Auctions without commitment in the auto-bidding world: supplemental material

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4.1 Uniform reserves

Theorem 3 strongly relies on the auctioneer's ability to readjust the reserve price at the query and bidder level. However, when the set of queries is large, readjusting the reserve prices for each query may turn out to be too expensive for the auctioneer. In this kind of situation, when the auctioneer is constrained to set a uniform reserve price, Theorem 4 shows that the bidder still prefers the tCPA format so long as the extra-buyers game is symmetric. We remark that this result does not rely on how the auctioneer readjusts the extra buyers' reserve prices.[1]

When dealing with uniform reserve prices, the key technical challenge compared to Theorem 3 is that the auctioneer cannot replicate the effect of the bidder's bidding on the remaining extra-buyers by setting personalized uniform reserve prices. Thus, when the auctioneer readjusts the bidder's reserve price not only the bidder's marginal bid changes but also the marginal bids of extra-buyers using a tCPA format. To tackle this problem, we assume that game for extra-buyers using the tCPA format is symmetric.

Definition 1. *The extra-buyers' game is tCPA-symmetric if for every ω_v and Extra-Buyers i, j using the tCPA format, we have that their final marginal bids b_i, b_j are the same.*[2]

Remark 1. *When there is only one extra-buyer is in the auction, the game is tCPA-symmetric.*

We are now in position to present Theorem 4.

Theorem 4. *Suppose that the auctioneer is constrained to set a uniform reserve price to the bidder and that the extra-buyers game is tCPA-symmetric. Then, $u^*(tCPA|v) > u^*(mCPA|v)$.*

[1] This weaker condition on how the auctioneer behaves with the extra-buyers allows to include cases like when some extra-buyers are budgeted constrained, and hence, the auctioneer cannot readjust their reserve prices.

[2] A sufficient condition for those marginal bids to be the same is that (i) both bidders have the same target ($T_i = T_j$) and (ii) that for every query x there exists a query x' such that $q_i(x) = q_j(x')$.

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The proof of Theorem 4 is similar in spirit to that of Theorem 3, but now we can not use the power of the personalized per-query reserve prices to perform the step where we"simulate world T in world M". Instead of such a simulation, we show that in a tCPA-symmetric game, there is a structural property of the bidding behavior in equilibrium, which allows us to prove the result.

4.2 Proof of Theorem 4

Before proving the theorem, we require the following technical lemma.

Lemma 4 (No Swapping Lemma)**.** *Consider two final marginal bids profile $(b_0, \boldsymbol{b})$ and $(b_0', \boldsymbol{b}')$ and a particular realization ω_v. Consider a directed graph $G = (V, E)$, where each vertex v is one of the participants on the auction and the edge $e = (i, j)$ exists if and only if for profile $(b_0, \boldsymbol{b})$ agent i wins query x and for the profile $(b_0', \boldsymbol{b}')$ agent j wins query x. Then the graph G is acyclical.*

Proof. Suppose that for some ω_v, G contains a cycle.

Then consider the sequence of bidders in the auction $i_1, \dots, i_k, i_1$ creating the cycle. Let x_{i_j} the query that i_j wins with the bid profile $(b_0, \boldsymbol{b})$ and that i_{j+1} wins when the bid profile is $(b_0', \boldsymbol{b}')$. Observe that $b_{i_j} q_{i_j}(x_{i_j}) > b_{i_{j+1}} q_{i_{j+1}}(x_{i_j})$. Therefore, we derive that

$$b_{i_1} > b_{i_2} \frac{q_{i_2}(x_{i_1})}{q_{i_1}(x_{i_1})} > b_{i_1} \prod_{j=1}^{k} \frac{q_{i_{j+1}}(x_{i_j})}{q_{i_j}(x_{i_j})}. \tag{1}$$

where $k + 1 = i$.

Using the same logic for the bidding profile $(b_0', \boldsymbol{b}')$ we obtain that

$$b_{i_1}' < b_{i_2}' \frac{q_{i_2}(x_{i_1})}{q_{i_1}(x_{i_1})} < b_{i_1}' \prod_{j=1}^{k} \frac{q_{i_{j+1}}(x_{i_j})}{q_{i_j}(x_{i_j})}. \tag{2}$$

Equations (1) and (2) generate a contradiction. We conclude that G does not have any cycle. □

We are now in a position to prove Theorem 4.

Proof of Theorem 4. The proof strategy is similar to Theorem 3. Given an arbitrary marginal bid b_{mCPA}, we want to show that if the bidder submits $b_{\text{mCPA}} \in [0, v]$ using the mCPA format, the bidder can weakly improves her payoff by bidding a target $T = b_{\text{mCPA}}$ with the tCPA format for every realization ω_v. Furthermore, the inequality is strict for a positive measure of ω_v. We split our proof into the following steps.

Let $X^0_{\text{mCPA}}(\omega_v)$ and $X^0_{\text{mCPA}}(\omega_v)$ the subset of queries that the bidder obtains in the mCPA and tCPA cases, respectively. We assert that $X^0_{\text{mCPA}}(\omega_v) \subseteq X^0_{\text{tCPA}}(\omega_v)$. Suppose for the sake of a contradiction that is not true. Then, consider $x \in X^0_{\text{mCPA}}(\omega_v) \setminus X^0_{\text{mCPA}}(\omega_v)$. Let b_{mCPA} and b'_{mCPA} the bidder's final marginal bid for the mCPA and tCPA cases, respectively. From Lemma 1 we have that $b'_{\text{mCPA}} \geq b_{\text{mCPA}}$. Therefore, since the bidder is losing query x in the tCPA case an extra-buyer i has increased her final marginal bid from b_i to b_i' and wins query

x. Thus, for the graph G described in Lemma 4 we have an edge $e = (0, i)$.

Because the marginal bid of Extra-Buyer i is not constant, it implies that she is using the tCPA format. We assert that there is a query $x' \in X^0_{\text{mCPA}}(\omega_v) \setminus X^0_{\text{tCPA}}(\omega_v)$. Because mCPA extra-buyers do not change their marginal bids and tCPA-extra buyers have a higher marginal bid in the tCPA case (since $b'_i > b_i$ and the game of tCPA-buyers is symmetric), the cost for every query is (weakly) higher in the tCPA case than in the mCPA case. In particular, the average cost-per-acquisition (CPA) of wining queries $X^0_{\text{mCPA}}(\omega_v)$ is at least T_i. Moreover, because the queries in $X \setminus X^0_{\text{mCPA}}(\omega_v)$ have a CPA higher than the T_i (since $b_i \geq T_i$ but the Extra-Buyer i did not win those queries in the mCPA case), we have that in order to win query x and, at the same time, keep an average CPA no more than T_i, Extra-Buyer i is loosing a query $x' \in X^0_{\text{mCPA}}(\omega_v) \setminus X^0_{\text{tCPA}}(\omega_v)$. If the winner of that query is the bidder, then the edge $e = (i, 0)$ belongs to G. From Lemma 4 this is not possible. Suppose the winner of the query is a different Extra-Buyer k. In that case, we can reiterate the logic of this paragraph and either obtain a cycle on G or, again, find another extra buyer winning a new query. Because there is a finite set of extra-buyers, at some point in the iteration G will have a cycle. This is a contradiction. Therefore, $X^0_{\text{mCPA}}(\omega_v) \subseteq X^0_{\text{tCPA}}(\omega_v)$.

To finish the proof, we follow the same reasoning as the one used for the proof of Theorem 3. We use Step 3. to show the weak inequality for every ω_v and Step 4 to show that the inequality is strict for a positive measure of ω_v. $\square$