1 Waves

Maxwell Equations:

$$\nabla \cdot \vec{E} = 0 \tag{1}$$

$$\nabla \cdot \vec{B} = 0 \tag{2}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \tag{3}$$

$$\nabla \times \vec{B} = \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t} \tag{4}$$

Plane waves propagate in one direction and have uniform $\vec{E}$ fields perpendicular to the direction of propagation.
1D Scalar Wave equation:

$$\frac{\partial^2 E_x}{\partial t^2} = \frac{1}{\mu_0 \epsilon_0} \frac{\partial^2 E_x}{\partial z^2}$$

3D wave equation:

$$\nabla^2 \psi = \frac{1}{v^2} \frac{\partial^2 \psi}{\partial t^2}$$

Wave vector:

$$\vec{k} = k_x \hat{x} + k_y \hat{y} + k_z \hat{z}, \quad \left|\vec{k}\right| = \frac{2\pi}{\lambda}$$

$\vec{k}$ points in the direction of propagation.

$$v = \frac{\omega}{k} = c$$

$$\frac{\partial}{\partial x} \rightarrow ik$$

$$\frac{\partial^2}{\partial x^2} \rightarrow = kx^2$$

$$\nabla^2 \rightarrow -\left(k - x^2 + k_y^2 + k_z^2\right)$$

$$\nabla \cdot \rightarrow i\vec{k} \cdot$$

$$\nabla \times \rightarrow i\vec{k} \times$$

$$E_{x'} = E_{x'_0} \cos(\omega t + \phi_x)$$

$$E_{y'} = E_{y'_0} \cos(\omega t + \phi_y)$$

$\vec{E}$ field can be either linearly or elliptically polarized:

$$\phi_y - \phi_x = 0, \pi, 2\pi \ldots \quad \rightarrow \quad \text{linearly polarized}$$

Otherwise, we have elliptical polarization.

2 Radiation and Scattering

Poynting flux:

$$\vec{S} = \frac{1}{\mu_0}\vec{E} \times \vec{B}$$

Radiation Pressure [$\frac{F}{L^3}$]:

$$u = u_E + u_B = \frac{1}{2}\epsilon_0 \left|\vec{E}\right|^2 + \frac{1}{2\mu_0}\left|\vec{E}\right|^2 = \epsilon_0 \left|\vec{E}\right|^2$$

Flux of $\vec{u}$:

$$\vec{F} = u \cdot \vec{v} = u \cdot c\hat{k}$$

Intensity [$\frac{\text{Watts}}{m^2}$]:

$$I = \langle \vec{S} \rangle = u \cdot c$$

Radiation component of $\vec{E}_k$:

$$E_\theta = \frac{qa \sin\theta}{4\pi\epsilon_0 c^2 R}$$

Larmor Power [watts]:

$$P = \frac{q^2 a^2}{6\pi\epsilon_0 c^3} = \frac{q^4 E_0^2}{6\pi\epsilon_0 m^2 c^3}\cos^2(\omega t)$$

$$\langle P \rangle = \frac{q^4 E_0^2}{12\pi\epsilon_0 m^2 c^3}$$

Thomson cross section:

$$\sigma_{th} = \frac{8\pi}{3}\left(\frac{q^2}{4\pi\epsilon_0 mc^2}\right)^2$$

Mean free path [m]:

$$\lambda_{mfp} = l = \frac{1}{\sigma_{th} n_e}$$

Rayleigh cross section:

$$\sigma_{th}\left(\frac{\omega^2}{\omega_0^2 - \omega^2}\right)^2$$

Where $\omega_0^2 = k/m$ and k is the spring constant.
Index of refraction for a dilute gas:

$$n = 1 + \frac{Nq^2}{2\epsilon_0 m_e}\frac{1}{\omega_0^2 - \omega^2}$$

Dipole moment:

$$\vec{p} = q\vec{d}$$

Average moment over all molecules in a material element:

$$\langle \vec{p} \rangle = \frac{1}{N}\sum_i \vec{P}_i$$

Relative permitivity:

$$\frac{\epsilon}{\epsilon_0} = 1 + \chi_e$$

For optics, we generally let $\mu_0 = \mu$.
Bulk Magnetization:

$$\vec{M} = \chi_m \vec{H}$$

Relative permeability:

$$\frac{\mu}{\mu_0} = 1 + \chi_m$$

Maxwell Equations in Material:

$$\vec{D} = \epsilon \vec{E} \tag{5}$$

$$\nabla \cdot \vec{D} = 0 \tag{6}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \tag{7}$$

$$\vec{B} = \mu \vec{H} \tag{8}$$

$$\nabla \cdot \vec{B} = 0 \tag{9}$$

$$\nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t} \tag{10}$$

Poynting Flux in Material:

$$\vec{S} = \vec{E} \times \vec{H}$$

Radiation Pressure in Material:

$$u = \frac{1}{2}\vec{E} \cdot \vec{D} + \frac{1}{2}\vec{H} \cdot \vec{B}$$

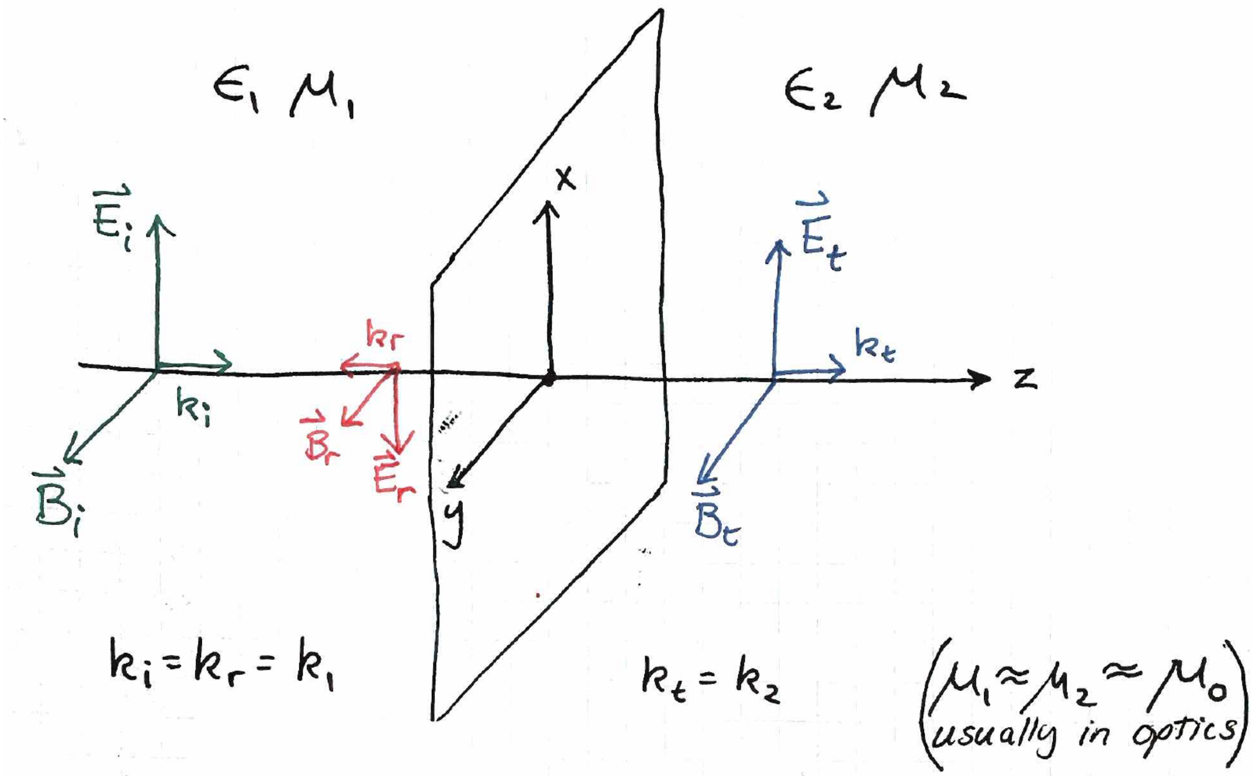
Index of Refraction Optics:

$$n = \frac{c}{v} = \sqrt{\frac{\mu\epsilon}{\mu_0\epsilon_0}} = \sqrt{\frac{\epsilon}{\epsilon_0}} = \sqrt{\chi_e}$$

Impedance:

$$Z = \sqrt{\frac{\mu}{\epsilon}}$$

3 Reflection and Refraction



$$E_i = E_{0i}\, e^{i(\omega t - k_1 z)} \qquad E_t = E_{0t}\, e^{i(\omega t - k_2 z)}$$

$$B_i = B_{0i}\, e^{i(\omega t - k_1 z)} \qquad B_t = B_{0t}\, e^{i(\omega t - k_2 z)}$$

$$E_r = -E_{0r}\, e^{i(\omega t + k_1 z)}$$

$$B_r = B_{0r}\, e^{i(\omega t + k_1 z)}$$

where $k_n = \frac{\omega}{v_n} = \omega\sqrt{\mu_n \epsilon_n}$

$$E_r = E_i \left[\frac{Z_1 - Z_2}{Z_1 + Z_2}\right]$$

and

$$E_t = E_i \left[\frac{2Z_2}{Z_1 + Z_2}\right]$$

With $Z_i = \mu_i v_i = \frac{\mu_i c}{n_i}$ gives:

$$E_r = E_i \left[\frac{n_2 - n_1}{n_1 + n_2} \right]$$

and

$$E_t = E_i \left[\frac{2n_1}{n_1 + n_2} \right]$$

Reflection coefficient:

$$R = \left| \frac{E_r}{E_i} \right|^2 = \left(\frac{n_2 - n_1}{n_2 + n_1} \right)^2$$

Transmission coefficient:

$$T = \frac{n_2 \cos\theta_2}{n_1 \cos\theta_1} \left(\frac{E_t}{E_i} \right)^2 = \frac{4 n_1 n_2}{(n_1 + n_2)^2}$$

Snell's Law:

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

Fresnel Formulae:

$$\frac{E_{ot}}{E_{oi}} = 1 - \frac{E_{or}}{E_{oi}} = 1 - \left(- \frac{\cos\theta_1 - \sqrt{\left(\frac{n_2}{n_1}\right)^2 - \sin^2\theta_1}}{\cos\theta_1 + \sqrt{\left(\frac{n_2}{n_1}\right)^2 - \sin^2\theta_1}} \right) \tag{11}$$

$$\frac{E_{ot}}{E_{oi}} = \frac{n_1}{n_2} \left[1 + \frac{E_{or}}{E_{oi}} \right] = 1 + \left(\frac{\left(\frac{n_2}{n_1}\right)^2 \cos\theta_1 - \sqrt{\left(\frac{n_2}{n_1}\right)^2 - \sin^2\theta_1}}{\left(\frac{n_2}{n_1}\right)^2 \cos\theta_1 + \sqrt{\left(\frac{n_2}{n_1}\right)^2 - \sin^2\theta_1}} \right) \tag{12}$$

Where (11) is when E is perpendicular to the incidence plane, and (12) is when E is parallel to the incidence plane.
Brewster's Angle:

$$\theta_B = \tan^{-1}\left(\frac{n_2}{n_1} \right)$$

When the reflection and transmission are perpendicular to each other, and there will be 0 reflection.
Total internal reflection when $\theta_1 > \theta_{crit}$, where

$$\sin\theta_{crit} = \frac{n_2}{n_1}$$

e-folding scale:

$$z_0 = \frac{1}{k^2} \left(\frac{\sin^2\theta_1}{\sin^2\theta_{crit}} - 1 \right)^{-1/2}$$