Finite difference method
Heat equation:
$$Ut = Uxx$$

$$\frac{\mathcal{U}_{i,t+1} - \mathcal{U}_{i,t}}{\Delta t} = \frac{\frac{\mathcal{U}_{i+1,t} - \mathcal{U}_{i,t}}{\Delta x} - \frac{\mathcal{U}_{i,t} - \mathcal{U}_{i-1,t}}{\Delta x}}{\Delta x}$$

$$\frac{U_{i,t+1} - U_{i,t} - U_{i,t} - U_{i,t} + U_{i-1,t}}{\Delta x} = \frac{U_{i,t+1} - U_{i,t} - U_{i,t} - U_{i,t}}{\Delta x}$$

$$\frac{U_{i,t+1}-U_{i,t}}{\Delta t} = \frac{U_{i+1,t}-2U_{i,t}+U_{i-1,t}}{(\Delta x)^2}$$

$$\frac{U_{i_{1}+1}-U_{i, t}}{\Delta t} = \frac{U_{i_{1}, t+1}-U_{i_{1}, t+1}}{\Delta x} - \frac{U_{i_{1}, t+1}-U_{i_{2}, t+1}}{\Delta x}$$

$$= \frac{U_{i+1, t+1}-U_{i_{1}, t+1}-U_{i_{1}, t+1}+U_{i_{2}, t+1}}{(\Delta x)^{2}}$$

$$= \frac{U_{i+1, t+1}-2U_{i_{1}, t+1}+U_{i_{2}, t+1}}{(\Delta x)^{2}}$$

$$\Delta t$$
 $(\Delta x)^{-}$

$$\frac{\mathcal{U}_{i,t+1} - \mathcal{U}_{i,t}}{\Delta t} = \frac{1}{2} \left(\frac{\mathcal{U}_{i+1,t+1} - 2\mathcal{U}_{i,t+1} + \mathcal{U}_{i-1,t+1}}{(\Delta X)^2} + \frac{\mathcal{U}_{i+1,t} - 2\mathcal{U}_{i,t} + \mathcal{U}_{i-1,t}}{(\Delta X)^2} \right)$$

define
$$K = \frac{\Delta t}{(\Delta x)^2}$$

$$(2+2k)$$
 $U_{i,k+1}$ - K $U_{i-1,k+1}$ - K $U_{i+1,k+1}$ = $(2-2k)$ $U_{i,k}$ + K $U_{i-1,k}$ + K $U_{i+1,k}$ This is a linear dynamical system of equations:

$$\begin{bmatrix}
2 + 2k - k & 0 \\
- k & 2 + 2k - k
\end{bmatrix}
\begin{bmatrix}
u_1 + n \\
v_2 + n
\end{bmatrix} =
\begin{bmatrix}
2 - 2k & k & 0 \\
k & 2 - 2k & k
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\begin{bmatrix}
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$$A U_{t+1} = B U_t$$

where
$$6 = 1 \left(\begin{array}{c} u_0^t + u_0^{t+1} \\ u_N + u_N \end{array} \right)$$

$$(2+2k) \mathcal{U}_{i,k+1} - \mathcal{K} \mathcal{U}_{i-1,k+1} - \mathcal{K} \mathcal{U}_{i+1,k+1} = (2-2k) \mathcal{U}_{i,k} + \mathcal{K} \mathcal{U}_{i-1,k} + \mathcal{K} \mathcal{U}_{i+1,k} + \mathcal{K} \mathcal{U}_{i+1,k}$$

$$(2+2k)U_{1},t+1-kU_{0},t+1-kU_{2},t+1=(2-2k)U_{1},t+kU_{0},t+kU_{2},t$$

$$(2+2k)U_{1,k+1} - ku_{2,k+1} = (2-2k)u_{1,t} + ku_{2,t} + (ku_{0,t} + ku_{0,k+1})$$

the same for the bast interior i = N-1