Group exercise 1

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Task 1 – Proof of 2.2 a

We are to prove that

$$\mathrm{E}\big[u(w-X)\big] \leq \mathrm{E}\big[u(w-Y)\big] \iff \mathrm{E}\big[b+a\cdot u(w-X)\big] \leq \mathrm{E}\big[b+a\cdot u(w-Y)\big], \tag{1}$$

for $a \in \mathbb{R}^+$ and $b \in \mathbb{R}$. Here, $\mathbb{R}^+ = (0, \infty)$. We begin to prove '$\Longrightarrow$':

$$\begin{aligned}
&\mathrm{E}\big[u(w-X)\big] \leq \mathrm{E}\big[u(w-Y)\big] \overset{a>0}{\Longrightarrow} a\cdot\mathrm{E}\big[u(w-X)\big] \leq a\cdot\mathrm{E}\big[u(w-Y)\big] \\
&\Longrightarrow \mathrm{E}\big[b\big] + a\cdot\mathrm{E}\big[u(w-X)\big] \leq \mathrm{E}\big[b\big] + a\cdot\mathrm{E}\big[u(w-Y)\big] \\
&\Longrightarrow \mathrm{E}\big[b+a\cdot u(w-X)\big] \leq \mathrm{E}\big[b+a\cdot u(w-Y)\big]\square
\end{aligned} \tag{2}$$

We now prove '$\Longleftarrow$':

$$\begin{aligned}
&\mathrm{E}\big[b+a\cdot u(w-X)\big] \leq \mathrm{E}\big[b+a\cdot u(w-Y)\big] \overset{\text{lin op}}{\Longrightarrow} \mathrm{E}\big[b\big] + \mathrm{E}\big[a\cdot u(w-X)\big] \leq \mathrm{E}\big[b\big] + \mathrm{E}\big[a\cdot u(w-Y)\big] \\
&\Longrightarrow \mathrm{E}\big[a\cdot u(w-X)\big] \leq \mathrm{E}\big[a\cdot u(w-Y)\big] \Longrightarrow a\cdot\mathrm{E}\big[u(w-X)\big] \leq a\cdot\mathrm{E}\big[u(w-Y)\big] \\
&\overset{a>0}{\Longrightarrow} \mathrm{E}\big[u(w-X)\big] \leq \mathrm{E}\big[u(w-Y)\big]\square
\end{aligned} \tag{3}$$

We have now proved that

$$\mathrm{E}\big[u(w-X)\big] \leq \mathrm{E}\big[u(w-Y)\big] \iff \mathrm{E}\big[b+a\cdot u(w-X)\big] \leq \mathrm{E}\big[b+a\cdot u(w-Y)\big]. \tag{4}$$

Task 2 – Exercise 1.2.4

Let $u(x) = \sqrt{x},\ x \geq 0 \Rightarrow u(w-Z) = \sqrt{w-Z},\ w \geq Z$. This person will prefer X to Y if the expected utility evaluated at X is larger than at Y. So, we need to show that

$$\mathrm{E}\big[u(X)\big] > \mathrm{E}\big[u(Y)\big] \Rightarrow \mathrm{E}\big[\sqrt{X}\big] > \mathrm{E}\big[\sqrt{Y}\big] \Rightarrow \sum P(X=x)\cdot\sqrt{x} > \sum P(X=y)\cdot\sqrt{y} \tag{5}$$

This is straight forward as we know the PMFs for X and Y:

$$\begin{aligned}
&\sum P(X=x)\cdot\sqrt{x} > \sum P(X=y)\cdot\sqrt{y} \Rightarrow \frac{1}{2}\Big[20+30\Big] > \frac{1}{5}\Big[30+80\Big] \\
&\Rightarrow 25 \geq 22 \Rightarrow \text{prefer X to Y}
\end{aligned}$$

He is risk-averse because $u''(x) < 0$. A risk-averse person that now have determined that he want to go for X rather than Y, would decline for values of w where facing X is better than facing $\mathrm{E}\big[X\big]$;

$$25 = \mathrm{E}\big[u(X)\big] > u\Big(\mathrm{E}\big[X\big]\Big) = \sqrt{w} \Rightarrow w < 625$$

For example, a linear utility function would results in this person preffering Y over X.

Task 3 – Exercise 1.2.8

Nothing about current wealth are stated in the text, so we assume that $w = 0$ for simplicity. After winning the first time he win 1. Then the utility function is $u(1)$. Assuming that the the chance of winning is 0.5, we can see that the expected utility playing a second round is $\mathrm{E}\big[u(2)\big]$. The choice of yes, signify that

$$\mathrm{E}\big[u(2)\big] > u(1) \Rightarrow \frac{1}{2}(u(0) + u(2)) > 1 \Rightarrow u(2) > 2 \tag{6}$$

If he plays a third round, his expected utility is $\mathrm{E}\big[u(4)\big]$. Saying yes to this, as he does, entails that

$$\mathrm{E}\big[u(4)\big] = u(2) \Rightarrow \frac{1}{2}(u(0) + u(4)) = u(2) \Rightarrow u(4) = 2u(2) > 4 \tag{7}$$