PH-291 Physics Lab
Professor Corn-Agostini
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Lab # 4: Optical Instruments

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Read and sign Academic Integrity Statement:

I hereby attest that I have not given or received any unauthorized assistance on this assignment.

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Grading Rubric

CATEGORY	**POINTS**	**GRADE**
Purpose	2	
Data	6	
Theory & Calculations (includes Q1 and Q2)	6	
Results & Analysis	4	
Conclusion	2	
Total	20	

Contents

1 Purpose

In this lab, Bessel's Method will be used to determine the focal lengths of two converging lenses. The focal length of each length is approximated using the thin lens equation (Eq. 1). Here, the lenses are convex, meaning that f is positive. This approximate focal length is used to determine the distance between the source and the screen (D). D must be sufficiently large so that the light rays will converge and form a real image. If D is too small, a virtual image will be formed, which only exists within the brain. A clear image is formed by each lens in two positions, the difference of which (d) is used in Bessel's Method to calculate the focal lengths. The angular magnification of the two lenses will then be calculated and used to construct a simple telescope, which is used to test the expected magnification value.

2 Data

Approximate Focal Length:	5.50 cm
Instrumental Error:	0.05 cm
Approximate Focal Length:	5.50 ± 0.05 cm

Table 1: Approximate Focal Length of Lens F1

Approximate Focal Length:	10.00 cm
Instrumental Error:	0.05 cm
Approximate Focal Length:	10.00 ± 0.05 cm

Table 2: Approximate Focal Length of Lens C

These approximate focal lengths were measured using a meter stick. The distance from the source to the lens was taken to be infinite, which allowed us to use the thin lens equation to approximate our focal lengths. Using the approximate focal lengths of each lens, the optical bench was then set up. The distance between the source and the screen (D) must be 4 times greater than the focal length.

Measurement	Position 1 (cm)	Position 2 (cm)	D (cm)	d (cm)
1	14.00	7.85	21.70	6.15
2	13.80	7.70	21.70	6.10
3	14.00	8.10	21.70	5.90
Instrumental Error: 0.05 cm				
Random Error (d): 0.13 cm				

Table 3: Optical Bench Data for Lens F1

Measurement	Position 1 (cm)	Position 2 (cm)	D (cm)	d (cm)
1	27.10	17.30	41.70	9.80
2	26.80	16.80	41.70	10.00
3	26.40	16.30	41.70	10.10
Instrumental Error: 0.05 cm				
Random Error (d): 0.15 cm				

Table 4: Optical Bench Data for Lens C

Line 1 Length (cm)	**Line 2 Length (cm)**	**Angular Magnification**
20.00	9.50	2.11
Instrumental Error: 0.05 cm		
Experimental Angular Magnification 2.11±0.05		

Table 5: Experimental Angular Magnification

The calculated angular magnification value is compared to the experimental magnification factor to confirm that the telescope functions as expected.

3 Calculations

Part A: Bessel's Method for Determining Focal Length

1. Thin Lens Equation

$$\frac{1}{p} + \frac{1}{i} = \frac{1}{f}$$

Where p is the distance between the source and the lens, i is the distance between the image and the lens, and f is the focal length of the lens.

2. Focal Length of Lens ($\boldsymbol{f}$)

$$f = \frac{D^2 - d^2}{4D}$$

Initially, the focal length of each lens is approximated, which allows us to find our value for D, the distance between the source and screen. D must be greater than $4f$, where f is our approximate focal length. Each lens has two positions where a sharp image is displayed, and the difference between these two positions provides our d value. From there, an accurate focal length can be determined for each lens using this equation. The error of the mean values were propagated through this calculation using the following equations.

Error Propagation:

2.1 Partial Derivative of Eq. 2 w.r.t $\boldsymbol{d}$

$$\frac{\partial f}{\partial d} = \frac{-d}{2D}$$

2.2 Partial Derivative of Eq. 2 w.r.t $\boldsymbol{D}$

$$\frac{\partial f}{\partial D} = \frac{D^2 + d^2}{4D^2}$$

Each of these partial derivatives are used in the final calculation of the total error associated with f. Because d and D are dependent variables we use the following equation to find the total error.

2.3 Total Error Associated with the Focal Length ($\boldsymbol{f}$)

$$\delta f = \left|\frac{\partial f}{\partial d}\delta d\right| + \left|\frac{\partial f}{\partial D}\delta D\right|$$

The error associated with the mean values were calculated (shown in Tables 6 and 7). This provides our final error associated with f.

3. Angular Magnification ($\boldsymbol{m_\theta}$)

$$m_\theta = -\frac{f_{obj}}{f_{eye}}$$

We proceed to calculate the angular magnification (m_θ), which relies on the focal lengths of both lenses. f_{obj} is the focal length of the objective lens, which is the lens with a longer focal length. The lens with the short focal length acts as the eyepiece, f_{eye}. The total error of m_θ is found using the following equations.

Error Propagation:

3.1 Partial Derivative of Eq. 3 w.r.t $\boldsymbol{f_{obj}}$

$$\frac{\partial m_\theta}{\partial f_{obj}} = \frac{-1}{f_{eye}}$$

3.2 Partial Derivative of Eq. 3 w.r.t $\boldsymbol{f_{eye}}$

$$\frac{\partial m_\theta}{\partial f_{eye}} = \frac{f_{obj}}{f_{eye}^2}$$

Each of these partial derivatives are used in calculating the total error in the following equation. Both f_{eye} and f_{obj} are independent, which allows us to use the following equation.
3.3 Total Error Associated with the Angular Magnification ($\boldsymbol{m_\theta}$)

$$\delta m_\theta = \sqrt{\left(\frac{\partial m_\theta}{\partial f_{obj}}\delta f_{obj}\right)^2 + \left(\frac{\partial m_\theta}{\partial f_{eye}}\delta f_{eye}\right)^2}$$

As in Equation 1.3, this equation gives us the total error associated with (m_θ), shown in the results section (Table 8).

4 Results

f	$\frac{\partial f}{\partial d}$	$\frac{\partial f}{\partial D}$	Error of f
5.00	-0.14	0.27	0.03
$\boldsymbol{f : 5.00 \pm 0.03}$			

Table 6: f Final Value for Lens F1 by Bessel's Method

f	$\frac{\partial f}{\partial d}$	$\frac{\partial f}{\partial D}$	Error of f
9.83	-0.12	0.26	0.03
$\boldsymbol{f : 9.83 \pm 0.03}$			

Table 7: f Final Value for Lens C by Bessel's Method

m_θ	$\frac{\partial m_\theta}{\partial f_{obj}}$	$\frac{\partial m_\theta}{\partial f_{eye}}$	Error of m_θ
-1.965	-0.200	0.393	0.014
$\boldsymbol{m_\theta : -1.965 \pm 0.014}$			

Table 8: m_θ Final Value

The approximate focal length of lens F1 was 5.50 ± 0.05 cm, which does not lie within the same uncertainty range as the calculated focal length for lens F1 of 5.00 ± 0.13 cm. This could be attributed to the approximation process, which takes the distance between the source and the lens to be infinite, while it instead was a large finite value (discussed further in questions section). Additionally, the approximation required the light source to be directly above the lens and the paper to lie flat on the floor. The measurements for approximate focal lengths may vary if the light or the paper is not properly positioned. Another significant source of error arose due to the need for human perception. Depending on the experimenter's eyesight, they may see the different levels of focus compared to their partner. This would produce variation in the focal length measurements and calculations depending on which experimenter viewed the image to determine the point at which the image would be sharpest. In setting up the optical bench, the source, lens, and screen all needed to be aligned vertically. This also provides a source of error, in that if they are not properly aligned, the calculated focal lengths would be different.

The telescope construction involved placing the lenses into clips onto a meter stick. These clips fit snugly on the meter stick, but the lenses required tape to hold them in place. Additionally, the telescope should have been held straight to ensure vertical alignment of the lenses, but due to the length of the meter stick, it was somewhat challenging to keep everything in line. The lines were drawn based on the calculated magnification but were not exact due to difficulty getting perfect lines with the chalk. This gave an experimental magnification of 2.11 ± 0.05, as opposed to the theoretical magnification of 1.965 ± 0.014.

The negative sign reported in Table 8 reflects the inversion of the final image seen by the observer. The experimental and theoretical magnification values do not lie within the same error range, which can be attributed to the sources of error discussed above. If the lines had been redrawn to collect additional measurements, the experimental and theoretical values would be more similar.

5 Conclusion

Using Bessel's method, a focal length for Lens F1 was calculated to be 5.00 ± 0.03 cm, and 9.83 ± 0.03 cm for Lens C. These calculated focal lengths provided a magnification factor of 1.965 ± 0.014. Based on the calculated magnification factor and focal lengths, a simple telescope was constructed, which provided an experimental magnification factor of 2.11 ± 0.05. Although these magnification factors do not lie in the same error range, further experimental measurements could provide more similar experimental and theoretical magnification factors. The difference between the experimental and theoretical values can b attributed to human error, as described above. If error could be reduced, Bessel's Method will provide for more accurate focal length and magnification factor values.

6 Questions

1. Derive the equation $f = \frac{D^2 - d^2}{4D}$ and explain why we need the subsidiary condition $D > 4f$.

Starting with the thin lens equation (Eq. 1),

$$\frac{1}{p} + \frac{1}{i} = \frac{1}{f}$$

Since i is the distance between the image and the lens, we can rewrite the equation,

$$i = D - p$$

$$\frac{1}{p} + \frac{1}{D - p} = \frac{1}{f}$$

We then rewrite as the following equation

$$Df = pD - p^2$$

which can be solved via the quadratic equation

$$p = \frac{D \pm \sqrt{D^2 - 4Df}}{2}$$

Note here that if $D > 4f$ the discriminant is positive, meaning that for all other cases an imaginary answer will be given for p. We can denote the two positions of p by d, which is the difference between the two positions

$$d = \frac{D + \sqrt{D^2 - 4Df}}{2} - \frac{D - \sqrt{D^2 - 4Df}}{2} = \sqrt{D^2 - 4fD}$$

Using this value of d, we can solve for f

$$d^2 = D^2 - 4fD$$

$$D^2 - d^2 = 4fD$$

$$\boxed{f = \frac{D^2 - d^2}{4D}}$$

2. To get the approximate focal length of the lens we approximated the distance from the lens to the source to be infinite when it was actually finite (but large). Use the thin lens equation to show that this will slightly overestimate the focal length of the lens.

Starting with the thin lens equation (Eq. 1),

$$\frac{1}{p} + \frac{1}{i} = \frac{1}{f}$$

Since we take $p \to \infty$, $\frac{1}{p} \to 0$, which provides us with the following relationship

$$0 + \frac{1}{i} = \frac{1}{f} \Rightarrow f = i$$

Substituting back into the thin lens equation gives

$$\frac{1}{p} + \frac{1}{f} = \frac{1}{f}$$

which implies that a finite p value will make for a slight overestimation of the focal length.