LPF Internship Progress Report (Week 3 of 10)

Introduction

This report explains work done in an attempt to extract information about the space craft and thrusters on the LPF by analyzing simulated thruster command signals and space craft acceleration data (both linear and angular) relative to two free-falling test masses. In simulations, eight thrusters are commanded to generate thrust oscillating at distinct frequencies; these frequencies are used to identify a particular thruster's contribution to the motion of the space craft relative to the test masses. The space craft's response to each thruster is then used to determine thruster time delays (the time difference between a thruster command and its response), calibration (the ratio of measured to commanded thrust) and orientation (the altitude and azimuth in which the thruster points, relative to the mechanical frame of the space craft). Orientation calculations assume knowledge about the positions of the test masses and the space craft's center of mass relative to the mechanical frame of the space craft, and calibration calculations assume knowledge about the mass of the space craft.

Isolating Signals

Each thruster (indexed by i) is commanded to generate a thrust $T^{(i)}$ given by

$$T^{(i)} = F^{(i)} \sin\left(\omega^{(i)} t + \varphi^{(i)}\right) + A_{\mathrm{c}}^{(i)}, \tag{1}$$

where $F^{(i)}$ is a time-independent amplitude, $\omega^{(i)}$ is an angular frequency unique to each thruster, $\varphi^{(i)}$ is some signal phase, and $A_{\mathrm{c}}^{(i)}$ is the sum of all commands sent to the thruster by the space craft's attitude control system (which are generally aperiodic). Given their total commanded signal, each thruster's characteristic angular frequency $\omega^{(i)}$ and amplitude $F^{(i)}$ are identified from the signal's amplitude spectrum. The amplitudes for all thrusters in all simulations thus analyzed are about 500 nN at frequencies ~ 10 mHz (with bandwidths of ~ 100 μHz).

After identifying thruster signal frequencies, all time-series acceleration data is filtered through a low-pass filter at a cutoff frequency of 30 mHz to mitigate aliasing from high frequency noise, downsampled from 10 Hz to 100 mHz, and filtered through band-pass filters at the characteristic frequencies of the thrusters. Downsampling prior to using the band-pass filter reduces the dynamic range on filter coefficients, and hence reduces numerical errors in the filter. The resulting signals for each data set from each band-pass filter are the isolated response of each measured quantity, and are stored independently.

Thruster command signals are processed in the same manner (by low-pass filtering, downsampling, and band-pass filtering each thruster at its own frequency) to account for amplitude changes and phase shifts due to filtering.

Time Delay

A transfer function of the post-processing thruster signals to accelerations is calculated to find their relative phase difference $\Delta\varphi \in [0, \pi]$ at the frequency of the thruster signal. The direction of acceleration in response to a positive thruster signal $(+/-)$ is given by whether the magnitude $|\Delta\varphi|$ is less $(+)$ or greater $(-)$ than $\pi/2$. The phase

$$\Delta\varphi' = \begin{cases} \Delta\varphi & \Delta\varphi \le \pi/2 \\ \pi - \Delta\varphi & \Delta\varphi > \pi/2 \end{cases} \tag{2}$$

is then taken as the direction-independent phase difference in the signals, and used to find the time delay Δt of the thrusters by the relation

$$\Delta\varphi' = \omega \Delta t \implies \Delta t = \frac{\Delta\varphi'}{\omega}. \tag{3}$$

As each acceleration data set yields its own delay for each thruster, delays from all data sets at each thruster frequency are averaged to find the delays of individual thrusters. Delays vary by simulation, but are typically in the range 0.3–1 s.

Orientation and Calibration

Finding the calibration and orientation of the thrusters requires first finding the contribution to linear acceleration $\mathbf{a}_{\mathrm{sc}}^{(i)}$ of the space craft by the thruster indexed by i, which is given by

$$\mathbf{a}_{\mathrm{sc}}^{(i)} = \mathbf{a}_{\mathrm{tm}}^{(i)} - \boldsymbol{\alpha}_{\mathrm{tm}}^{(i)} \times \mathbf{r}_{\mathrm{tm}}, \tag{4}$$

where $\mathbf{a}_{\mathrm{tm}}^{(i)}$ and $\boldsymbol{\alpha}_{\mathrm{tm}}^{(i)}$ are the contributions to linear and angular acceleration of the space craft by thruster i and $\mathbf{r}_{\mathrm{tm}}$ is the position of a test mass relative to the space craft's center of mass. Neglected in this equation are corrections for the Coriolis and centrifugal forces in the rotating frame of the test mass, which were found to be of orders 10^{-6} and 10^{-8} respectively smaller than the included terms. It is also assumed that changes in $\mathbf{r}_{\mathrm{tm}}$ are negligibly small, so that it can be taken as constant, and that the space craft's center of mass is constant relative to the mechanical frame of the space craft, so that accelerations relative to the mechanical frame (i.e. acceleration data) are equal to accelerations relative to the center of mass.

After finding $\mathbf{a}_{\rm sc}^{(i)}$, a covariance matrix $\mathbf{V}^{(i)}$ of its components along the axes of the space craft's mechanical frame is computed, with elements $V_{k\ell}^{(i)}$ given by:

$$V_{k\ell}^{(i)} = \left\langle \left(a_k^{(i)} - \left\langle a_k^{(i)} \right\rangle \right) \left(a_\ell^{(i)} - \left\langle a_\ell^{(i)} \right\rangle \right) \right\rangle . \quad (5)$$

Here $a_n^{(i)} = \mathbf{a}_{\rm sc}^{(i)} \cdot \mathbf{x}_n$, where $\{ \mathbf{x}_n \}$ are the basis vectors of the mechanical frame, and $\langle \chi \rangle$ denotes the expectation value of the variable χ. The eigenvectors of $\mathbf{V}^{(i)}$ define the principal axes of $\mathbf{a}_{\rm sc}^{(i)}$ in the basis of the mechanical frame, with corresponding eigenvalues equal to the variance of $\mathbf{a}_{\rm sc}^{(i)}$ along the respective principle axes. As $\mathbf{a}_{\rm sc}^{(i)}$ is an acceleration in response to the thrust provided by thruster i, the eigenvector corresponding to the largest eigenvalue of $\mathbf{V}^{(i)}$ is parallel to the direction of thrust; motion in orthogonal directions (along the remaining eigenvectors) are due to signal noise, error, or otherwise unaccounted for effects at the frequency of thruster i. This eigenvector thus points in the direction of thrust (of thruster i) up to sign; the ambiguity in sign is resolved using knowledge about the direction of accelerations given a positive thrust or by approximate knowledge of thruster orientations to find the orientation of each thruster. Pitch (the angle above the x-y plane) and azimuth (the angle in the x-y plane, from the x axis to the y axis) are thus determined to an accuracy of about 1 microradian.

Given that $\mathbf{a}_{\rm sc}^{(i)}$ is sinusoidal, its amplitude $a_{\rm sc}^{(i)}$ is given in terms of the variance $\left(\sigma_{\rm sc}^{(i)}\right)^2$ along its principal axis by

$$a_{\rm sc}^{(i)} = \sqrt{2} \left(\sigma_{\rm sc}^{(i)} \right)^2 . \quad (6)$$

The calibration $C^{(i)}$ of thruster i is then given in terms of the mass $m_{\rm sc}$ of the space craft by

$$C^{(i)} = \frac{m_{\rm sc} a_{\rm sc}^{(i)}}{F^{(i)}} . \quad (7)$$

These measured calibrations typically lie in the range 0.97–1.03 (whereas all simulated calibrations are exactly 1).

Further Directions

There are currently several current pursuits in this work. Firstly, there are some unaccounted for discrepancies in the measured time delays. Secondly, the calibrations of all thrusters are set to unity in simulations. While $1-3\%$ relative errors in deduced calibrations may attributed to error, these are larger than one would expect, and from qualitative inspection of the actual numbers there appears to be some systematic errors in the results. Thirdly, the behaviors of all $\mathbf{a}_{\rm sc}^{(i)}$ in the respective plane perpendicular to their principal axes appear to be regular and periodic; such behavior is not expected to occur absent of error (motion in this plane is expected to be random and unpredictable), and is not presently understood. Finally, the assumption that the center of mass of the space craft is stationary relative to the mechanical frame may not be robust. It may be possible to deduce the center of mass of the space craft from thruster inputs and response data if the locations of the thrusters are known in the mechanical frame. There is thus an effort to see what information can be deduced from data about the center of mass if its location is unknown.