PH-291 Physics Lab
Professor Corn-Agostini
Fall 2022

Lab # 1: Statistics of Measurement

Your Name: Perla Berkovitz

Your Lab Section: PH-291-E

Your Lab Instructor: Professor Corn-Agostini

Your Lab Partner's Name: Sharon Sitt

Read and sign Academic Integrity Statement:

I hereby attest that I have not given or received any unauthorized assistance on this assignment.

_____

Sign here

Grading Rubric

SECTION	POINTS	GRADE	COMMENTS..................................
Purpose	1		
Data	3		
Calculations	3		
Results	3		
Conclusion	1		
Answers	4		
Total	15		

Contents

1 Purpose

In this lab, various measurement devices are used to find the volume of a cylindrical disc and the density of an aluminum plate. A sample set is taken for each measurement to provide the most precise result given the uncertainty of each measurement device. While some devices might be more precise than others, all physical measurements have an innate degree of physical uncertainty. Calculating the uncertainty of the results allows the experimental data to have meaning.

2   Data

Right Reading (mm)	Left Reading (mm)	Length (mm)
39.0	0	39.0
39.5	0	39.5
39.0	0	39.0
39.5	0	39.5
39.0	0	39.0
39.5	0	39.5
Diameter (μ) (mm)	**Random Error (mm)**	**Instrumental Error (mm)**
39.25	0.112	± 0.5

Table 1: Diameter of the Cylindrical Disk (Linear Scale)

Right Reading (mm)	Left Reading (mm)	Length (mm)
3.5	0	3.5
3.5	0	3.5
3.5	0	3.5
3.0	0	3.0
3.5	0	3.5
3.5	0	3.5
Thickness (μ) (mm)	**Random Error (mm)**	**Instrumental Error (mm)**
3.416	0.083	± 0.5

Table 2: Thickness of the Cylindrical Disk (Linear Scale)

Length (mm)	Corrected Length (mm)
39.6	39.6
39.64	39.64
39.76	39.76
39.6	39.6
39.74	39.74
39.64	39.64
Diameter (μ) (mm)	**Zero Error (mm)**
39.66	0
Instrumental Error (mm)	**Random Error (mm)**
± 0.02	0.028

Table 3: Diameter of the Cylindrical Disk (Vernier Caliper)

Length (mm)	Corrected Length (mm)
3.2	3.2
3.16	3.16
3.16	3.16
3.2	3.2
3.24	3.24
3.3	3.3
Thickness (μ) (mm)	**Zero Error (mm)**
3.21	0
Instrumental Error (mm)	**Random Error (mm)**
± 0.02	0.022

Table 4: Thickness of the Cylindrical Disk (Vernier Caliper)

Length (mm)	Corrected Length (mm)
41.5	41.5
41.5	41.5
41.64	41.64
41.68	41.68
41.66	41.66
41.56	41.56
Length (μ) (mm)	**Zero Error (mm)**
41.59	0
Instrumental Error (mm)	**Random Error (mm)**
±0.02	0.033

Table 5: Length of the Rectangular Object (Vernier Caliper)

Length (mm)	Corrected Length (mm)
38.26	38.26
38.24	38.24
38.26	38.26
38.24	38.24
38.24	38.24
38.24	38.24
Width (μ) (mm)	**Zero Error (mm)**
38.25	0
Instrumental Error (mm)	**Random Error (mm)**
±0.02	0.004

Table 6: Width of the Rectangular Object (Vernier Caliper)

Length (mm)	Corrected Length (mm)
3.11	3.13
3.2	3.22
3.17	3.19
3.105	3.125
3.135	3.155
3.135	3.155
Thickness (μ) (mm)	**Zero Error (mm)**
3.16	-0.02
Instrumental Error (mm)	**Random Error (mm)**
±0.005	0.015

Table 7: Thickness of the Rectangular Object (Micrometer)

Length (mm)	Corrected Length (mm)
3.15	3.17
3.16	3.18
3.23	3.25
3.16	3.18
3.185	3.205
3.15	3.17
Thickness (μ) (mm)	**Zero Error (mm)**
3.19	-0.02
Instrumental Error (mm)	**Random Error (mm)**
±0.005	0.013

Table 8: Thickness of the Cylindrical Disk (Micrometer)

Mass (g)	Corrected Mass (g)
10.5	10.5
10.5	10.5
10.6	10.6
10.5	10.5
10.6	10.6
10.5	10.5
Mass (μ) (g)	**Zero Error (g)**
10.53	0
Instrumental Error (g)	**Random Error (g)**
0.05	0.021

Table 9: Mass of the Cylindrical Disk (Digital Balance)

Mass (g)	Corrected Mass (g)
13	13
13	13
13	13
13	13
12.9	12.9
13	13
Mass (μ) (g)	**Zero Error (g)**
12.98	0
Instrumental Error (g)	**Random Error (g)**
0.05	0.017

Table 10: Mass of the Rectangular Object (Digital Balance)

Corrected Length (mm)
6.54
7.285
4.87
5.74
6.945
6.23
6.165
6.12
6.09
6.175
5.93
6.06
7.37
6.915
6.015
7.35
5.85
6.05
5.21
6.085
Width (μ) (mm)
6.25

Table 11: Width of Pieces in Bottle #10

Corrected Length (mm)
5.905
6.75
5.83
6.1
7.14
6.04
5.15
5.615
6.195
5.91
4.75
5.66
5.825
4.55
6.305
6.215
6.205
6.955
6.21
6.25
Width (μ) (mm)
5.98

Table 12: Width of Pieces in Bottle #17

3   Calculations

1. **Volume of a cylindrical disc:**

$$V_c = \pi \times \left(\frac{d}{2}\right)^2 \times h$$

   where r is the radius of the disc and h is the thickness of the disc.

   Sample Calculation: $V_c = \pi \times \left(\dfrac{3.9 \text{ cm}}{2}\right)^2 \times 0.35 \text{ cm} = \boxed{4.18 \text{ cm}^3}$

2. **Volume of a rectangular object:**

$$V_r = l \times w \times h$$

   where l and w are the length and width respectively, and h is the thickness of the object.

   Sample Calculation: $V_r = 4.15 \text{ cm} \times 3.83 \text{ cm} \times 0.31 \text{ cm} = \boxed{4.97 \text{ cm}^3}$

3. **Computing density:**

$$\rho = \frac{m}{V}$$

   where m and V are the mass and volume of each object.

   Sample Calculation: $\rho = \dfrac{10.5 \text{ g}}{4.18 \text{ cm}^3} = \boxed{2.51 \text{ g/cm}^3}$

4. **Sample Mean:**

$$\bar{x} = \frac{\sum\limits_{i=1}^{n} x_i}{n}$$

   where x_i represents each value and n is the total number of entries

   Sample Calculation: $\bar{x} = \dfrac{\sum\limits_{i=1}^{20} x_i}{20} = \boxed{6.25 \text{ mm}}$

5. **Sample Standard Deviation:**

$$S_x^2 = \frac{1}{N_x - 1} \sum_{i=1}^{N_x} (x_i - \bar{x})^2$$

   where S_x^2 is the sample variance and S_x is the sample standard deviation for each sample set. N_x is the number of samples in the set.

   Sample Calculation: $S_x = \sqrt{\dfrac{\sum\limits_{i=1}^{20} (x_i - \bar{x})^2}{19}} = \boxed{0.66 \text{ mm}}$

6. **Standard Deviation of the Mean (SDOM):**

$$\sigma_x = \frac{S_x}{\sqrt{N_x}}$$

This also provides us with the random error of each sample set.

Sample Calculation: $\sigma_x = \dfrac{0.66 \text{ mm}}{\sqrt{20}} = \boxed{0.15 \text{ mm}}$

7. **Test Statistic Value:**

$$t = \frac{|\bar{x} - \bar{y}|}{\sqrt{\sigma_{\bar{x}}^2 + \sigma_{\bar{y}}^2}}$$

Sample Calculation: $t = \dfrac{|6.25 - 5.98|}{\sqrt{0.15^2 + 0.14^2}} = \boxed{1.32}$

8. **Percent Improvement of Accuracy:**

$$\%_{\text{Improvement}} = 100 - \frac{IE_{\text{Vernier Caliper}}}{IE_{\text{Linear Scale}}} \times 100 = 100 - \frac{0.02}{0.5} \times 100 = \boxed{96\%}$$

where IE refers to the instrumental error of each measurement instrument used.

9. **Error Propagation (Independent Variables):**

$$\delta f = \sqrt{\left(\frac{\partial f}{\partial x}\delta x\right)^2 + \left(\frac{\partial f}{\partial y}\delta y\right)^2 + \cdots + \left(\frac{\partial f}{\partial z}\delta z\right)^2}$$

where the variables x, y, z are independent and used to determine the value of f. δf provides the error of the function $f(x, y, \ldots, z)$.

Sample Calculation for V_r: $\delta V_r = \sqrt{(0.048\%)^2 + (0.052\%)^2 + (0.16\%)^2)} = \boxed{0.175\%}$

4 Results

Diameter (cm)	Thickness (cm)	Mass (g)	Volume (cm³)	Density (g/cm³)
3.9	0.35	4.18	10.5	2.51
3.95	0.35	4.29	10.5	2.45
3.9	0.35	4.18	10.6	2.54
3.95	0.3	3.68	10.5	2.86
3.9	0.35	4.18	10.6	2.54
3.95	0.35	4.29	10.5	2.45

Table 13: Cylindrical Disk Results (Linear Scale)

Diameter (cm)	Thickness (cm)	Mass (g)	Volume (cm³)	Density (g/cm³)
3.96	0.32	3.94	10.5	2.66
3.964	0.316	3.90	10.5	2.69
3.976	0.316	3.92	10.6	2.70
3.96	0.32	3.94	10.5	2.66
3.974	0.324	4.02	10.6	2.64
3.964	0.33	4.07	10.5	2.58

Table 14: Cylindrical Disk Results (Vernier Caliper)

Diameter (cm)	Thickness (cm)	Mass (g)	Volume (cm³)	Density (g/cm³)
3.96	0.317	3.90	10.5	2.69
3.964	0.318	3.92	10.5	2.68
3.976	0.325	4.04	10.6	2.63
3.96	0.318	3.92	10.5	2.68
3.974	0.3205	3.98	10.6	2.67
3.964	0.317	3.91	10.5	2.68

Table 15: Cylindrical Disk Results (Micrometer)

Diameter (cm)	Thickness (cm)	Volume (cm³)	Mass (g)	Density (g/cm³)
1.28%	14.29%	14.40%	0.48%	14.41%
1.27%	14.29%	14.40%	0.48%	14.41%
1.28%	14.29%	14.40%	0.47%	14.41%
1.27%	16.67%	16.76%	0.48%	16.77%
1.28%	14.29%	14.40%	0.47%	14.41%
1.27%	14.29%	14.40%	0.48%	14.41%

Table 16: Cylindrical Disk Error Propagation (Linear Scale)

Diameter (cm)	Thickness (cm)	Volume (cm³)	Mass (g)	Density (g/cm³)
0.05%	0.63%	0.63%	0.48%	0.79%
0.05%	0.63%	0.64%	0.48%	0.80%
0.05%	0.63%	0.64%	0.47%	0.79%
0.05%	0.63%	0.63%	0.48%	0.79%
0.05%	0.62%	0.62%	0.47%	0.78%
0.05%	0.61%	0.61%	0.48%	0.77%

Table 17: Cylindrical Disk Error Propagation (Vernier Caliper)

Diameter (cm)	Thickness (cm)	Volume (cm³)	Mass (g)	Density (g/cm³)
0.05%	0.16%	0.17%	0.48%	0.51%
0.05%	0.16%	0.17%	0.48%	0.51%
0.05%	0.15%	0.17%	0.47%	0.50%
0.05%	0.16%	0.17%	0.48%	0.51%
0.05%	0.16%	0.17%	0.47%	0.50%
0.05%	0.16%	0.17%	0.48%	0.51%

Table 18: Cylindrical Disk Error Propagation (Micrometer)

Length (cm)	Width (cm)	Thickness (cm)	Mass (g)	Volume (cm³)	Density (g/cm³)
4.15	3.826	0.313	13.0	4.97	2.62
4.15	3.824	0.322	13.0	5.11	2.54
4.164	3.826	0.319	13.0	5.08	2.56
4.168	3.824	0.3125	13.0	4.98	2.61
4.166	3.824	0.3155	12.9	5.03	2.57
4.156	3.824	0.3155	13.0	5.01	2.59

Table 19: Rectangular Plate Results
(Vernier Caliper (Length and Width) and Micrometer (Thickness))

Length (cm)	Width (cm)	Thickness (cm)	Volume (cm³)	Mass (g)	Density (g/cm³)
0.048%	0.052%	0.160%	0.175%	0.385%	0.422%
0.048%	0.052%	0.155%	0.171%	0.385%	0.421%
0.048%	0.052%	0.157%	0.172%	0.385%	0.421%
0.048%	0.052%	0.160%	0.175%	0.385%	0.423%
0.048%	0.052%	0.158%	0.174%	0.388%	0.425%
0.048%	0.052%	0.158%	0.174%	0.385%	0.422%

Table 20: Rectangular Plate Error Propagation
(Vernier Caliper (Length and Width) and Micrometer (Thickness))

Bottle #	$\bar{x}$ (mm)	S_x	σ_x
10	6.24	0.66	0.15
17	5.98	0.64	0.14
Test Statistic: 1.32			

Table 21: Statistics for Bottles 10 and 17

Bottle #10 Histogram

Bottle #17 Histogram
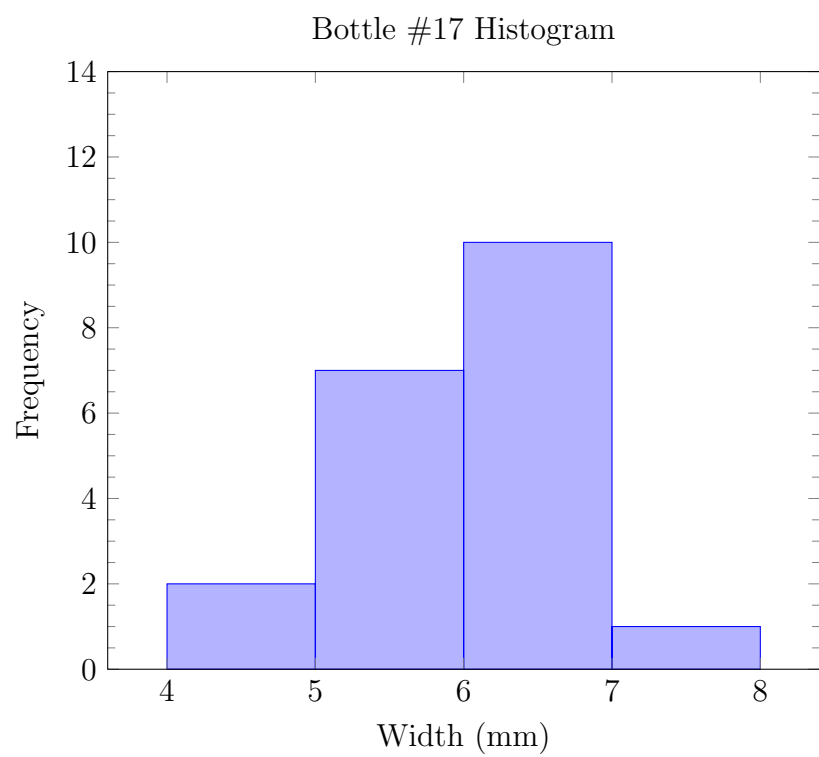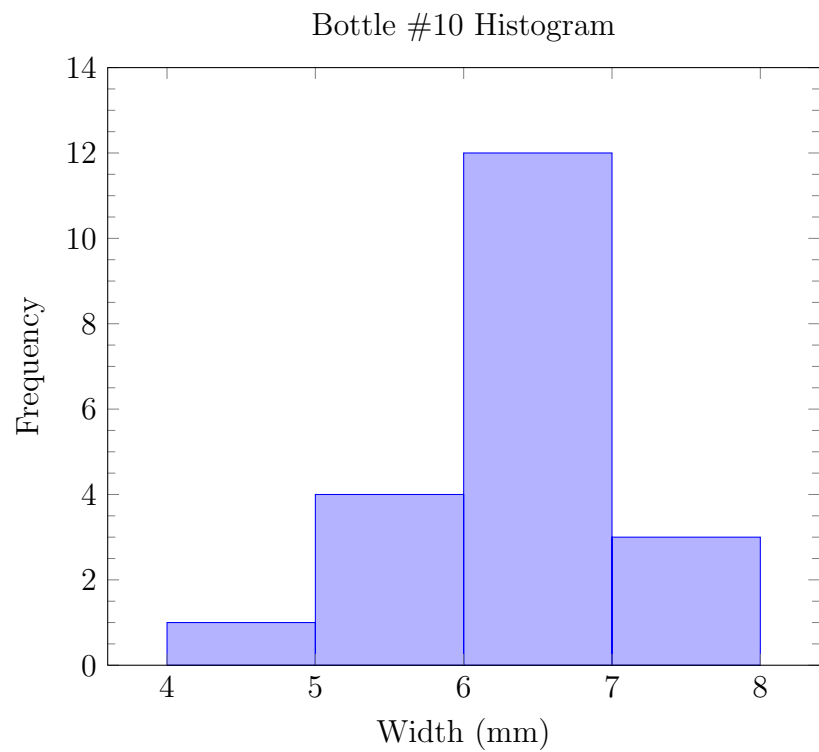
5 Conclusion

In the first part of the experiment, the volume of a cylindrical disc was calculated using measurements taken by a linear scale and then by a vernier caliper. The vernier caliper was a more accurate measurement device than the linear scale due to its lower instrumental error. As expected, the overall error in calculating the volume, which included both the instrumental and random error, was reduced relative to the change in instrumental error. The calculated volume using measurements from the linear scale had an average error of 14.8%, while the measurements from the vernier calipers provided an error of 0.63%, which shows a 95.8% decrease in error.

Aluminum alloy 6061 is known to have a density of 2.7 g/cm^3, which corresponds to the calculated densities of both the rectangular and cylindrical plates. The density value calculated using measurements from the micrometer was 2.67 ± 0.01 g/cm^3, which is only 1.1% lower than the known value. The slight difference in calculated densities can be attributed to instrumental error, random error, and imperfections in the plates. The vernier calipers and the micrometer are made of steel, which is significantly harder than aluminum. If the measurement devices were clamped too tightly around the plate, they may eat into the material and provide inaccurate results.

The final part of the experiment compares the dimensions of acrylic pieces in two separate bottles. The null hypothesis states that the averages lengths of pieces in each bottle will be equal. The histograms for each bottle don't appear to follow a particular distribution. In bottle 10, the average length was 6.25 ± 0.005 mm. In bottle 17, the average length was 5.98 ± 0.005 mm. These numbers do not initially appear to be equal, however by using the t-test, a p-value of 40.9% was obtained which allows us to fail to reject the null-hypothesis.

6 Discussion Questions

1. Compare the two error values. Did the better instrument produce a corresponding reduction in error value? (For example, if the better instrument had a 20% less instrumental error, did your final result show a 20% reduction in error?) If not, why?

   The better instrument, being the one with a greater accuracy, produced a correspondingly lower error. For example, when comparing the calculated volumes using the linear scale and the vernier calipers, the error decreased by about 96%, which is the same as the decrease in instrumental error.

2. Do the density values agree with one another and the standard value? (Remember, you will have to include the uncertainties when you answer this question).

   The standard value of the density of Aluminum alloy 6061 is 2.7 g/cm^3. The density calculated using the linear scale measurement of the cylindrical disc was 2.56 ± 0.4 g/cm^3. The density calculated from vernier caliper measurements of the cylindrical disc was 2.66 ± 0.02 g/cm^3. The measurements of the disc with the micrometer provided a density of 2.67± 0.01 g/cm^3. Finally, the measurements of the rectangular plate provided a density of 2.58 ± 0.01 g/cm^3. These experimental values agree with one another within the error range, but are slightly lower than the standard value.

3. Comment if your data sets follow a normal distribution (the table in the margin shows what a normal distribution should be like).

   The data from bottles 10 and 17 does not follow a normal distribution. Both histograms do not seem to follow any specific distribution pattern, which corresponds to variety of lengths in each bottle.

4. Using the t value decide if your data supports the null hypotheses at the 5% significance level.

   The expected t value at 5% significance level with 19 degrees of freedom is 1.729. The calculated value was 1.32. The p-value, which is the difference between the two t-values, is 0.409 (40.9%) which means that the p-value supports the null hypothesis.