1.2.4 He prefers X to Y $\Leftrightarrow E(u(X)) > E(u(Y))$

$$\Leftrightarrow 0.5\sqrt{400} + 0.5\sqrt{900} > 0.6\sqrt{100} + 0.4\sqrt{1600}$$

$$\Leftrightarrow 0.5\times 20 + 0.5\times 30 > 0.6\times 10 + 0.4\times 40$$

$$\Leftrightarrow 10 + 15 > 6 + 16$$

$$\Leftrightarrow 25 > 22$$

He declines the offer $\Leftrightarrow E(u(X)) < u(w)$

$$\Leftrightarrow 25 < \sqrt{w}$$

$$\Leftrightarrow w > 625$$

If u is linear ($u(w) = w$), then

$$E(u(X)) = 0.5\times 400 + 0.5\times 900 = 650$$

$$E(u(Y)) = 0.6\times 100 + 0.4\times 1600 = 700$$

so $E(u(X)) < E(u(Y))$ i.e. he prefers Y

1.2.8 No mention of capital $\rightarrow$ assume $w = 0$

We select u / $u(0) = 0$ and $u(1) = 1$

We assume probability of winning a round is 0.5

The 1^{st} choice means that $E(u(2)) > u(1)$

$$\Rightarrow 0.5\,\underset{0}{\underset{\parallel}{u(0)}} + 0.5\,u(2) > \underset{1}{\underset{\parallel}{u(1)}}$$

$$\Rightarrow u(2) > 2$$

The 2^{nd} choice means that $E(u(4)) = u(2)$

$$\Rightarrow 0.5\,u(0) + 0.5\,u(4) = u(2)$$

$$\Rightarrow u(4) = 2\,u(2)$$

The 3^{rd} choice means that $E(u(8)) < u(4)$

$$\Rightarrow 0.5\,u(0) + 0.5\,u(8) < u(4)$$

$$\Rightarrow u(8) < 2\,u(4)$$

1.3.6 With exponential utility, $P(\alpha) = \frac{1}{\alpha} \log(m_X(\alpha))$

For $X \sim N(\mu, \sigma^2)$, $m_X(\alpha) = e^{\mu\alpha + \frac{\sigma^2\alpha^2}{2}}$

i.e $P(\alpha) = \frac{1}{\alpha}\left(\mu\alpha + \frac{\sigma^2\alpha^2}{2}\right) = \mu + \frac{\sigma^2\alpha}{2}$

$X \sim N(400, 25\,000)$, $Y \sim N(420, 20\,000)$, $\alpha = 0.001$

$P_X(\alpha) = 400 + \frac{25\,000 \times 0.001}{2} = 400 + 12.5 = 412.5$

$P_Y(\alpha) = 420 + \frac{20\,000 \times 0.001}{2} = 420 + 10 = 430$

The premium for Y is higher.

$P_X(\alpha) > P_Y(\alpha) \iff 400 + \frac{25\,000\alpha}{2} > 420 + \frac{20\,000\alpha}{2}$

$\iff 12\,500\alpha > 20 + 10\,000\alpha$

$\iff 2500\alpha > 20$

$\iff 125\alpha > 1$

$\iff \alpha > \frac{1}{125} = 0.008$

The premium for X is higher when $\alpha > 0.008$

1.3.8 From 1.3.6, $P(\alpha) = \mu + \frac{\sigma^2 \alpha}{2}$ for exponential family

For a risk with distribution $N(1000, 100^2)$, the premium is

$$P(\alpha) = 1000 + \frac{100^2 \alpha}{2}.$$

$$P(\alpha) \geq 1250 \iff \frac{100^2 \alpha}{2} \geq 250$$

$$\iff 5000\alpha \geq 250$$

$$\iff \alpha \geq \frac{50}{1000} = 0.05$$

If $\dim(X) = \text{money}$, then $\dim(\mu_X) = \text{money} = \dim(\sigma_X^2)$.

$P_X(\alpha)$ is a premium, so $\dim P_X(\alpha) = \text{money}$.

But $P_X(\alpha) = \mu_X + \frac{\sigma_X^2 \alpha}{2}$, so $\alpha = (P_X(\alpha) - \mu_X) \times \frac{2}{\sigma_X^2}$

i.e. $\dim(\alpha) = \frac{\text{money}}{\text{money}^2} = \text{money}^{-1}$