelength determined in the single slit diffractions, we were then able to calculate the thickness of a hair.

3 Calculations

Part A: Determination of a Laser's Wavelength

1. Intensity due to the interference of N sources

$$I(\delta) = I_0 \left[\frac{\sin(\frac{N\delta}{2})}{\sin(\frac{\delta}{2})} \right]^2 \quad \text{where} \quad \delta = \frac{2\pi}{\lambda} d \sin\theta$$

Here, d is the distance between slits, λ is the wavelength of light, and θ is the angle between the center of the incident beam of light and the point of interest on the screen. In our experiment, we were interested in the peaks of the interference pattern.

1.1 Determining $\boldsymbol{\theta}$

$$\tan\theta = \frac{x_m}{D} \Rightarrow \theta = \arctan\frac{x_m}{D} \quad m = 0, \pm 1 \pm 2, \ldots$$

Where D is the distance between the grating and the screen, and x_m represents the distance between the center and each of the principal maxima.

Error Propagation:

1.1.1 Partial Derivative of Eq. 1.1 w.r.t $\boldsymbol{D}$

$$\frac{\partial\theta}{\partial D} = \frac{-x_m}{D^2 + x_m^2}$$

1.1.2 Partial Derivative of Eq 1.1 w.r.t $\boldsymbol{x_m}$

$$\frac{\partial\theta}{\partial x_m} = \frac{D}{x_m^2 + D^2}$$

Using the above partial derivatives, we can calculate the total error associated with θ. Because D and x_m are dependant variables, we use the following equation to compute the total error.

1.1.3 Total Error Associated with $\boldsymbol{\theta}$

$$\delta\theta = \left|\frac{\partial\theta}{\partial D}\delta D\right| + \left|\frac{\partial\theta}{\partial x_m}\delta x_m\right|$$

2. Principal Maxima Locations

$$\sin\theta = \frac{m\lambda}{d} \quad m = 0, \pm 1, \pm 2, \ldots$$

Here m refers to each of the principal maxima. For example in Table 1, the x_{+1} column refers to the $m = 1$ maxima

Error Propagation:

2.1 Partial Derivative of Eq. 2 w.r.t *d*

$$\frac{\partial \lambda}{\partial d} = \frac{\sin\theta}{m}$$

2.2 Partial Derivative of Eq. 2 w.r.t *θ*

$$\frac{\partial f}{\partial \theta} = \frac{d}{m}\cos\theta$$

2.3 Partial Derivative of Eq. 2 w.r.t *m*

$$\frac{\partial f}{\partial m} = \frac{-d\sin\theta}{m^2}$$

Each of these partial derivatives are used in the final calculation of the total error associated with λ. These variables are independent, which allows us to use the following equation to calculate the total error.

2.4 Total Error Associated with the Wavelength (λ)

$$\delta\lambda = \sqrt{\left(\frac{\partial \lambda}{\partial d}\delta d\right)^2 + \left(\frac{\partial \lambda}{\partial \theta}\delta\theta\right)^2 + \left(\frac{\partial \lambda}{\partial m}\delta m\right)^2}$$

In our experiment, m represents the number of the principal maxima, so there is no error associated with it. The total error associated with the mean values were calculated (shown in the Results section). This provides our final error associated with λ.

Part B: Single Slit Diffraction

3. Intensity due to a Single Slit

$$I(\beta) = I_0\left(\frac{\sin\beta}{\beta}\right)^2 \quad \textit{where} \quad \beta = \frac{\pi}{\lambda}a\sin\theta$$

Here, a is the width of the slit and λ and θ are as given in Equation 1.

4. Minima of the Diffraction Pattern Locations

$$\sin\theta = \frac{p\lambda}{a} \quad p = 0, \pm 1, \pm 2, \ldots$$

This equation matches with Equation 2, where a is analogous to d and p is analogous to m. When determining the error in calculating the wavelengths for a single slit, the Standard Deviation of the Mean is calculated.

4.1 Error Calculation: Standard Deviation of the Mean (SDOM)

$$\sigma_{\bar{x}} = \frac{S_x}{\sqrt{N}}$$

Where S_x is the standard deviation of the data and N is the number of data points. For our single slit wavelength calculations, 12 values for the wavelength were calculated, meaning $N = 12$.

Determination of a Hair's Thickness

5. Slit Thickness

$$a = \frac{\lambda p}{\sin\theta}$$

Rearranging Equation 4 to solve for the slit width gives this equation, which allows us to calculate the thickness of the hair, given the value of λ. To determine the error of the hair's thickness, the SDOM was calculated, using Equation 4.1, where $N = 6$, since there were thickness values for the first, second, and third order peaks.

4 Results

	$\boldsymbol{x_{+1}}$	$\boldsymbol{x_{-1}}$
$\boldsymbol{\theta}$ **(rad)**	0.738	0.726
$\boldsymbol{\delta\theta}$ **(rad)**	0.006	0.005
$\boldsymbol{\lambda}$ **(cm)**	$6.727 * 10^{-5}$	$6.641 * 10^{-5}$
$\boldsymbol{\delta\lambda}$ **(cm)**	$4.0 * 10^{-7}$	$4.5 * 10^{-7}$
Experimental Wavelength: $\mathbf{6.685 * 10^{-5} \pm 4.3 * 10^{-7}}$ **cm**		

Table 6: Wavelength determination with grating

Here, $\delta\theta$ and $\delta\lambda$ are the error values for θ and λ respectively, determined via error propagation. The experimental wavelength error reported is half the range of the two calculated wavelengths. Since the experimental error is the average of the two propagated errors, we can assume for future calculations that the SDOM of the measurements will be sufficient to provide the uncertainty associated with the final calculated value.

	$\boldsymbol{x_{+1}}$	$\boldsymbol{x_{-1}}$	$\boldsymbol{x_{+2}}$	$\boldsymbol{x_{-2}}$
$\boldsymbol{\theta}$ **(rad)**	0.003	0.003	0.006	0.007
$\boldsymbol{\lambda}$ **(cm)**	$5.694 * 10^{-5}$	$6.383 * 10^{-5}$	$6.363 * 10^{-5}$	$6.839 * 10^{-5}$

Table 7: Wavelength Determination with 0.2 mm Slit Width

	$\boldsymbol{x_{+1}}$	$\boldsymbol{x_{-1}}$	$\boldsymbol{x_{+2}}$	$\boldsymbol{x_{-2}}$
$\boldsymbol{\theta}$ **(rad)**	0.002	0.002	0.004	0.004
$\boldsymbol{\lambda}$ **(cm)**	$4.552 * 10^{-5}$	$5.549 * 10^{-5}$	$5.560 * 10^{-5}$	$5.452 * 10^{-5}$

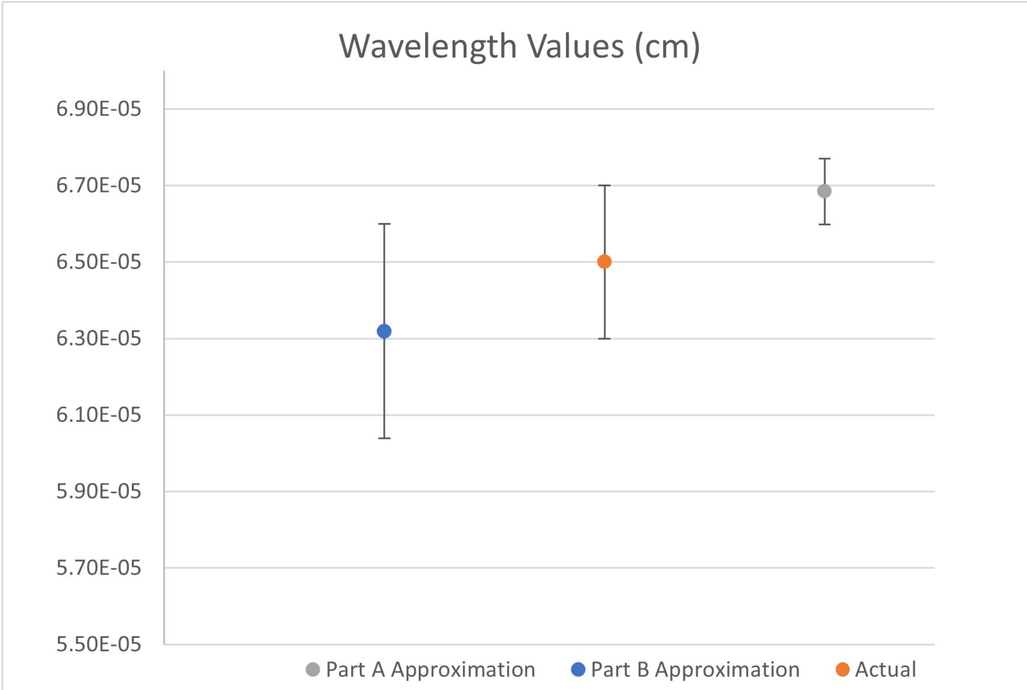
Table 8: Wavelength Determination with 0.3 mm Slit Width

	$\boldsymbol{x_{+1}}$	$\boldsymbol{x_{-1}}$	$\boldsymbol{x_{+2}}$	$\boldsymbol{x_{-2}}$
$\boldsymbol{\theta}$ **(rad)**	0.002	0.002	0.004	0.004
$\boldsymbol{\lambda}$ **(cm)**	$8.008 * 10^{-5}$	$6.640 * 10^{-5}$	$7.232 * 10^{-5}$	$7.563 * 10^{-5}$

Table 9: Wavelength Determination with 0.4 mm Slit Width

$$\boxed{\boldsymbol{\lambda = 6.320 * 10^{-5} \pm 2.8 * 10^{-6}}\ \mathbf{cm}}$$

Final Value of $\boldsymbol{\lambda}$



Plot 1: Mean Wavelength Values with Associated Uncertainty

Plot 1 gives the final values of the wavelength as determined in parts A and B alongside the accepted value. Although the wavelength of Part B has a larger uncertainty than Part A ($2.8 * 10^{-6}$ cm as opposed to $4.3 * 10^{-7}$ cm), both wavelength values lie within the range of accepted values for a red laser. The mean values of Part A and Part B differed by $3.652 * 10^{-6}$ cm. The accepted value lies almost exactly in between the estimates of Part A and B, with Part A being off by $1.85 * 10^{-6}$ cm, and Part B off by $1.80 * 10^{-6}$ cm. The mean of Part A and Part B is equivalent to the average accepted value for the wavelength of a red laser beam of $6.50 * 10^{-5}$ cm. For both part A and B, error was introduced when measuring the distance between the central maximum and outer maxima. The maxima were marked using a mechanical pencil with a lead thickness of 0.38 mm. While this thickness is relatively thin compared to most other pencils, this still made it difficult to mark down the maxima. Part B of the experiment used the minima, rather than the maxima, which required marking the edges of the maxima and finding the center. As the single slit became wider, this became more challenging as the maxima grew closer to each other. The midpoint of the maxima was estimated and then marked using the same mechanical pencil. The distance between the central maxima and outer maxima/minima was measured using a pair of vernier calipers. As the tick marks for the maxima/minima were approximations of the centers of each peak, the measurements given by the calipers had an additional uncertainty of 0.1 cm. This uncertainty, alongside difficulty measuring pencil markings contributed to overall error in the wavelength values.

	x_{+1}	x_{-1}	x_{+2}	x_{-2}	x_{+3}	x_{-3}
θ (rad)	0.006	0.007	0.013	0.013	0.018	0.019
a (cm)	0.011	0.009	0.010	0.009	0.007	0.007
a	0.009 ± 0.0006 **cm**					

Table 10: Hair Thickness Determination

Plot 2: Hair Thickness With Associated Uncertainty

Plot 2 shows the final value of the thickness of a hair as determined via Babinet's principle. Due to how fine hair is, alongside the wide variety of hair types, it has been difficult for researchers to determine an exact value for the thickness of a hair. Thin hair can be at a minimum thickness of 15μm, while thicker hair can have a diameter of 181μm. The hair we measured in class had a diameter of 90μm, putting it towards the middle of the expected range. Our value was offset from the average accepted value by about 12μm, but still remained within the accepted range of values. This estimation has similar sources of uncertainty to the wavelength determinations of Part A and B, as the same process was followed in order to collect data on the maxima/minima produced by the slit.

5 Conclusion

The wavelength of a red laser beam was calculated using an N-slit interference pattern to be $6.685*10^{-5} \pm 4.3*10^{-7}$ cm. Using a single slit interference pattern, with various slit widths, the wavelength was determined to be $6.320*10^{-5} \pm 2.8*10^{-6}$ cm. Both of these wavelength values lie within the expected wavelength of red light, which varies between 630 nm and 670 nm. Based on the calculated wavelength of the single slit diffraction, the thickness of a human hair was determined using Babinet's principle to be 0.009 ± 0.0006 cm. This thickness lies within the accepted thickness of a hair, which varies between 15μm and 181μm based on the type of hair used. Although the experimental values of wavelength and hair thickness lie within their accepted ranges, there were many sources of error within the experiment. Further experimental measurements can provide greater accuracy for the wavelength and thickness calculations. Once the sources of error are reduced, a more accurate value can be calculated via both the N-slit and single slit diffraction.

6 Questions

1. Explain why $(\sin\theta = \frac{m\lambda}{d}, m \in \mathbb{Z})$ is so. You will have to take the appropriate limits for Equation 1 $\left(I(\delta) = I_0 \left[\frac{\sin\frac{N\delta}{2}}{\sin\frac{\delta}{2}}\right]^2, \delta = \frac{2\pi}{\lambda} d\sin\theta\right)$

This equation comes from the limit of the maxima of the intensity for an N-slit diffraction pattern, given in the above equation.

$$\lim_{\delta\to\infty} \frac{\sin(N\delta)}{\sin\delta} = \frac{0}{0}$$

Using L'Hopital's Rule

$$\lim_{\delta\to\infty} \frac{N\cos(N\delta)}{\cos\delta} = \frac{-N}{-1} = N$$

$$I(\delta)\text{at maxima } (\pi, 2\pi, \dots) = I_0 N^2$$

$$\delta = \pi, 2\pi, \dots, m\pi, m \in \mathbb{Z}$$

$$\sin\theta = \frac{\delta\lambda}{2\pi d} = \frac{m\pi\lambda}{2\pi d} = \frac{m\lambda}{d} \quad m \in \mathbb{Z} \quad \text{as desired.}$$

2. Explain how we arrive at $\left(\sin\theta = \frac{p\lambda}{a}, p = \pm1, \pm2, \dots\right)$ from Equation 3 $\left(I(\beta) = I_0 \left(\frac{\sin\beta}{\beta}\right)^2\right)$

This equation comes from the minima of the intensity for a single slit diffraction pattern, which occurs when $\frac{\pi}{\lambda} a\sin\theta = \pi, 2\pi, \dots, m\pi, \quad m \in \mathbb{Z}$.

$$\frac{\pi}{\lambda} a\sin\theta = m\pi$$

$$\sin\theta = \frac{m\pi\lambda}{\pi a} = \frac{m\lambda}{a} \quad \text{as desired.}$$