Cryptocurrencies and the Velocity of Money

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Abstract

The velocity of money denotes the intensity with which its tokens circulate. Velocity is central to the quantity theory of money, which relates it to the general price level. While the theory motivated countless empirical studies to include velocity as price determinant, few find a significant relationship in the short or medium run. Since the velocity of money is generally unobservable, these studies were limited to use proxy variables, leaving it unclear whether the lacking relationship refutes the theory or the proxies. Cryptocurrencies on public blockchains, however, visibly record all transactions, and thus allow to measure—rather than approximate—velocity. This paper evaluates most suggested proxies for velocity, and also proposes a novel measurement approach. We introduce velocity measures for UTXO-based cryptocurrencies focused on the subset of the money supply effectively in use for the processing of transactions. By separating circulating from hoarded or lost tokens, this approach explicitly addresses the hybrid use of cryptocurrencies as media of exchange and as stores of value, a major distinction in recently proposed theoretical pricing models. Implementing all measures for Bitcoin, we provide public, re-usable code based on the BlockSci blockchain parser. We show that each of the velocity estimators is approximated best by the simple ratio of on-chain transaction volume to total coin supply. Moreover, "coin days destroyed," if used as an approximation for velocity, shows considerable discrepancy from the other approaches.

1 Introduction

Velocity of money plays a key role in traditional monetary economics since [1]. Broadly speaking, velocity of money denotes the average number of transactions per monetary unit within a certain time period.[1] In the quantity theory of money, velocity is related to the price level. While empirical studies frequently apply this concept to cryptocurrencies, surprisingly few find a significant relationship between velocity and prices. We take this discrepancy as occasion to evaluate current approaches to quantify the velocity of money for cryptocurrencies, and propose a novel one.

Until recently, meaningful measures for velocity of cryptocurrencies did not exist, and most studies resorted to proxy variables.[2] Recent years saw first advances to measure—instead of approximate—the velocity of money. [2] and [3] first considered the quantity equation of money by [1] to measure velocity as the ratio of transaction volume and money supply. [2] modified this approach to create a measure handling the change transactions in cryptocurrency systems. While [2] and later [4] focused on adjusting the transaction volume in the above ratio, we complement their approach by adjusting the money supply.

Money in effective circulation should be differentiated from money held for long-term investment or speculation. Not only does the total monetary aggregate contain technically dysfunctional money (burnt coins), a major portion of cryptocurrency is stored unused over long time periods (compare [5] or [4]). Economists like [1], [6] or [7] have argued to exclude such funds and focus on money in circulation. To our knowledge, [3] and [2] were first to apply this distinction in theoretical cryptocurrency pricing models. Both link feedback effects from speculation and price levels to a reduction of coins in effective circulation. [3] explicitly define the velocity of money as based on the component of coin supply in effective circulation. In this paper, we operationalize this definition for velocity measurement.

In implementing this concept, we make common implicit assumptions explicit. For example, the separation of money into *hoarded* or *circulating* depends on the choice of a time window. Tokens can be defined as circulating if moved within

[1] We refer to its definition arising from the "transaction form" of the quantity theory of money.

[2] We use *measure* and *estimator* interchangeably but contrast them to the terms *proxy variable* or *approximating variable*. The former quantify the concept of interest directly (e.g. length with a yardstick), while the latter rely on the quantification of a distinct concept which is assumed to be correlated with the one sought (e.g. wealth via horsepower of owned cars).

the last day, month, year or any other period. The choice of [2] and [4], defining money in circulation as the total coin supply, implies an infinite time window. The other extreme might be a very restrictive definition requiring coins to be moved within the period for which velocity is measured. As the optimal time-window might depend on the respective use case, we operationalize a velocity measure for UTXO-based[3] cryptocurrencies as a function of the respective time-window.

Subsequently, we apply our approach to Bitcoin and compare a variety of potential proxy variables to measures characterizing the two extremes of the design space. Measuring the goodness of fit from a variety of perspectives, we show that the most common proxy-variable, *coin days destroyed* (CDD)[4] in the vast majority of tests shows higher approximation errors than the simple ratio of unadjusted, on-chain transaction volume and total coin supply as shown by a series of Model Confidence Set (MCS) tests. As the majority of research opted for CDD, our results might suggest a reason for the unexpectedly missing relation between velocity and prices in most studies.

Our implementation is based on the open-source blockchain parser *BlockSci*[5]. The codebase to calculate the evaluated velocity measures for UTXO-based cryptocurrencies is re-usable and will be openly available after publication. Summarizing, concerning cryptocurrencies as field of research we offer the following contributions:

- a holistic review of approaches to quantify velocity[6]
- novel measures based on money in circulation
- empirical evaluation of common approximation methods

2 Literature review

Since the early times of economic research on cryptocurrencies, velocity of money has received attention in theoretical pricing models and empirical studies of price determinants.

Empirically, CDD is commonly used as proxy variable in regressions of cryptocurrency return patterns. Based on the quantity equation, these studies expect a significant positive relationship between prices and the chosen proxy. While [8] and [9] confirm the hypothesis, more often it is rejected [10, 11, 12, 13, 14]. Following [1], [2] estimate velocity as the ratio of adjusted on-chain transaction volume to the total Bitcoin supply when modeling the Bitcoin price. Additionally they, employ the ratio of off-chain transaction volume and coin supply (both denominated in USD) as a velocity estimator.

Also theoretically, cryptocurrency pricing models refer to the velocity of money. [3] use velocity as central building block of their pricing model. They decompose the velocity of money into a part for monetary units used as medium of exchange and a part for those used as long-term investment. The paper does not specify, however, how this decomposition could be implemented. Our paper is the first to offer an operationalization for UTXO-based cryptocurrencies.

[2] present a measure of velocity of Bitcoins acknowledging the need for adjustments for artificial transaction volume generated by change money. [4] adopt the same concept but provide deeper insights into its technical configuration as a byproduct of introducing a new blockchain parser for UTXO-based cryptocurrencies.

Recognizing the need for a more precise method, [16] proposed a cryptocurrency's *turnover* as derivation of CDD. We compare this approach to the other methods and show that, compared to CDD, the measure is indeed closer to the velocity estimates in many tests.

3 Theoretical concepts of measures for the velocity of money

To facilitate an informed discussion of measures for the velocity of money, we first clarify the use of technical terms for the theoretical concepts.

Broadly speaking, velocity of money is defined as the average number of turnovers per monetary unit within a time period. This definition stems from the *transaction form* of the quantity theory of money as formalized by [1].[7] The central equation of the theory, incorporating a time period p, corresponds to money flows given by the product of money supply M_p and velocity V_p, with flows of real transactions, given by the scalar product $\langle P_p, T_p \rangle$ of prices P_p and transaction volumes T_p. With $n \in \mathbb{N}$ and refining the notation in [1], this amounts to

$$M_p V_p = \langle P_p, T_p \rangle \text{ with } M_p, V_p \in \mathbb{R}, \text{ and } P_p, T_p \in \mathbb{R}^n. \quad (1)$$

The scalar product on the right-hand side of the equation is referred to as the *price sum*. In this product, $P_p = (P_{p_1}, P_{p_2}, \cdots, P_{p_n})$ denotes a vector with prices P_{p_t} of transacted goods and services in transaction t during period p. Transaction volumes T_p are given in units of goods and services. They are conceptualized as the vector $T_p = (T_{p_1}, T_{p_2}, \cdots, T_{p_n})$ with volume T_{p_t} in transaction t in period p. On the left-hand side, M_p stands for the number of all units of money supply available in period p. V_p denotes the velocity of money.[8] While T_p, V_p and P_p are measured over a time period, M_p is a point-in-time measure. To simplify, we assume

[3] Descendants of Bitcoin's approach to build a transaction graph are known as *UTXO-based cryptocurrencies*. See Section 4 for details.

[4] The variable is discussed in Section 9.1.1.

[5] https://citp.github.io/BlockSci/index.html

[6] Refer to Appendix 11 for a condensed summary of all approaches.

[7] While the transaction form highlights the use of money as a medium of exchange, other forms stress different characteristics. All forms, however, relate money flows to real transactions [cf. 17].

[8] To see this equations in a less abstract way, imagine a economy with only 2 gold coins, Alice, Bob and sheep Eve. Alice owns the 2 gold coins and Bob the sheep Eve. In 2019, Alice buys Eve from Bob for 1 gold coin. Later in the year, Bob regrets his decision and buys Eve back for the same price—Alice receives her coin back. The quantity equation states the following:

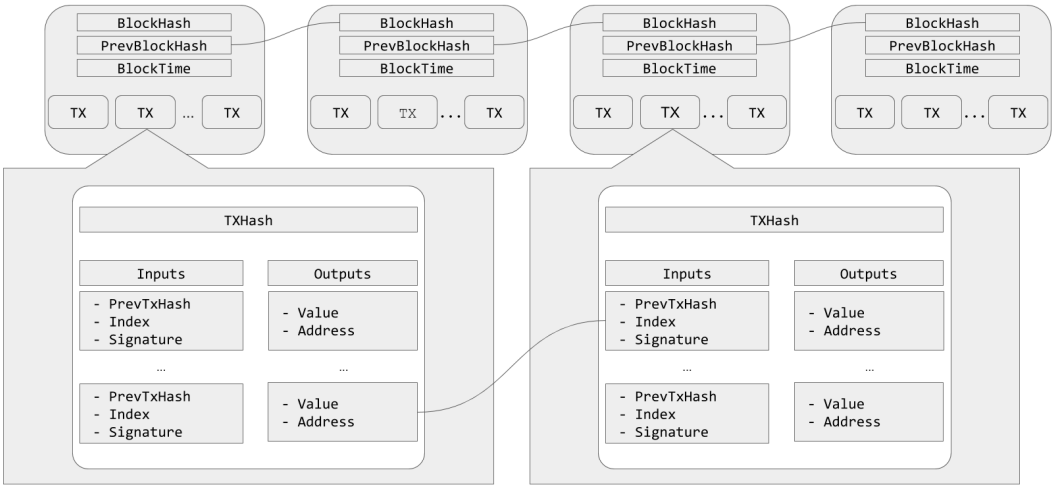


Figure 1: Blockchain and Transactions (adapted from [15]).

the money supply is fixed during period p and record it at the period's beginning $p_{\texttt{start}}$.

To develop intuitive insight for velocity V_p, the measure can be viewed as the weighted average number of turnovers for monetary units as in [17]. To get weights, these units are sorted into groups $g \in \mathbb{G}_p$ with respect to their numbers of turnovers v_{p_g} during period p. Velocity V_p is then defined as

$$V_p = \sum_{g \in \mathbb{G}_p} \left(v_{p_g} \cdot \frac{N_{p_g}}{\sum_{g \in \mathbb{G}_p} N_{p_g}} \right), \tag{2}$$

with N_{p_g} monetary units in group g in period p. The velocity is thus simply the sum of all the turnover numbers v_{p_g}, weighted by the respective fractions.[9] While this definition makes the concept of velocity intuitive, it cannot be used for measuring velocity in practice. Turnover numbers for monetary units are neither recorded for fiat currencies, nor can they be inferred unambiguously for UTXO-based cryptocurrencies by counting transfers of coins (compare Section 4). In practice, the velocity of money is thus backed out of Equation (1), leading to $V_p = \langle P_p, T_p \rangle / M_p$.

For cryptocurrencies, this seems like a simple task, just using on-chain transaction volume and total coin supply. However, due to their technical implementation, transaction volumes recorded on-chain are distorted. The next section therefore discusses the relevant subtleties of UTXO-based cryptocurrency systems.

$2\texttt{coins} \cdot V_{2019} = \langle \begin{pmatrix} 1 \frac{\texttt{coins}}{\texttt{sheep}} \\ 1 \frac{\texttt{coins}}{\texttt{sheep}} \end{pmatrix}, \begin{pmatrix} 1\texttt{sheep} \\ 1\texttt{sheep} \end{pmatrix} \rangle = 2\texttt{coins}.$

Now the velocity V_{2019} of the economy's money can be backed out as 1.

[9] Returning to the example in footnote 8, one coin was turned over twice—and one not at all. Thus $V_{2019} = 0 \cdot \frac{1\texttt{coins}}{2\texttt{coins}} + 2 \cdot \frac{1\texttt{coins}}{2\texttt{coins}}$. This shows that the introduced form of the quantity equation is an implicit definition of velocity, not a testable statement (compare [17]).

4 utxo-based cryptocurrencies

Bitcoin builds its transaction graph chaining transaction outputs. This approach has been followed by many cryptocurrency projects, which are referred to as *UTXO-based cryptocurrencies*. "UTXO" refers here to the "coins" that can be spend—*unspent transaction outputs*.[10] While UTXO-based cryptocurrencies record only transactions, account-based cryptocurrencies store a balance for each generated address on their blockchain. For a complete explanation of the mechanics, we refer the interested reader to [15] and restrict this section to features relevant for the calculation of velocity measures.

UTXO-based cryptocurrency protocols ensure an ordered transaction history using a linked chain of so called *blocks* (compare Figure 1). These blocks contain a hash (***BlockHash***) fingerprinting the information of all the transactions recorded in the block, a timestamp (***BlockTime***) and the hash of the previous block (***PrevBlockHash***). This constellation of hashes and timestamps establishes pointers that are determining the order of blocks. The process of creating new blocks, called *mining*, creates new monetary units in so called *coinbase transactions*. Creating blocks, often requires solving a computational demanding puzzle (*proof-of-work*) or proving stake in the existing coin supply (*proof-of-stake*).

Generally, transactions contain a hash as identifier (***TxHash***) as well as inputs and outputs. Coinbase transactions deviate by just including an output but no input. This effectively increases the amount of spendable outputs, and thus money in the system. Inputs are recorded by linking them back to outputs of a previous transaction identified by an index (***PrevTxHash***). Outputs can be sent to addresses (***Address***) that are generated by a set of a public and a private key. Unspent previous outputs can only be used as inputs,

[10] For a detailed explanation of UTXO-based and account-based cryptocurrency systems refer to [18].

when proof of ownership has been provided (***Signature***). To generate this proof, the private key that belongs to the public key having generated the output receiving address is required. Transaction inputs can only be spent as a whole. If a part of an input is to be retained, an output needs to be formed linking back to a address controlled by the spender. These addresses are known as *change addresses* and we will refer to the respective outputs as *change outputs* hereafter.

Generalizing, all outputs sent to an address controlled by the sender are, as in [4], referred to as *self-churn*. As the concept of *user identities* does not exist in UTXO-based cryptocurrencies, there is no simple way to clearly separate self-churn from outputs transferred to third parties (compare [19]). In addition to missing identities, any assignment between the inputs and outputs of a certain transaction is undetermined. Unspent outputs are fully fungible, so that there is no technical link between individual outputs and inputs. Although it would be helpful for calculating velocity measures, thus, it is hardly feasible to follow transaction links due to splitting and rejoining transactions in combination with the unspecified assignment of in- and outputs of each transaction.

5 Self-churn and Clustering

As discussed in the last section, by construction many cryptocurrency transactions contain outputs linked back to the sender himself, returning the change money to an address in control of the sender. Not only these change outputs but also all other self-churn ought to be excluded from the transaction volume: "What is desired is the rate at which money is used for purchasing goods, not for making change." ([20]).

5.1 Transaction volumes inflated by self-churn

While the transaction volume in principle can be calculated accumulating the output values $\texttt{valOut}(o)$[11] of all transactions t recorded within period p, this does yield a inflated aggregate $\langle P_p{}', T_p{}' \rangle$. Hence, this can be formalized as

$$\langle P_p{}', T_p{}' \rangle = \sum_{t \in \mathbb{T}_p} \sum_{o \in \mathbb{O}_t} \texttt{valOut}(o), \tag{3}$$

where $o \in \mathbb{O}_{p_t}$ denotes the set of all outputs of transaction t in period p, this transaction volume needs to be adjusted. Defining $\mathbb{O}^{\texttt{selfchurn}}$ as the set of all self-churn outputs, the accumulated transaction volume from these outputs O_p can thus be calculated from the individual self-churn outputs $c \in \mathbb{O}^{\texttt{selfchurn}}$ as

$$O_p = \sum_{t \in \mathbb{T}_p} \sum_{o \in \mathbb{O}^{\texttt{selfchurn}}} \texttt{valOut}(o). \tag{4}$$

[11] For simplified notation, we include many variables as functions of transactions, outputs or inputs. For example, function $\texttt{valOut}(o)$ extracts the output value of ouput o of transaction t in period p.

Given the above adjustments, a "deflated" transaction volume can be calculated as $\langle P_p, T_p \rangle = \langle P_p{}', T_p{}' \rangle - O_p$.

5.2 Adjustment heuristics to deflate transaction volumes

Practically, as discussed in Section 4, only addresses but no identities are part of the information written into the transaction ledger. This makes it hard to decide, which output is to be considered self-churning in order to calculate a deflated transaction volume. However, statistical properties can be used to determine whether an output is likely to belong to the same individual user as the transactions inputs (compare [19]). These properties are often called *heuristics* and can be used to create *user-clusters* of addresses. Outputs are classified as self-churn if the cluster of their destination address equals the cluster of their input addresses.

As our emprical analysis builds on a blockchain parser proposed in [4], we follow their choice of heuristics. Their study employes one heuristic that has been proposed first in [19] and one heuristic accounting for *peeling chains*. According to [4], peeling chains are transaction patterns that split large unspent transaction outputs into smaller amounts in a chain of transactions. Manual inspection lead [4] to the conclusion that outputs that are created and spent during a relatively short time period are often belonging to the same user cluster. The heuristics used are thus:

1. All inputs used in a transaction are most likely from one person. [12]
2. Outputs that are created and spent within 4 blocks, are classified as self-churn transactions.

6 Velocity defined on the total money supply

Equipped with all necessary basics from both realms—economics and computer science—recently suggested methods of velocity measurement for cryptocurrencies are to be summarized. Both approaches utilize the total coin supply as measure of the money supply M_p in equation (1). In this paper, we denote total coin supply suggested in the approaches as $M_{\texttt{total}\,p}$, which is calculated as the aggregated sum of evermined coins at the beginning of period p, which itself is a deterministic function of the block height, to our knowledge, for all UTXO-system based cryptocurrencies.[13]

Transforming the quantity equation of money (1), it seems intuitive to calculate the velocity of money by simply dividing the unadjusted Bitcoin transaction volume $\langle P_p{}' T_p{}' \rangle$ by the

[12] Note that [4] added the additional restrictions that this heuristic is not applied to coinjoin transactions which are used to obfuscate transaction paths. Coinjoin transaction are classified as in [21].

[13] As discussed in Section 3, we assume that the money supply is fixed during period p and just use the amount of monetary units in the beginning of the period. Neither [4] nor [2] clearly specify their adopted choice.

total coin supply $M_{\texttt{total}p}$ and in fact, this has been described in [3] and adopted by [22]. The measure can be formalized as

$$V^{\texttt{msr}}_{\texttt{triv}p} = \frac{\langle P_p{}', T_p{}' \rangle}{M_{\texttt{total}p}}. \tag{5}$$

$V^{\texttt{msr}}_{\texttt{triv}p}$ offers the advantages of providing a theoretically sound interpretation and trivial calculation. Moreover, data for calculating measure $V^{\texttt{msr}}_{\texttt{triv}p}$ (the simple on-chain transaction volume and the total coin supply) are widely available.[14] However, the result would show major distortions: Self-churn transactions of the Bitcoin protocol would lead to an overestimation of the used transaction volume.

The estimation of the velocity measure proposed by [2] and [4] is similar to the above. Here, however, the price sum cleaned from self-churn is used. For every period p it is thus calculated as the ratio of price sum $\langle P_p, T_p \rangle$ divided by the complete money supply $M_{\texttt{total}p}$. Velocity $V^{\texttt{msr}}_{\texttt{total}}$ can thus be estimated as

$$V^{\texttt{msr}}_{\texttt{total}} = \frac{\langle P_p, T_p \rangle}{M_{\texttt{total}p}}. \tag{6}$$

Following Section 3, both measures in principal can interpreted as the turnover of coins during period p averaged over the complete coin supply.

7 Velocity defined on money in effective circulation

While the velocity measures proposed so far certainly brought advances in quantifying the velocity of money for cryptocurrencies, there are certain drawbacks associated with them. Instead of using money supply defined as "the aggregate of all monetary units issued over time" (denoted by $M_{\texttt{total}p}$), we operationalize a measure based on the component of money that circulates effectively. Denominating this circulating subcomponent $M_{\texttt{circ}p}$, this measure yields

$$V^{\texttt{msr}}_{\texttt{circ}p} = \frac{\langle P_p, T_p \rangle}{M_{\texttt{circ}p}}. \tag{7}$$

7.1 Formal derivation

To see why a simple exclusion of hoarded money does not violate the quantity theory of money, one might extend the summands in equation (2). Therefore, let h denote the group of money that incorporates all the monetary units that are being treated as investment and let $\mathbb{G}'_p := \mathbb{G}_p \setminus \{h\}$ denote all other groups. Thus, extending equation (2) yields

$$V^{\texttt{msr}}_{\texttt{total}p} = v_{p_h} \frac{N_{p_h}}{\left(N_{p_h} + \sum_{g \in \mathbb{G}'_p} N_{p_g}\right)} + \sum_{g \in \mathbb{G}'_p} v_{p_g} \frac{N_{p_g}}{\left(N_{p_h} + \sum_{g \in \mathbb{G}'_p} N_{p_g}\right)}. \tag{8}$$

Since h encompasses only those monetary units being treated as investment, the turnovers achieved of units in group h is $v_{p_h} = 0$ for period p.

Measure $V_{\texttt{circ}p}$ reduces the equation to

$$V_{\texttt{circ}p} = \sum_{g \in \mathbb{G}'_p} v_{p_g} \frac{N_{p_g}}{\sum_{g \in \mathbb{G}'_p} N_{p_g}}, \tag{9}$$

where group h, hoarded money, is not considered. Hence, the measure can be interpreted as the average number of turnovers that effectively circulating money units were able to achieve in period p. Dropping hoarded money in equation (9) is reflected in the quantity equation by redefining the money supply M_p in equation (1) to circulating money $M_{\texttt{circ}p}$ as well. Therefore, calculating velocity for money in circulation does not violate the theory, only its definition is adjusted.

7.2 Practical considerations and theoretical advantages

A first advantage offered by a velocity measure based on money in effective circulation, is its higher information density. One might ask whether the approaches $V^{\texttt{msr}}_{\texttt{triv}}$ and $V^{\texttt{msr}}_{\texttt{total}}$ can add information compared to transaction volumes $\langle P_p, T_p \rangle$ and $\langle P_p{}', T_p{}' \rangle$ respectively. Most UTXO-based cryptocurrencies implement their money supply as simple function of the block height[15]. Thus, the two former measures are very close to a merely scaled versions of their respective price-sum.

Moreover, the total coin supply used in $V^{\texttt{msr}}_{\texttt{triv}}$ and $V^{\texttt{msr}}_{\texttt{total}}$ includes money that is technical dysfunctional or being held unused as storage of wealth or speculation.[16] These components of the money supply surely do not fulfill one of the most important functions of money, i.e., being used as medium-of-exchange.[17]

Furthermore, the amount of money frozen in speculative investments might not be neutral to money flows and prices. Since the beginnings of monetary economics, currency speculation was associated with patterns in price levels. In [28] and [29], the illiquid component is used as reservoir for neutralizing demand shocks and excluded from the quantity equation

[14] Time series data can be downloaded for free from www.blockchair.com, www.blockchain.com, www.btc.com. Also www.blockwatch.cc and many others data broker provide the respective data.

[15] Note that additional difficulty adjustments for mining, lead to relativly constant time periods between blocks creation [15].

[16] Most studies agree, that cryptocurrencies are currently used as long-term speculative vehicle rather than as medium of exchange (compare e.g. [23, 24, 25, 26]).

[17] Compare [27] for a detailed discussion of the discrepancy arising from money as storage of wealth.

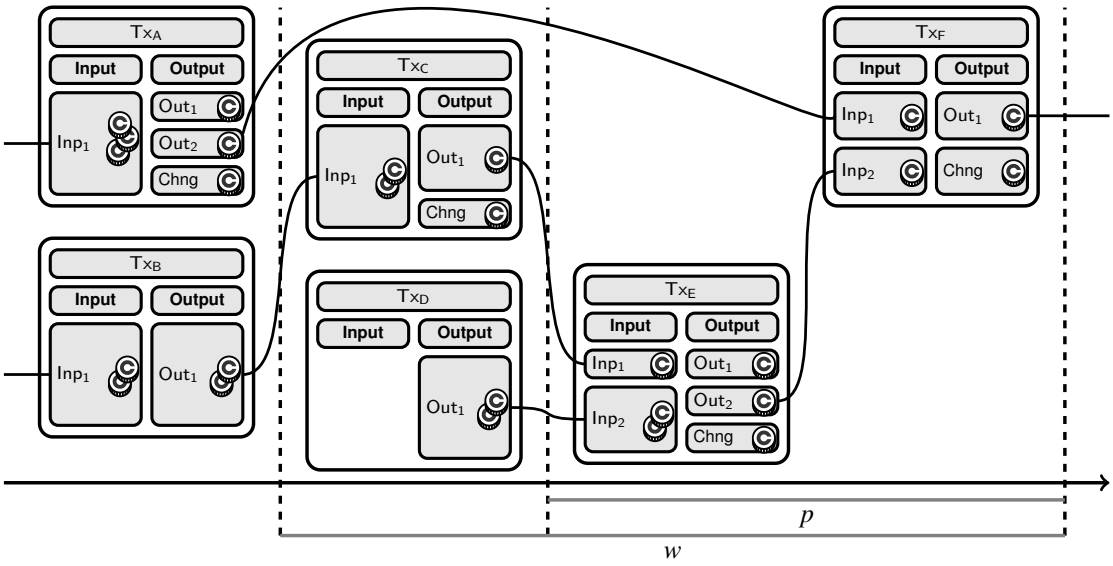


Figure 2: An example of a transaction chain.

of money. For [7] and [6] hoarded money, as destroyed money, is *leakage* that needs to be compensated to keep the general price level stable. In [1], the rise in market prices for one of the early fiat bank notes used in the U.S. is associated with an interrelation between speculation and circulating money supply as well. [1] concluded that "speculation acted as a regulator of the quantity of money".

Current research on cryptocurrency price processes has been inspired by this perception. We operationalize the velocity of money in effective circulation, which is also part of the model proposed in [3]. [3] directly utilize the quantity equation of money brought forth by [1]. They extend it, however, with a model for the demand of transactions in the equilibrium and for rational speculation and assume that a fiat currency is serving as denominator of value. While the authors use the quantity equation as basis, they deduct coins that are bought and hold merely as speculative investment and adjust it to cope with exchange rates between cryptocurrency and fiat money. Their modified version of equation (1) relates the exchange rate between the fiat- and cryptocurrency S_p to the velocity of cryptocurrency in effective circulation $V_{\mathtt{circ}p}$ and the volume of transactions $T_p{}^*$ denominated in the units of the cryptocurrency. This can be expressed as

$$S_p = \frac{T_p{}^* / V_{\mathtt{circ}p}}{M_{\mathtt{circ}p}}. \tag{10}$$

[3] then added speculators who are willing to purchase a part of the cryptocurrency supply, if the trading prices are below a risk-adjusted, discounted expected future price of the cryptocurrency. With an increasing aggregated value of speculative positions, the risk of additional speculative investments increases, lowering the price which speculators are willing to pay. On the other hand, increased speculative positions (and thus decreasing $M_{\mathtt{circ}p}$) increases market prices according to their modified quantity equation of money. A similar interrelation of money in circulation and speculation can be found in [2].

The described concepts, implicitly or explicitly, define velocity thus as being based on circulating money instead of based on the total money supply. Therefore, operationalizing this approach for the use in empirical pricing studies might add an interesting perspective.

8 A novel approach to measure velocity of money in effective circulation practically

Equipped with a conceptional understanding of the velocity of money in effective circulation, we propose algorithms to operationalize the segregation of money supply and calculate the respective velocity measure in practice.

8.1 Segregating money supply in practice

As a first step, a time-window has to be defined relative to which a certain monetary unit can be determined as *circulating* or not. Money might be referred to as circulating if it has been moved within the last day, month, year or any other conceivable time period. Let us denote this window w covering $[w_{\mathtt{start}}, w_{\mathtt{end}}]$.

To extract the component of money supply that circulated effectively within period w it proves useful to step trough every transaction recorded in period w. Transactions spending outputs generated before time window w or outputs from coinbase transactions might be interpreted as bringing an

amount of money into circulation that corresponds to the spent outputs value. All transactions using unspent transaction outputs generated within period w as input would then simply be re-spending already counted money supply.[18]

Note that time window w can be defined relative to period p by a maximum length α of look-back window w, where $w_{\mathtt{start}} = p_{\mathtt{start}} - \alpha$ and $w_{\mathtt{end}} = p_{\mathtt{end}}$. While measuring velocity still for period p covering $[p_{\mathtt{start}}, p_{\mathtt{end}}]$, money is considered *circulating* if having been moved in time window w. While the above approach is reasonable, certain particularities lead to differing variants of measuring money in effective circulation. Technically, two major characteristics of UTXO-based cryptocurrencies shape potential ways to measure the circulating money supply: Firstly, transactions can only spend prior transaction outputs in full—or not at all. Secondly, the exact link between the individual outputs and inputs of a transaction is unspecified, so that it is unclear which input converts to which output.

The first characteristic requires a choice with respect to the atomic unit of money drawn into circulation. Assuming a transaction generated change: Should the whole input be considered *money in circulation* because the complete input effectively moved—or should only the fraction sent to third parties be considered as such? We divide these choices and call the first *whole-bill-approach* (WB-approach) and the second *moved-coin-approach* (MC-approach). The above can be exemplified by transaction $\mathsf{Tx_C}$ in Figure 2. The moved-coin-approach would suggest, considering only output $\mathsf{Out_1}$ but not the change output. This makes sense, as this output captures the net-value that was intended to transfer to a third party. The WB-approach would classify the whole input of transaction $\mathsf{Tx_A}$ as money supply. This approach thus captures the amount of currency that in effectively has been moved and can be interpreted as having been available for processing transactions.

If the MC-approach is chosen, inputs are to be added to money in circulation net of change outputs. This can lead to ambiguous constellations if either an input generated within or, however, before window w might convert to the change output.

For an example, refer to transaction $\mathsf{Tx_F}$ in Figure 2. Transaction $\mathsf{Tx_F}$ has two inputs—$\mathsf{Inp_1}$, which originated before period w and $\mathsf{Inp_2}$ originating within that period. Assuming that only amounts sent to third parties are to be counted, it is unclear which of the two inputs converted to the change output. If input $\mathsf{Inp_1}$ is chosen, the transaction would not increase money in circulation at all. If, however, $\mathsf{Inp_2}$ would be converted to the change output, the transaction would trigger an increase by the value of $\mathsf{Inp_1}$.

To thwart the described ambiguities, an assignment rule between transaction inputs and outputs is required. Utilizing the terminology of cost accounting, we differentiate between "last-in-first-out" (LIFO) where oldest inputs get assigned to outputs first and "first-in-first-out" (FIFO) where it is the other way around.

Naturally, when the WB-approach is adopted, the above differentiation is unnecessary. The above, thus leads to three definitions of money in circulation depending on window length ω: Money in circulation for period w adopting the WB-approach ($M_{\mathtt{circWba}p[\alpha]}$) as well as money in circulation adopting the MC-approach assuming the rule LIFO ($M_{\mathtt{circMcaLifo}p[\alpha]}$) and adopting FIFO as assignment rule ($M_{\mathtt{circMcaFifo}p[\alpha]}$).

8.2 Defining velocity measures

Using the above definitions for money in circulation, three variations of the velocity measure conceptionalized in equation (7) can be calculated—each based on one of the money aggregates. All the measures can be interpreted as the average number of peer-to-peer coin turnovers that effectively circulating monetary units where able to achieve in period p but differ in their definitions w.r.t. *circulating monetary units* and assignment rules between transaction inputs and outputs. The first measure is based on $M_{\mathtt{circWba}p}$ and can simply be calculated as

$$V^{\mathtt{msr}}_{\mathtt{circWba}p[\alpha]} = \frac{\langle P_p, T_p \rangle}{M^{\mathtt{msr}}_{\mathtt{circWba}p[\alpha]}}. \tag{11}$$

Velocity measure $V_{\mathtt{circWba}p}$ might be favored if the data should reflect on-chain liquidity and a conservative measurement of coin turnover is to be used. Also the measure avoids additional assumptions as necessary when using the MC-approach to measure money in circulation. The second and third measure are based on $M_{\mathtt{circMcaFifo}p[\alpha]}$ and $M_{\mathtt{circMcaLifo}p[\alpha]}$ respectively. They can be calculated in a similar fashion:

$$V^{\mathtt{msr}}_{\mathtt{circMcaFifo}p[\alpha]} = \frac{\langle P_p, T_p \rangle}{M^{\mathtt{msr}}_{\mathtt{circMcaFifo}p[\alpha]}} \tag{12}$$

and

$$V^{\mathtt{msr}}_{\mathtt{circMcaLifo}p[\alpha]} = \frac{\langle P_p, T_p \rangle}{M^{\mathtt{msr}}_{\mathtt{circMcaLifo}p[\alpha]}}. \tag{13}$$

Velocity measures $V^{\mathtt{msr}}_{\mathtt{circMcaFifo}p}$ and $V^{\mathtt{msr}}_{\mathtt{circMcaLifo}p}$ might be favored if the data should be more stringent on the definition of money in circulation. Here, the monetary aggregate instead of counting "touched" transaction inputs but only the exact transaction outputs that have needed to achieve the transactions made in period p. This however comes at the cost of further assumptions.

[18] To develop an intuition, remember the example of sheep Eve being sold and re-bought by Bob within year 2019. This time Bitcoin was used and Alice held unspent outputs of 2 Bitcoin generated in transactions 5 years ago. The transaction formed by Alice to buy sheep Eve, let us call it *TX-1* spends her old output and generates a new one within year 2019 which now is controlled by Bob. Bob buys sheep Eve back and pays with the freshly generated output of 1 Bitcoin in a second transaction (*TX-2*). For $w = p = 2019$, one looks through all transactions during year 2019. The value of *TX-1* (1 Bitcoin) has been generated before 2019 and can be interpreted as money brought into circulation. *TX-2* however, used an outputs generated within 2019—and thus spent an already counted monetary unit.

8.3 Algorithmic implementation

Having described the segregation of money supply in a more general fashion, we now describe our precise technical approach and its implementation in detail.

The measurement of money in circulation adopting the WB-approach is summarized in Algorithm 1. For every period w we loop through all transactions $t \in \mathbb{T}_w$ in order to add transaction inputs to the money supply that either reference outputs from coinbase transactions, denoted by $\texttt{genByCoinbase}(i)$, or those outputs with timestamps, denoted by $\texttt{dateGen}(i)$, originating before the first timestamp $w_{\texttt{start}}$ of period w.

Adopting the MC-approach, the respective measurement of money in circulation is depicted in Algorithm 2. As in Algorithm 1, for time window w, a loop goes trough all transactions $t \in \mathbb{T}_p$ and adds transaction inputs based on the same core condition (compare Algorithm lines 2-15 and 1-6). This time, however, inputs are added step by step—only until the the full amount sent to third parties is matched. The order of inputs to add is determined using the LIFO or FIFO principle. For every transaction t in time window w, the amount sent to third parties is determined net of self-churn as $\texttt{valOut}^{\texttt{toOthers}}(t) = \sum_{o \in \mathbb{O}'_t} \texttt{valOut}(o)$ where $\mathbb{O}'_t$ denote the non-self-churn outputs of transaction t. If, now, there are only outputs identified as self-churn and thus $\texttt{valOut}^{\texttt{toOthers}}(t) = 0$, the algorithm jumps to then next transaction. If $\texttt{valOut}^{\texttt{toOthers}}(t) > 0$, the algorithm proceeds. Depending on the choice of the respective assignment rule, the input values are stored in a vector $\mathbb{I}_t^{\texttt{sort}}$; either ascending LIFO or descending FIFO w.r.t. to their timestamp of generation in a previous transaction. Then, looping through the inputs i of all the sorted transactions, input values $\texttt{valInp}(i)$ are added to the summand $M_{\texttt{circMca}p[\alpha]}(t)$ if they meet the core condition (compare line 17) already introduced in line 6 of Algorithm 1.[19]

Deviating, however, we introduce an additional condition: If the last added input would increase the summand $M_{\texttt{circMca}p[\alpha]}(t)$ so that it exceeds the value of outputs sent to third parties $\texttt{valOut}^{\texttt{toOthers}}(t)$, we only add the latter amount. This effectively adds only the necessary fraction of the inputs of transaction t, generated before window w or from coinbase transactions, that were required to match $\texttt{valOut}^{\texttt{toOthers}}(t)$. The components $M_{\texttt{circMca}p[\alpha]}(t)$ are consecutively summing up all respective transaction values to form money in circulation $M_{\texttt{circMca}p[\alpha]}$ for the given period p with respect to time window w as well as the applied sorting type.

9 Benchmarking popular proxy-variables for Bitcoin

In order to assess the goodness of fit of popular approximation techniques, we apply all discussed velocity estimators to the Bitcoin blockchain. The collected data spans a period between June 2013 until June 2019, starting with the rise of first cryptocurrency exchanges and reliable trading data around 2013.

[19] Summands $M_{\texttt{circMca}p[\alpha]}(t)$ can be interpreted as money drawn into effective circulation by transaction t.

Algorithm 1: Whole-bill-approach: Measurement $M_{\texttt{circWba}}$ within period p with look-back window w.

Data: $w_{\texttt{start}}$ /* Beginn of look-back window w */
Data: $\mathbb{T}_p$ /* Set of transactions in period p */
Result: $M_{\texttt{circWba}}$ /* Money supply to be estimated within period p */
1 $M_{\texttt{circWba}} \leftarrow 0$
2 **foreach** $t \in \mathbb{T}_p$ /* Loop through transactions t of period p */
3 **do**
4 **foreach** $i \in \mathbb{I}_t$ /* Loop through inputs i of transaction t */
5 **do**
6 **if** $\texttt{dateGen}(i) < w_{\texttt{start}}$ *or* $\texttt{genByCoinbase}(i)$ /* Check, whether i stems from a coinbase transaction generated before $w_{\texttt{start}}$ */
7 **then**
8 $M_{\texttt{circWba}} = M_{\texttt{circWba}} + \texttt{valInp}(i)$
9 **end**
10 **end**
11 **end**
12 return $M_{\texttt{circWba}}$ /* Return estimated money in circulation for WB-approach */

Algorithm 2: Moved-coin-approach: Measurement $M_{\texttt{circMca}}$ for type (FIFO, LIFO) within period p, window w.

Data: $w_{\texttt{start}}$ /* Beginn of look-back window w */
Data: $\mathbb{T}_p$ /* Set of transactions in period p */
Data: $\mathbb{O}^{\texttt{selfchurn}}$ /* Set of self-churning outputs */
Result: $M_{\texttt{circMca}}$ /* Money supply to be estimated in period p with window w */
1 $M_{\texttt{circMca}} \leftarrow 0$
2 **foreach** $t \in \mathbb{T}_p$ **do**
3 $M_{\texttt{circMca}}(t) \leftarrow 0$ /* Set summand of $M_{\texttt{circMca}}(t)$ for every transaction t */
4 $\mathbb{O}'_t \leftarrow \mathbb{O}_t \setminus O^{\texttt{selfchurn}}$ /* Determine the set of outputs of t excluding self-churning outputs */
5 $\texttt{valOut}^{\texttt{toOthers}}(t) \leftarrow \sum_{o \in \mathbb{O}'_t} \texttt{valOut}(o)$ /* Amount of money sent to third parties */
6 **if** $\texttt{valOut}^{\texttt{toOthers}}(t) = 0$ **then**
7 continue /* Skip current transaction and go to the next */
8 **end**
9 **if** *type = LIFO* **then**
10 $\mathbb{I}_t^{\texttt{sort}} \leftarrow \mathbb{I}_t$ sorted ascending w.r.t. generation time /* Here, $\mathbb{I}_t$ is the set of all inputs of t */
11 **else**
12 $\mathbb{I}_t^{\texttt{sort}} \leftarrow \mathbb{I}_t$ sorted descending w.r.t. generation time
13 **end**
14 **foreach** $i \in \mathbb{I}_t^{\texttt{sort}}$ **do**
15 **if** $\texttt{dateGen}(i) < w_{\texttt{start}}$ *or* $\texttt{genByCoinbase}(i)$ /* Check, whether i stems from a coinbase transaction generated before $w_{\texttt{start}}$ */
16 **then**
17 $M_{\texttt{circMca}}(t) \leftarrow M_{\texttt{circMca}}(t) + \texttt{valInp}(i)$
18 **if** $M_{\texttt{circMca}}(t) > \texttt{valOut}^{\texttt{toOthers}}(t)$ **then**
19 $M_{\texttt{circMca}}(t) \leftarrow \texttt{valOut}^{\texttt{toOthers}}(t)$ /* Use $\texttt{valOut}^{\texttt{toOthers}}(t)$ as upper cap for $M_{\texttt{circMca}}(t)$ */
20 **end**
21 break /* Break foreach-loop on line 14 and proceed with line 24 */
22 **end**
23 **end**
24 $M_{\texttt{circMca}} \leftarrow M_{\texttt{circMca}} + M_{\texttt{circMca}}(t)$
25 **end**
26 return $M_{\texttt{circMca}}$ /* Return estimated money in circulation for MC-approach */

We assume, that the availability of public price data marks the beginning of Bitcoin being used in a similar fashion as today.

To calculate the velocity measure based on money in circulation, we select the special case of a time window α equal to period $p = 1$ day.[20] Therefore, a *coin* is in circulation if it is transferred at least once within the day that velocity is calculated for. The daily measure thus can be interpreted as the average turnover of monetary units that are part of the daily circulating money supply. Also, for the following comparison between proxies and estimators, a choice of the opposite endpoint on the spectrum of time spans compared to [4] seems sensible.[21] However, neither the Blocksci parser nor our approaches are restricted to the above choices and can be extended to other UTXO-based cryptocurrencies and time windows.

9.1 Popular proxy-variables of the velocity of money for UTXO-based cryptocurrencies

We now turn to evaluating the approximation methods that, to our knowledge, have been applied so far.

9.1.1 Coin Days Destroyed

The measure *coin days destroyed* (CDD) was introduced in *Bitcointalk.org* in 2011.[22] For each input, CDD denote the product of the input value and the number of days since it was last spent. CDD then refers to the amount of coin days summed up over all transactions within a certain time period. This can be expressed as the sum of CDD over the transactions in the respective period p:

$$V^{\texttt{app}}_{\texttt{cdd}\,p} = \sum_{t \in \mathbb{T}_p} \texttt{cdd}(t), \tag{14}$$

where

$$\texttt{cdd}(t) = \sum_{i \in \mathbb{I}_t} \Delta(i) \cdot \texttt{valInp}(i) \tag{15}$$

with $\Delta(i)$ denoting the number of days since the respective input originated as output (or coinbase transaction) in a prior block, and $\texttt{valInp}(i)$ representing the value of input i of transaction t in period p. The measure puts much weight on the reactivation of long-dormant coins compared to ones frequently spun. This feature differs clearly from the concept of velocity.

[20] Note that we define days based on transaction block times and UTC as the timezone.

[21] [4] define money in circulation as the total coin supply, implying an infinite time window.

[22] https://bitcointalk.org/index.php?topic=6172.msg90789#msg90789

9.1.2 Turnover

[16] proposed *turnover* as proxy variable for velocity, arguing that turnover can be interpreted as "the average number of times the actively used [coins] can be expected to be spent in on-chain transactions" during a certain time period. The measure is constructed as the inverse of *dormancy*, $V^{\texttt{app}}_{\texttt{cdd}\,p}$, multiplied with the time period turnover is approximated for [cf. 16]:

$$V^{\texttt{app}}_{\texttt{turn}\,p} = \frac{1}{\texttt{dorm}_p} \cdot \delta, \tag{16}$$

with

$$\texttt{dorm}_p = \frac{V^{\texttt{app}}_{\texttt{cdd}\,p}}{\sum_{t \in \mathbb{T}_p} \sum_{i \in \mathbb{I}_t} \texttt{valInp}(i)}. \tag{17}$$

Here δ refers to the period for which an approximated turnover per coin in circulation is calculated. To illustrate the concept, assume today's transacted coins have stayed unused on average for 6 hours before their transaction. According to [16], one might expect that circulating coins are turned over $\frac{24\text{h}}{6\text{h}} = 4$ times per day on average. Turnover, however, remains an approximation depending on transactions distributed homogeneously over time.

9.2 Origin and description of approximation and measurement data

For the proxy variables, existing data sources have been tapped as far as possible. CDD data is gathered via API from *Blockwatch*[23]. For trading data, we use CoinMarketCap[24]. To implement the different velocity measures, however, access to the atomic units of cryptocurrency transactions is required. We rely on *BlockSci* introduced in [4] as an efficient, open source blockchain parser. Re-usable code and data of this paper will be publicly available after publication as well. Replicating the clustering approach of to [4], we excluded one unreasonably large cluster with over 297 million addresses. [25] Table 1 shows descriptive statistics of the approximation and measurement data. Here, $V^{\texttt{app}}_{\texttt{cdd}}$ denotes CDD in million coin days, while $V^{\texttt{app}}_{\texttt{turn}}$ denotes bitcoin *turnover* in expected on-chain coin transfers. For the two proxy-variables, the means exceed the median, suggesting outliers in the skewed distribution. According to $V^{\texttt{msr}}_{\texttt{triv}}$, monetary units of the total coin supply are turned over 0.11 times, while the more sophisticated measure $V^{\texttt{msr}}_{\texttt{total}}$ gives an average turnover of 0.04. The difference in the average times of turnover is explained by the fact that the deflated transaction volume used for $V^{\texttt{msr}}_{\texttt{total}}$ is strictly lower then the inflated one (compare Section 5). According to the measure $V^{\texttt{msr}}_{\texttt{circWba}}$, which is based on the

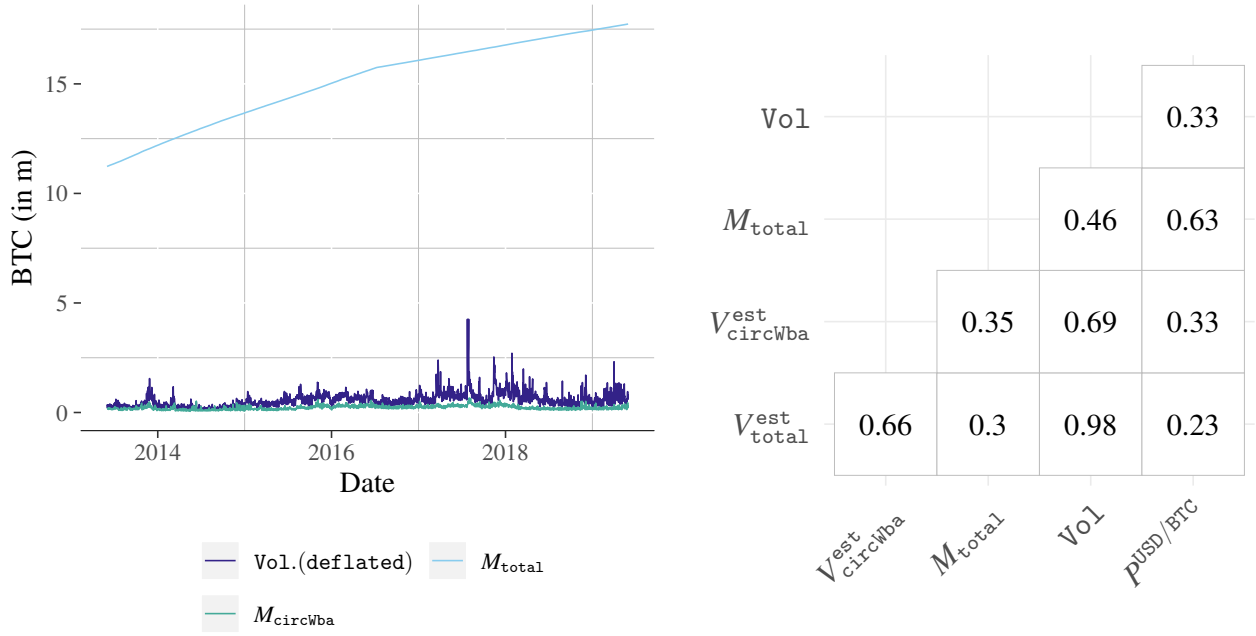
[23] https://blockwatch.cc

[24] https://coinmarketcap.com

[25] There are only 13 clusters with over 20000 addresses.

Table 1: Descriptives for approximation and measurement methods with $p = \alpha = 1$ day from 01.06.2013 to 01.06.2019.

	Obs.	Mean	Med.	Min.	Max.	Std. Dev.	Kurtosis
$V^{\text{app}}_{\text{cdd}}$	2158.00	9.81	5.73	0.87	361.99	16.27	136.68
$V^{\text{app}}_{\text{turnover}}$	2158.00	98.73	80.92	1.63	2635.37	93.91	255.95
$V^{\text{est}}_{\text{circMcaFifo}}$	2158.00	4.20	3.95	1.36	22.42	1.42	23.77
$V^{\text{est}}_{\text{circMcaLifo}}$	2158.00	4.20	3.95	1.36	22.42	1.42	23.77
$V^{\text{est}}_{\text{circWba}}$	2158.00	2.53	2.37	0.77	15.70	0.94	36.40
$V^{\text{est}}_{\text{naive}}$	2158.00	0.11	0.08	0.02	4.46	0.13	573.35
$V^{\text{est}}_{\text{total}}$	2158.00	0.04	0.03	0.01	0.46	0.02	88.55



(a) Components of velocity measures.

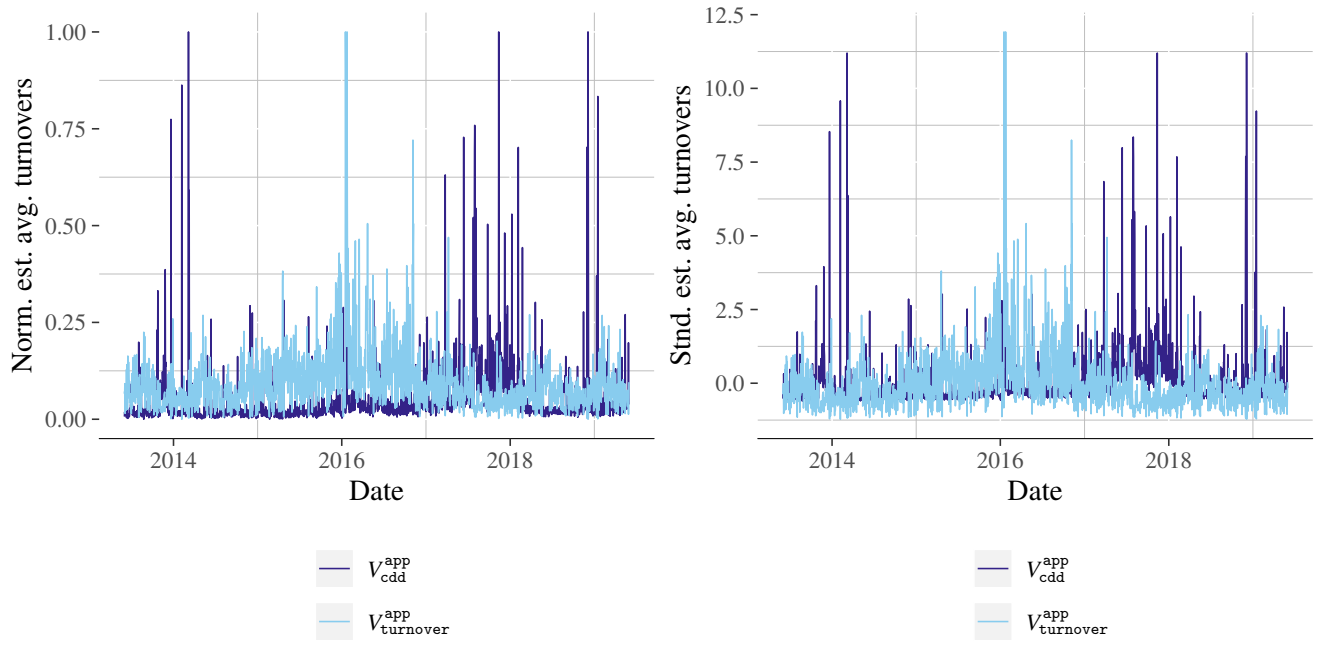
(b) Correlations between the measures and price.

Figure 3: Descriptives for approximation and measurement methods with $p = \alpha = 1$ day from 01.06.2013 to 01.06.2019.

WB-approach, coins in effective circulation during the day reach turnover of around 2.5. Assuming the clustering heuristics work well, coin transfers correspond to peer-to-peer hops. Accordingly, $V^{\text{msr}}_{\text{circWba}}$ estimates that monetary units in circulation change hands 2.5 times per day, while $V^{\text{msr}}_{\text{circMcaFifo}}$ and $V^{\text{msr}}_{\text{circMcaLifo}}$ give an estimate of around 4.2 peer-to-peer hops. The reason for the higher turnover estimate is the more conservative operationalization of the concept of *being in circulation* (compare Section 8.3).

The different levels can be disentangled by looking at the components of the velocity measures. Figure 3a exemplarily shows this for the components of $V^{\text{msr}}_{\text{total}}$ and $V^{\text{msr}}_{\text{circWba}}$. While M_{total} increases steadily over time, the subset of coins transacted at least once per day, with an average 1.5 % of the total supply, is minuscule but volatile in comparison.[26] The deflated on-chain transaction volume varies widely—always staying below the rigid total supply but above the supply in circulation. While the volatility in the transaction volume feeds fully into $V^{\text{msr}}_{\text{total}}$, for $V^{\text{msr}}_{\text{circWba}}$, the relation is less obvious. In Figure 3b, the components' co-variation with bitcoin price indicates that not only deflated on-chain transaction volume Vol, but also the monetary aggregates are positively correlated with price changes. While this provides only anecdotal evidence, it is consistent with the hypothesis that transaction volumes as well as the liquid component of the money supply rise with price, consequently lowering the correlation of the velocity measure $V^{\text{msr}}_{\text{circWba}}$ with price. This hypothesis is

[26] Compare appendix 5, 6, 7 and 8 for additional descriptive statistics on the dataset.



(a) Normalized variables of volatility approximation.

(b) Standardized variables of volatility approximation.

Figure 4: Time series plots for proxy-variables.

related to the broader question how far speculation might act as "regulator of the quantity of money" [1].

Comparing the maxima and minima, Table 1 shows that a comparison across time series requires scaling. We use two methods: normalization and standardization. Normalization is based on the usual $X^{norm} = (X - X_{min})/(X_{max} - X_{min}) \in [0,1]$. Standardization is based on Z-scores $x^{stand} = \frac{x-\mu}{\sigma}$, with mean μ and standard deviation σ. Both are affected by outliers [cf. 30], so we truncate at 10 standard deviations around the mean.[27] Figure 4 shows the time series of proxy-variables for velocity. The scaling leads to a visible difference, relevant when comparing approximation to measurement results. A first indication of the quality of the approximation variables is their diversity. Not only spikes but also general trends vary across methods. Figure 5 depicts the different velocity measures. Differences across measurements are smaller, highs and lows correspond more.

9.3 Assessing the goodness of fit of the proxy variables

We now compare and evaluate the different approximation methods. In contrast to our prior terminology, we treat the trivial velocity measure V^{msr}_{triv} as an approximation method, as its inputs (inflated, on-chain transaction volume and total coin supply) are similarly easy to gather as CDD or *turnover*.

[27] This procedure led to a maximum of 3 truncated values across time series.

In order to evaluate the performance of the different approximation methods the true velocity ought to be known. We assume here that the velocity measures suggested by [2] and [4] and the one in our paper are closer to this unknown than the different proxy-variables. As the results do not vary between the different measurement approaches, our results hold independent from choosing a certain velocity measure method as preferable.

9.3.1 Approximation errors

An intuitive approach for accessing approximation errors given by the different methods is the calculation of mean square errors and mean absolute errors. The MSE is the mean of squared deviations and thus punishes large deviations more rigorous. A large deviation is assigned higher punishment, than an equal sum of smaller deviations. The MAE, in contrast, does punish different levels of error linearly. Depeding on the application case of the approximation, both error measures might be considered. We applied the two error measures not only to the standardized and normalized time series, but also their first differences. Time series data used in econometric studies similar to the ones mentioned in Section 2 often use differenced data in order to avoid spurious regressions induced by common trends or cycles.

Table 2 shows that the above transformations differ in their assessment of the goodness of fit for a certain approximation method. When basing judgment on the normalized but oth-

erwise original dataset, the *turnover* $V^{\texttt{app}}_{\texttt{turn}}$ as defined in [16] achieves the lowest error for the approximation of the velocity measures based on an effectively circulating money supply. However, for almost all other constellations we find that the trivial measure $V^{\texttt{msr}}_{\texttt{triv}}$ provides the closest fit to all the different ways to measure velocity.

9.3.2 Model confidence set test

To understand, if the approximation methods are indeed significantly different, the MCS (model confidence set) test can be adopted. The test was developed by **(author?)** [31] for performance measurement of forecasting methods, but was proposed also for a more general setting. [31] uses equivalence tests and an elimination procedure, to determine which set of models significantly outperforms the rest of the models. For an intuitive understanding of the test one might think of a set M of models (here the different approximation methods). The models are indexed using i and $j \in 1,..,m$. When comparing the estimations of model i to true values, for each period p the model i leads to loss functions $L_{ip} = L(V^{\texttt{msr}}_p, V^{\texttt{app}}_{ip})$ where $V^{\texttt{msr}}_p$ generalizing denotes the chosen velocity measure and $V^{\texttt{app}}_{ip}$ the approximation method i.

To compare the performance of the models of set M, there are relative performance metrics d_{ijp} defined as

$$d_{ijp} = L_{ip} - L_{jp}$$

for all $i, j \in M$ where $i \neq j$. The loss function L used in the following will be just the squared errors, so that

$$d_{ijp} = \left(V^{\texttt{msr}}_p - V^{\texttt{app}}_{ip}\right)^2 - \left(V^{\texttt{msr}}_p - V^{\texttt{app}}_{jp}\right)^2.$$

The previous equations can be used, to express the relative performance of one model i in comparison to all the other models. This leads to

$$d_{i\cdot} = \frac{1}{m-1} \sum_{j \in M} d_{ij},$$

with $i = 1,...,m$. The intuition of the test is, that if the null hypothesis,

$$H_{0M} : E(d_{i\cdot}) = 0,\ \forall i \in M,$$

can be rejected for set M, then there are models in the set that are significantly outperforming the remaining ones. The above equivalence test is then followed by an elimination of the worst performing models. This two step procedure is repeated as long as the null hypothesis can be rejected. For a detailed description of the test the reader might refer to **(author?)** [31].

The results of the tests are displayed in Table 2 using the symbol † for model confidence sets at significance levels of 1 %. The results for the differenced dataset are very clear, so that the MCS tests already at the very tight 1 % significance levels are selecting a single winning approximation method for each constellation. The impression from Section 9.3.1 is confirmed at high significance levels. The measure $V^{\texttt{msr}}_{\texttt{triv}}$ is the single best element of even the 1 %-model confidence set for all constellations. For certain constellations of the raw dataset, the differences in the approximation errors are to small for the MCS-test to be able to pick a winner.[28] While for the normalized raw dataset the *turnover* $V^{\texttt{app}}_{\texttt{turn}}$ is the single best approximator with respect to measuring velocity using circulating money supply all but one of the constellations, for $V^{\texttt{msr}}_{\texttt{circWba}}$ the trivial measure is selected as member of the model confidence set as well. Is velocity measured as $V^{\texttt{msr}}_{\texttt{total}}$, however, the trivial measure $V^{\texttt{msr}}_{\texttt{triv}}$ again performs significantly better than all other proxies. For the standardized raw dataset, the differences mostly are to small to draw conclusions from the MCS-test. Summarizing, the support for the trivial measure $V^{\texttt{msr}}_{\texttt{triv}}$ is mostly confirmed by the MCS tests. Only if the application case demands trends over time and less focus on outliers than on smaller changes, *turnover* might be the better choice for approximation. Again however, $V^{\texttt{app}}_{\texttt{cdd}}$, as used in most studies is significantly outclassed in most of constellations.

9.3.3 Minzer-Zarnowitz regressions

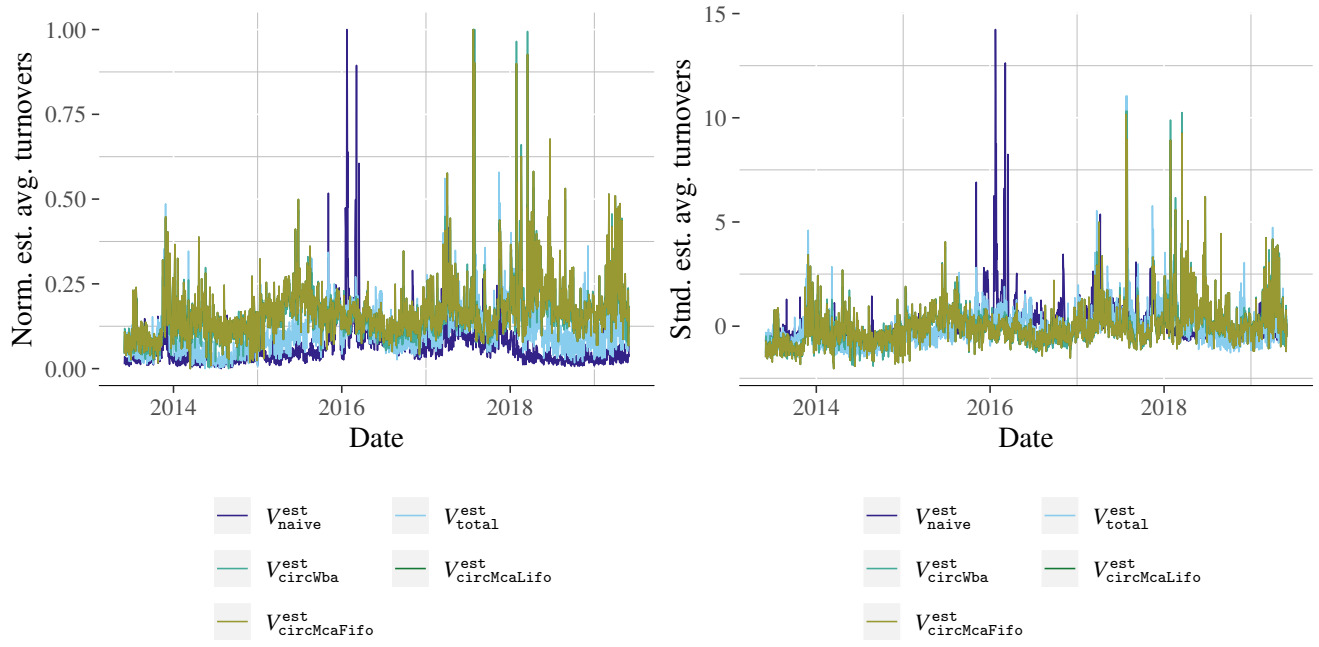
The Mincer-Zarnowitz (MZ) technique regresses estimates on true values in a simple ordinary-least-squares regression [32]. In order to avoid spurious regression results by common trends, only the first differences of standardization and normalization have been used to analyze the performance of approximation methods. The general structure for approximation method i could be expressed as:

$$\Delta V^{\texttt{msr}}_{ip} = \alpha_i + \beta_i \Delta V^{\texttt{app}}_{ip} + u_{ip}.$$

As benchmark one would expect an intercept of 0 and a β equal to 1 with an adjusted R^2 of 1 for a perfect approximation. The results of the MZ regressions in Table 3 confirm that the simple ratio of inflated on-chain transaction volume to total coin supply $V^{\texttt{msr}}_{\texttt{triv}}$ shows superior properties. While using this simple ratio leads to the adjusted R^2 above 0.04 up to 0.14, other proxy-variables show R^2 close to zero. This holds for all the different measurement methods of velocity. While in none of the approximations methods a significant intercept hints towards a bias in the approximations, the slope coefficients β for most of the constellations are significant and positive. Furthermore, with respect to the β coefficients, $V^{\texttt{msr}}_{\texttt{triv}}$ performs best. The latter shows coefficients between 0.35 to 0.51 for normalized and between 0.28 and 0.42 for standardized data and thus comes closer to 1 than the two approximation methods.

The Minzer-Zarnowitz regressions support the evidence in the prior sections. Again the trivial measure approximates

[28] While we show the results here only for the 1 % significance levels, the MCS-test for the 5 % or 10 % levels yields the same.



(a) Normalized variables of volatility measurement.

(b) Standardized variables of volatility approximation.

Figure 5: Time series plots for volatility measures with $p = \alpha = 1$ day.

the more sophisticated velocity measures better than the two approximation methods.

10 Conclusion

In this paper, we analyze approaches quantifying the velocity of money for cryptocurrencies; moreover we introduce novel measurement methods based on money in effective circulation. As a function of the time span for which money is considered *circulating effectively*, our results raise questions for future research: Can the relation of velocity to this time span be a determinant of price? What is the effect of including off-chain transactions?

In addition, we analyze goodness of fit for common velocity approximations. In most tests we find that the dominant proxy-variable, *coin days destroyed*, delivers higher approximation errors than the simple ratio of unadjusted, on-chain transaction volume to total coin supply.

On a broader scale, by publishing our code we hope to foster research on the central economic characteristics of cryptocurrencies.

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Table 2: Mean absolute error for normalized data of approximation methods compared to measurement methods with $p = \alpha =$ 1 day. Proxy-variables in 1 %-Model confidence set marked by †.

		Raw Data				First Differences			
		standardized		normalized		standardized		normalized	
Approximation	Estimator	MAE	MSE	MAE	MSE	MAE	MSE	MAE	MSE
$V^{\text{app}}_{\text{cdd}}$	$V^{\text{est}}_{\text{total}}$	2498.12	2498.12	26.74	26.74	3362.23	3362.23	23.41	23.41
$V^{\text{app}}_{\text{cdd}}$	$V^{\text{est}}_{\text{circWba}}$	† 3761.46	† 3761.46	49.96	49.96	4168.80	4168.80	29.16	29.16
$V^{\text{app}}_{\text{cdd}}$	$V^{\text{est}}_{\text{circMcaLifo}}$	† 4060.72	† 4060.72	57.06	57.06	4572.38	4572.38	31.98	31.98
$V^{\text{app}}_{\text{cdd}}$	$V^{\text{est}}_{\text{circMcaFifo}}$	† 4060.72	† 4060.72	57.06	57.06	4572.38	4572.38	31.98	31.98
$V^{\text{app}}_{\text{turnover}}$	$V^{\text{est}}_{\text{total}}$	4248.54	4248.54	27.28	27.28	3209.76	3209.76	19.27	19.27
$V^{\text{app}}_{\text{turnover}}$	$V^{\text{est}}_{\text{circWba}}$	4220.90	4220.90	† 35.09	† 35.09	3613.65	3613.65	22.37	22.37
$V^{\text{app}}_{\text{turnover}}$	$V^{\text{est}}_{\text{circMcaLifo}}$	† 4088.53	† 4088.53	† 37.45	† 37.45	3767.41	3767.41	23.52	23.52
$V^{\text{app}}_{\text{turnover}}$	$V^{\text{est}}_{\text{circMcaFifo}}$	† 4088.53	† 4088.53	† 37.45	† 37.45	3767.41	3767.41	23.52	23.52
$V^{\text{app}}_{\text{naive}}$	$V^{\text{est}}_{\text{total}}$	† 2324.34	† 2324.34	† 19.55	† 19.55	† 1406.98	† 1406.98	† 7.83	† 7.83
$V^{\text{app}}_{\text{naive}}$	$V^{\text{est}}_{\text{circWba}}$	† 3802.87	† 3802.87	† 40.28	40.28	† 2111.50	† 2111.50	† 12.50	† 12.50
$V^{\text{app}}_{\text{naive}}$	$V^{\text{est}}_{\text{circMcaLifo}}$	† 3877.53	† 3877.53	45.23	45.23	† 2374.48	† 2374.48	† 14.28	† 14.28
$V^{\text{app}}_{\text{naive}}$	$V^{\text{est}}_{\text{circMcaFifo}}$	† 3877.53	† 3877.53	45.23	45.23	† 2374.48	† 2374.48	† 14.28	† 14.28

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Table 3: Minzer-Zarnowitz regressions for standardized and normalized approximation and measurement data with $p = \alpha = 1$ day.

		Normalized						Standardized					
Approximation	Estimator	R^2_{adj}	α	p^α	β	p^β	p^{F-Test}	R^2_{adj}	α	p^α	β	p^β	p^{F-Test}
ΔV^{app}_{cdd}	ΔV^{est}_{total}	0.03	0.00	0.97	0.12	0.00^{***}	0.00^{***}	0.03	0.00	0.97	0.12	0.00^{***}	0.00^{***}
ΔV^{app}_{cdd}	$\Delta V^{est}_{circWba}$	0.01	0.00	0.97	0.06	0.00^{***}	0.00^{***}	0.01	0.00	0.97	0.06	0.00^{***}	0.00^{***}
ΔV^{app}_{cdd}	$\Delta V^{est}_{circMcaLifo}$	0.00	0.00	0.98	0.03	0.11	0.00^{***}	0.00	0.00	0.98	0.03	0.11	0.00^{***}
ΔV^{app}_{cdd}	$\Delta V^{est}_{circMcaFifo}$	0.00	0.00	0.98	0.03	0.11	0.00^{***}	0.00	0.00	0.98	0.03	0.11	0.00^{***}
$\Delta V^{app}_{turnover}$	ΔV^{est}_{total}	0.00	0.00	0.97	0.03	0.09^{*}	0.00^{***}	0.00	0.00	0.97	0.03	0.09^{*}	0.00^{***}
$\Delta V^{app}_{turnover}$	$\Delta V^{est}_{circWba}$	0.00	0.00	0.97	0.05	0.03^{**}	0.00^{***}	0.00	0.00	0.97	0.04	0.03^{**}	0.00^{***}
$\Delta V^{app}_{turnover}$	$\Delta V^{est}_{circMcaLifo}$	0.00	0.00	0.98	0.06	0.00^{***}	0.00^{***}	0.00	0.00	0.98	0.06	0.00^{***}	0.00^{***}
$\Delta V^{app}_{turnover}$	$\Delta V^{est}_{circMcaFifo}$	0.00	0.00	0.98	0.06	0.00^{***}	0.00^{***}	0.00	0.00	0.98	0.06	0.00^{***}	0.00^{***}
ΔV^{app}_{naive}	ΔV^{est}_{total}	0.14	0.00	0.97	0.51	0.00^{***}	0.00^{***}	0.14	0.00	0.97	0.42	0.00^{***}	0.00^{***}
ΔV^{app}_{naive}	$\Delta V^{est}_{circWba}$	0.05	0.00	0.98	0.37	0.00^{***}	0.00^{***}	0.05	0.00	0.98	0.30	0.00^{***}	0.00^{***}
ΔV^{app}_{naive}	$\Delta V^{est}_{circMcaLifo}$	0.04	0.00	0.99	0.35	0.00^{***}	0.00^{***}	0.04	0.00	0.99	0.28	0.00^{***}	0.00^{***}
ΔV^{app}_{naive}	$\Delta V^{est}_{circMcaFifo}$	0.04	0.00	0.99	0.35	0.00^{***}	0.00^{***}	0.04	0.00	0.99	0.28	0.00^{***}	0.00^{***}

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11 Summary of on-chain velocity measures and proxy-variables

Table 4: Summary of on-chain velocity measures and proxy-variables.

	Proxy-Variables		**Measures**				
Type			**non-segregated money supply**		**segregated money supply**		
Notation	$V^{\texttt{app}}_{\texttt{cdd}\,p}$	$V^{\texttt{app}}_{\texttt{turn}\,p}$	$V^{\texttt{msr}}_{\texttt{triv}\,p}$	$V^{\texttt{msr}}_{\texttt{total}}$	$V^{\texttt{msr}}_{\texttt{circWba}\,p[\alpha]}$	$V^{\texttt{msr}}_{\texttt{circMcaLifo}\,p[\alpha]}$	$V^{\texttt{msr}}_{\texttt{circMcaFifo}\,p[\alpha]}$
Description	Coin days destroyed	Turnover	Trivial velocity measure	Velocity measure based on total money supply	Based on money in effective circulation using the Whole Bill Approach	Based on money in effective circulation using the Moved Coin Approach (FIFO)	Based on money in effective circulation using the Moved Coin Approach (LIFO)
Formula	$\sum_{t\in\mathbb{T}_p}\texttt{cdd}(t)$	$\frac{1}{\texttt{dorm}_p}\cdot\delta$	$\frac{\langle P_p{}',T_p{}'\rangle}{M_{\texttt{total}\,p}}$	$\frac{\langle P_p,T_p\rangle}{M_{\texttt{total}\,p}}$	$\frac{\langle P_p,T_p\rangle}{M^{\texttt{msr}}_{\texttt{circWba}\,p[\alpha]}}$	$\frac{\langle P_p,T_p\rangle}{M^{\texttt{msr}}_{\texttt{circMcaLifo}\,p[\alpha]}}$	$\frac{\langle P_p,T_p\rangle}{M^{\texttt{msr}}_{\texttt{circMcaFifo}\,p[\alpha]}}$
Related Section	9.1.1	9.1.2	6	6	8.2	8.2	8.2
Sources	*Bitcointalk.org*, 2011	[16]	[3]	[2, 4]	novel	novel	novel
Considered in	[8, 9, 10, 11, 12, 13, 14]	[16]	[3]	[2, 4]			

t	...	an individual transaction
$\mathbb{T}_p$	...	the set of all transactions in period p
$\texttt{cdd}(t)$	...	coin days destroyed for transaction t as defined in Section 9.1
$\texttt{dorm}_p$	...	dormancy in period p as defined in Section 9.1.2
δ	...	the period for which an approximated turnover per coin in circulation is calculated
$\langle P_p{}', T_p{}'\rangle$	...	the inflated price-sum
$\langle P_p, T_p\rangle$	...	the deflated price-sum
$M_{\texttt{total}\,p}$	...	the complete, ever mined coin supply
$M^{\texttt{msr}}_{\texttt{circWba}\,p[\alpha]}$	...	money in effective circulation using the Whole Bill Approach
$M^{\texttt{msr}}_{\texttt{circMcaFifo}\,p[\alpha]}$	...	money in effective circulation using the Moved Coin Approach adopting first-in-first-out assignment between inputs and outputs
$M^{\texttt{msr}}_{\texttt{circMcaLifo}\,p[\alpha]}$	...	money in effective circulation using the Moved Coin Approach adopting last-in-first-out assignment between inputs and outputs

12 Descriptives of the entirety of raw data

Table 5: Basic descriptive statistics based on Bitcoin data from 01.06.2013 to 01.06.2019 with $p = \alpha = 1$ day. Variables marked by "*" are scaled down by factor 1.000.000.

	Obs.	Mean	Med.	Min.	Max.	Std. Dev.	Kurtosis
$M_{\texttt{circMcaFifo}}$	2158.00	0.14	0.13	0.03	0.52	0.06	2.88
$M_{\texttt{circMcaLifo}}$	2158.00	0.14	0.13	0.03	0.52	0.06	2.88
$M_{\texttt{circWba}}$	2158.00	0.23	0.22	0.06	0.66	0.09	0.75
$M_{\texttt{total}}$	2158.00	15.11	15.62	11.23	17.73	1.88	-1.04
$P^{\texttt{USD/BTC}}$	2158.00	2514.02	644.87	67.81	19535.70	3384.55	3.25
$\texttt{Vol.(deflated)}$	2158.00	0.58	0.52	0.10	4.26	0.33	18.10
$\texttt{Vol.(inflated)}$	2158.00	1.65	1.28	0.26	21.91	1.45	60.17
$V^{\texttt{app}}_{\texttt{cdd}}$	2158.00	9.70	5.73	0.87	172.48	14.54	52.74
$V^{\texttt{app}}_{\texttt{turnover}}$	2158.00	97.98	80.92	1.63	1037.82	78.89	30.09
$V^{\texttt{est}}_{\texttt{circMcaFifo}}$	2158.00	4.20	3.95	1.36	18.39	1.40	17.00
$V^{\texttt{est}}_{\texttt{circMcaLifo}}$	2158.00	4.20	3.95	1.36	18.39	1.40	17.00
$V^{\texttt{est}}_{\texttt{circWba}}$	2158.00	2.53	2.37	0.77	11.96	0.91	22.64
$V^{\texttt{est}}_{\texttt{naive}}$	2158.00	0.11	0.08	0.02	1.43	0.09	66.38
$V^{\texttt{est}}_{\texttt{total}}$	2158.00	0.04	0.03	0.01	0.26	0.02	18.45

13 Descriptives of the normalized proxy-variables and measurements

Table 6: Basic descriptive statistics based on normalized velocity measurement and approximation data from 01.06.2013 to 01.06.2019 with $p = \alpha = 1$ day.

	Obs.	Mean	Med.	Min.	Max.	Std. Dev.	Kurtosis
$V^{\texttt{app}}_{\texttt{cdd}}$	2158.00	0.02	0.01	0.00	1.00	0.05	136.68
$V^{\texttt{app}}_{\texttt{turnover}}$	2158.00	0.04	0.03	0.00	1.00	0.04	255.95
$V^{\texttt{est}}_{\texttt{circMcaFifo}}$	2158.00	0.14	0.12	0.00	1.00	0.07	23.77
$V^{\texttt{est}}_{\texttt{circMcaLifo}}$	2158.00	0.14	0.12	0.00	1.00	0.07	23.77
$V^{\texttt{est}}_{\texttt{circWba}}$	2158.00	0.12	0.11	0.00	1.00	0.06	36.40
$V^{\texttt{est}}_{\texttt{naive}}$	2158.00	0.02	0.01	0.00	1.00	0.03	573.35
$V^{\texttt{est}}_{\texttt{total}}$	2158.00	0.07	0.06	0.00	1.00	0.05	88.55

14 Descriptives of the standardized proxy-variables and measurements

Table 7: Basic descriptive statistics based on standardized velocity measurement and approximation data from 01.06.2013 to 01.06.2019 with $p = \alpha = 1$ day.

	Obs.	Mean	Med.	Min.	Max.	Std. Dev.	Kurtosis
$V^{\texttt{app}}_{\texttt{cdd}}$	2158.00	-0.00	-0.27	-0.61	11.20	1.00	52.74
$V^{\texttt{app}}_{\texttt{turnover}}$	2158.00	-0.00	-0.22	-1.22	11.91	1.00	30.09
$V^{\texttt{est}}_{\texttt{circMcaFifo}}$	2158.00	-0.00	-0.18	-2.04	10.16	1.00	17.00
$V^{\texttt{est}}_{\texttt{circMcaLifo}}$	2158.00	-0.00	-0.18	-2.04	10.16	1.00	17.00
$V^{\texttt{est}}_{\texttt{circWba}}$	2158.00	0.00	-0.18	-1.92	10.32	1.00	22.64
$V^{\texttt{est}}_{\texttt{naive}}$	2158.00	0.00	-0.24	-0.94	14.23	1.00	66.38
$V^{\texttt{est}}_{\texttt{total}}$	2158.00	-0.00	-0.18	-1.49	11.05	1.00	18.45

15 Correlations of the entirety of raw data

Table 8: Correlations based on Bitcoin data from 01.06.2013 to 01.06.2019 with $p = \alpha = 1$ day

	$V^{\texttt{est}}_{\texttt{naive}}$	$V^{\texttt{est}}_{\texttt{total}}$	$V^{\texttt{est}}_{\texttt{circWba}}$	$V^{\texttt{est}}_{\texttt{circMcaLifo}}$	$V^{\texttt{est}}_{\texttt{circMcaFifo}}$	$V^{\texttt{app}}_{\texttt{cdd}}$	$V^{\texttt{app}}_{\texttt{turnover}}$	$V^{\texttt{app}}_{\texttt{naive}}$	$M_{\texttt{total}}$	$M_{\texttt{circMcaLifo}}$	$M_{\texttt{circMcaFifo}}$	$M_{\texttt{circWba}}$	$\texttt{Vol.(inflated)}$	$\texttt{Vol.(deflated)}$	$P^{\texttt{USD/BTC}}$
$V^{\texttt{est}}_{\texttt{naive}}$	1	0.46	0.12	0.1	0.1	0.19	0.54	1	0.15	0.47	0.47	0.51	0.99	0.44	-0.07
$V^{\texttt{est}}_{\texttt{total}}$	0.46	1	0.66	0.57	0.57	0.42	0.02	0.46	0.3	0.74	0.74	0.72	0.48	0.98	0.23
$V^{\texttt{est}}_{\texttt{circWba}}$	0.12	0.66	1	0.94	0.94	0.13	0.02	0.12	0.35	0.12	0.12	0.05	0.15	0.69	0.33
$V^{\texttt{est}}_{\texttt{circMcaLifo}}$	0.1	0.57	0.94	1	1	0.06	0.05	0.1	0.31	-0.03	-0.03	-0.02	0.13	0.6	0.3
$V^{\texttt{est}}_{\texttt{circMcaFifo}}$	0.1	0.57	0.94	1	1	0.06	0.05	0.1	0.31	-0.03	-0.03	-0.02	0.13	0.6	0.3
$V^{\texttt{app}}_{\texttt{cdd}}$	0.19	0.42	0.13	0.06	0.06	1	-0.3	0.19	0.17	0.48	0.48	0.44	0.21	0.43	0.17
$V^{\texttt{app}}_{\texttt{turnover}}$	0.54	0.02	0.02	0.05	0.05	-0.3	1	0.54	0	-0.01	-0.01	0.02	0.53	0	-0.24
$V^{\texttt{app}}_{\texttt{naive}}$	1	0.46	0.12	0.1	0.1	0.19	0.54	1	0.15	0.47	0.47	0.51	0.99	0.44	-0.07
$M_{\texttt{total}}$	0.15	0.3	0.35	0.31	0.31	0.17	0	0.15	1	0.37	0.37	0.38	0.24	0.46	0.63
$M_{\texttt{circMcaLifo}}$	0.47	0.74	0.12	-0.03	-0.03	0.48	-0.01	0.47	0.37	1	1	0.95	0.5	0.74	0.2
$M_{\texttt{circMcaFifo}}$	0.47	0.74	0.12	-0.03	-0.03	0.48	-0.01	0.47	0.37	1	1	0.95	0.5	0.74	0.2
$M_{\texttt{circWba}}$	0.51	0.72	0.05	-0.02	-0.02	0.44	0.02	0.51	0.38	0.95	0.95	1	0.54	0.72	0.18
$\texttt{Vol.(inflated)}$	0.99	0.48	0.15	0.13	0.13	0.21	0.53	0.99	0.24	0.5	0.5	0.54	1	0.48	0
$\texttt{Vol.(deflated)}$	0.44	0.98	0.69	0.6	0.6	0.43	0	0.44	0.46	0.74	0.74	0.72	0.48	1	0.33
$P^{\texttt{USD/BTC}}$	-0.07	0.23	0.33	0.3	0.3	0.17	-0.24	-0.07	0.63	0.2	0.2	0.18	0	0.33	1