# Implementing Path-Coloring Algorithms for Planar Graphs

Aven Bross

Department of Computer Science

University of Alaska

Fairbanks, AK 99775-6670

dabross@alaska.edu

Glenn G. Chappell

Department of Computer Science

University of Alaska

Fairbanks, AK 99775-6670

chappellg@member.ams.org

Chris Hartman

Department of Computer Science

University of Alaska

Fairbanks, AK 99775-6670

cmhartman@alaska.edu

May 29, 2017

**Abstract**

ZZZ

## 1   Introduction

All graphs will be finite, simple, and undirected.

A *path coloring* of a graph $G$ is a vertex coloring (not necessarily proper) of $G$ so that each color class induces a *linear forest*, that is, a forest of maximum degree at most 2, or, equivalently, a disjoint union of paths. A graph $G$ is *path k-colorable* if $G$ admits a path coloring using $k$ colors.

Broere & Mynhardt conjectured [1, Conj. 16] that every planar graph is path 3-colorable. This was proven independently by Poh [6, Thm. 2] and by Goddard [4, Thm. 1].

**Theorem 1.1** (Poh 1990, Goddard 1991). *Let $G$ be a planar graph. Then $G$ is path 3-colorable.*   □

1

Hartman [5, Thm. 4.1] (see also Chappell & Hartman [3, Thm. 2.1]) proved a list-coloring generalization of Theorem 1.1. A graph $G$ is *path $k$-choosable* if, whenever each vertex of $G$ is assigned a list of $k$ colors, there exists a path coloring of $G$ in which each vertex receives a color from its list.

**Theorem 1.2** (Hartman 1997)**.** *Let $G$ be a planar graph. Then $G$ is path $3$-choosable.*  □

A slightly weaker result was proven by Škrekovski [7, Thm. 2.2b] using essentially the same technique.

We discuss two efficient path-coloring algorithms based on proofs of the above results.

Section 2 covers an algorithm based on Poh's proof of Theorem 1.1. The algorithm is given a planar graph; it produces a path coloring of the given graph using three colors.

Section 3 covers an algorithm based Hartman's proof of Theorem 1.2, along with a similar proof of a weaker result by Škrekovski. The algorithm is given a planar graph and an assignment of a list of three colors to each vertex; it produces a path coloring of the given graph in which each vertex receives a color from its list.

Implementations of both algorithms are available; see Bross [2].

# 2    Path Coloring: the Poh Algorithm

ZZZ

# 3    Path List Coloring: the Hartman-Škrekovski Algorithm

ZZZ

# References

[1] I. Broere and C. M. Mynhardt, Generalized colorings of outerplanar and planar graphs, *Graph theory with applications to algorithms and computer science* (Kalamazoo, Mich., 1984), 151–161, Wiley-Intersci. Publ., Wiley, New York, 1985.

[2] A. Bross, *Path coloring algorithms on plane graphs*, Masters project, University of Alaska Fairbanks, 2017, available from
`http://github.com/permutationlock/path_coloring_bgl`.

[3] G. G. Chappell and C. Hartman, Path choosabiility of planar graphs, in preparation.

[4] W. Goddard, Acyclic colorings of planar graphs, *Discrete Math.* **91** (1991), no. 1, 91–94.

[5] C. M. Hartman, *Extremal Problems in Graph Theory*, Ph.D. Thesis, University of Illinois, 1997.

[6] K. S. Poh, On the linear vertex-arboricity of a planar graph, *J. Graph Theory* **14** (1990), no. 1, 73–75.

[7] R. Škrekovski, List improper colourings of planar graphs, *Combin. Probab. Comput.* **8** (1999), no. 3, 293–299.