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Title : Rubik’s Cube – An Application of Group theory

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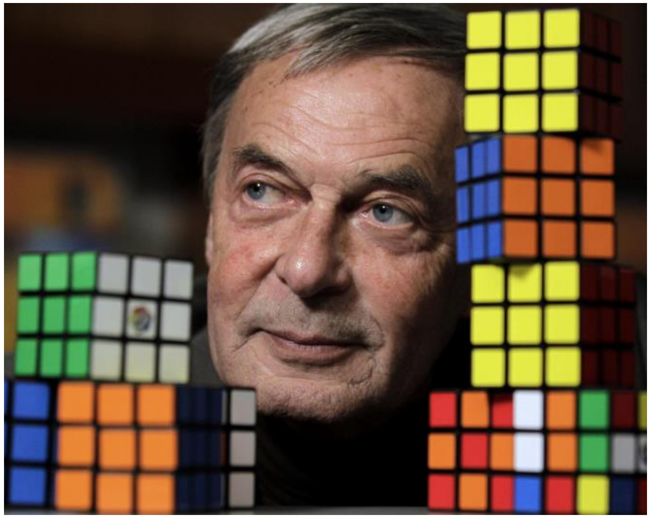
Introduction

- From 1971 to 1979, Rubik was a professor of architecture at the Budapest College of Applied Arts (Iparm˝uvészeti F˝oiskola).
- It was during his time there that he built designs for a three-dimensional puzzle and completed the first working prototype of the Rubik’s Cube in 1974, applying for a patent on the puzzle in 1975.
- Today Rubik’s Cube is more popular and speed cubing competitions are held through the World Cube Association where different types of cube to solve and participant attempt to solve the cube as fast as possible



WCA and PUZZLES

Figure : WCA (World Cube Association)

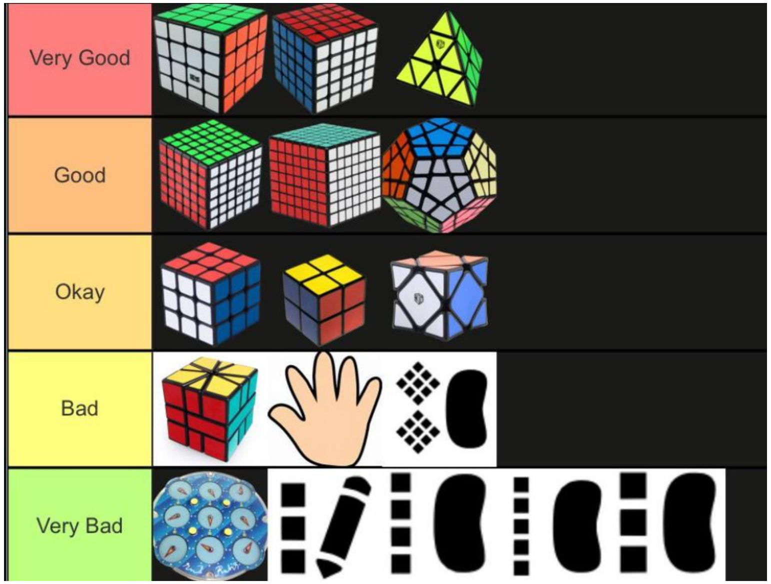


Figure : All WCA puzzles

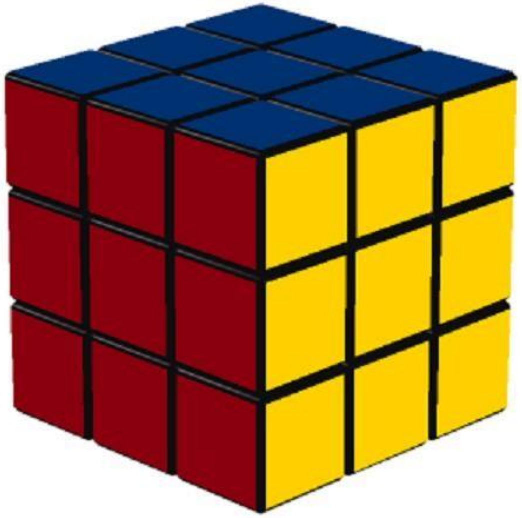
Singmaster Notation

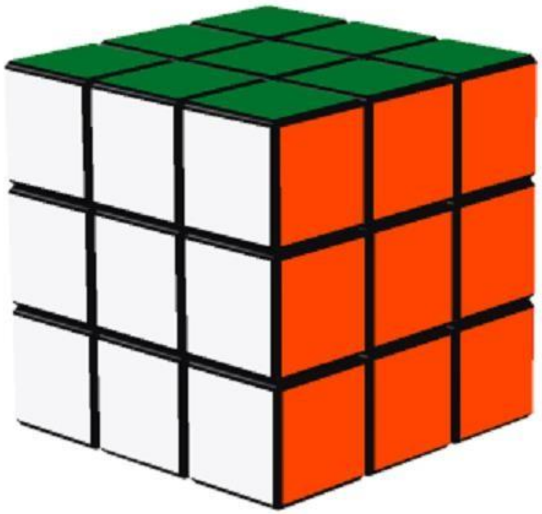
Many 3×3×3 Rubik's Cube enthusiasts use a notation developed by David Singmaster to denote a sequence of moves, referred to as "Singmaster notation" .

- F (Front): the side currently facing the solver
- B (Back): the side opposite the front
- U (Up): the side above or on top of the front side
- D (Down): the side opposite the top, underneath the Cube
- L (Left): the side directly to the left of the front
- R (Right): the side directly to the right of the front
- x (rotate): rotate the entire Cube on R
- y (rotate): rotate the entire Cube on U
- z (rotate): rotate the entire Cube on F

Rubik's Cube

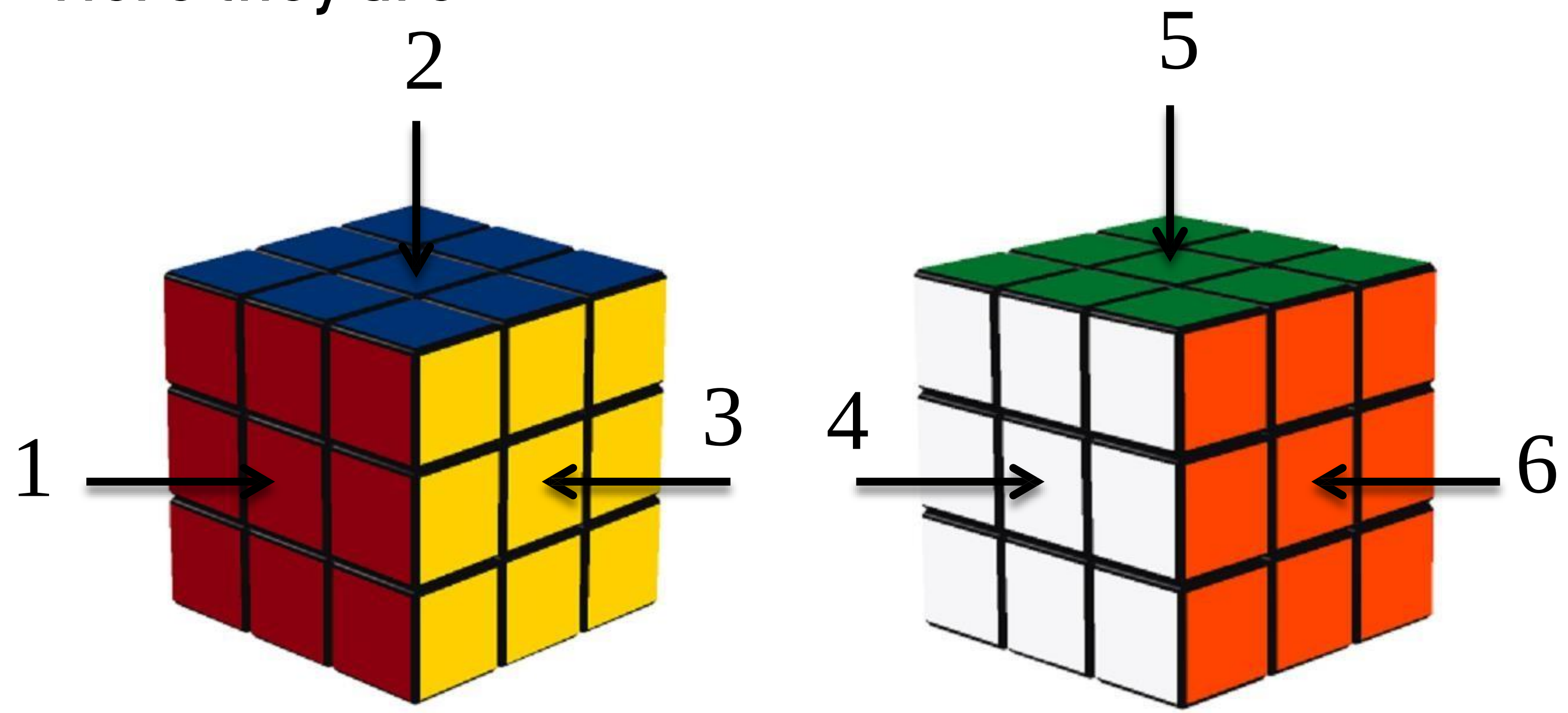
- There are six center cubies





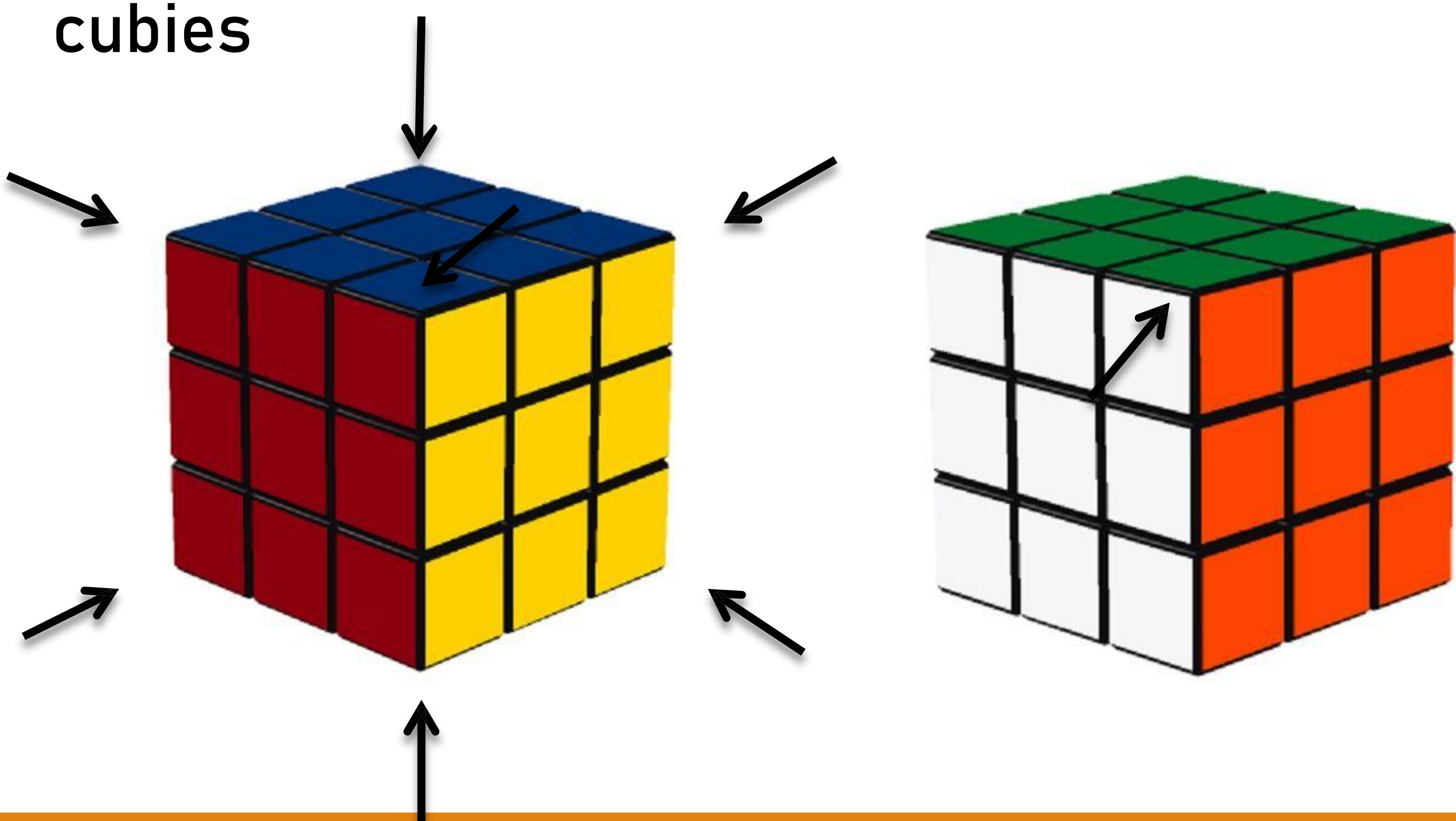
Rubik’s Cube

- Here they are



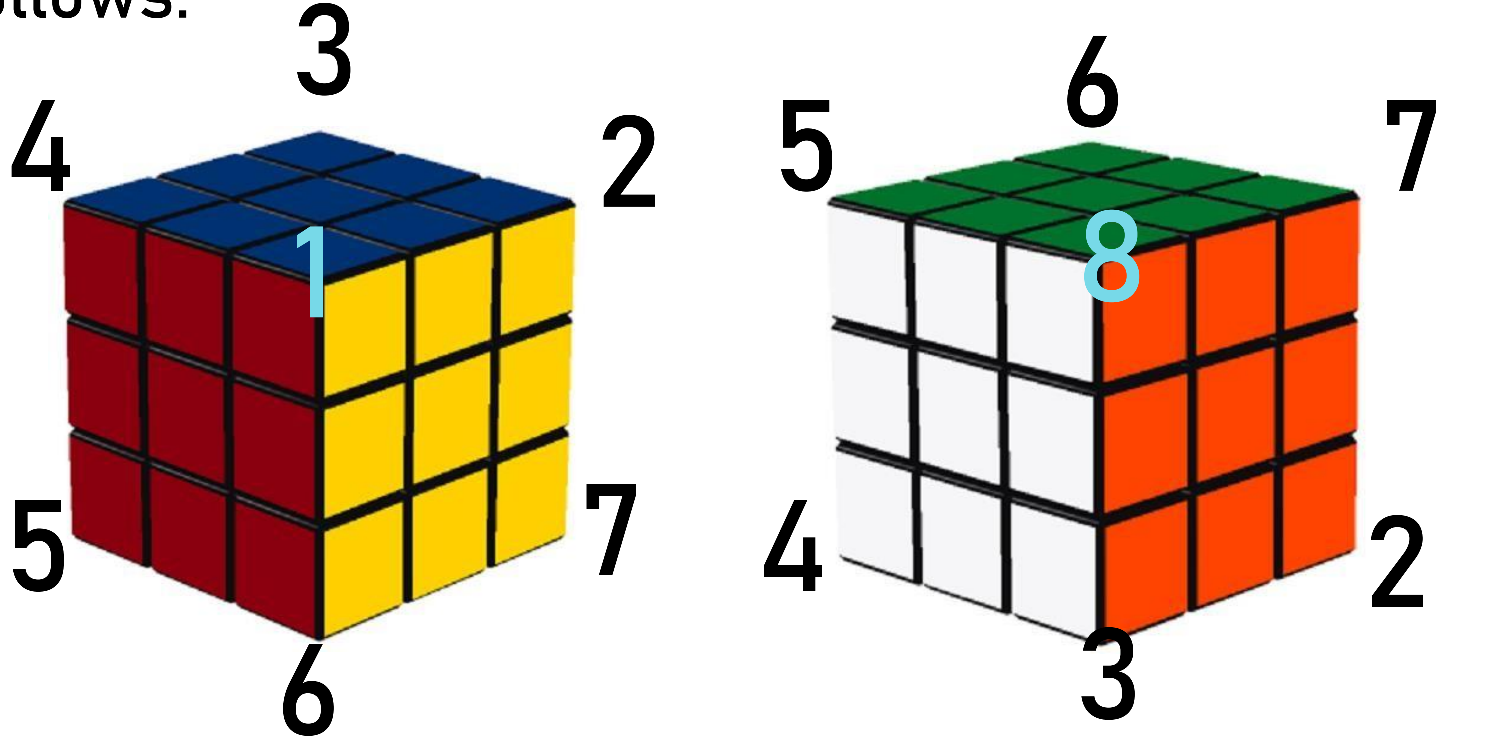
Rubik's Cube

- There are 8 corner cubies



Rubik's Cube

- We number the corner cubies as follows:



Rubik's Cube

- There are 8! ways to rearrange corner cubies.

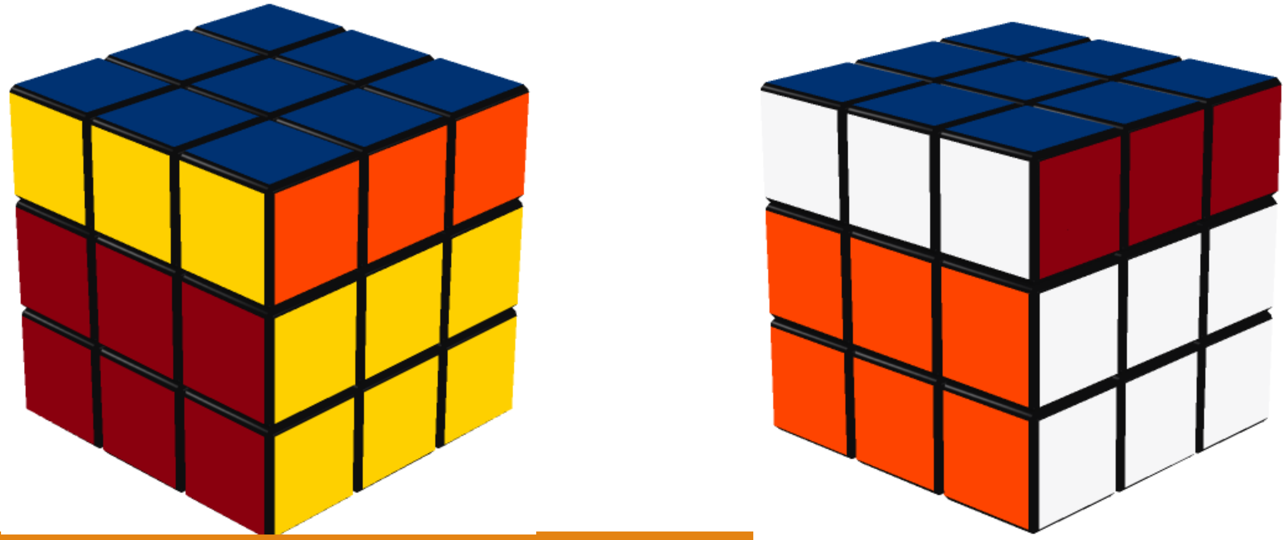
- Each corresponds to a permutation of the form:

    - (1, 2, 3, 4, 5, 6, 7, 8)

Rubik’s Cube

The images below correspond to an upper or U twist on the Rubik’s cube The U twist permutation has the form:

$$(1, 2, 3, 4, 5, 6, 7, 8)$$
$$\downarrow$$
$$(2, 3, 4, 1, 5, 6, 7, 8)$$



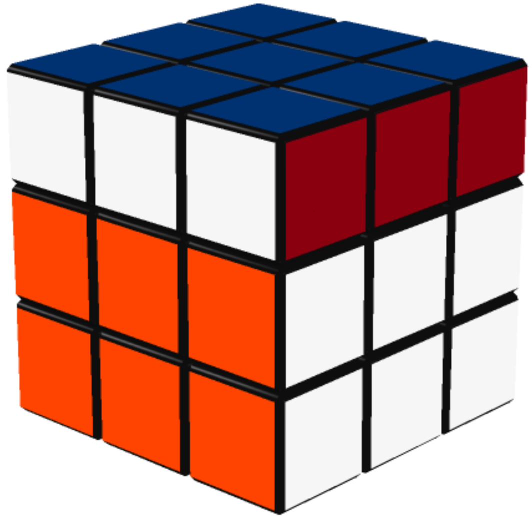
Rubik's Cube

We then take the permutation

$$(2, 3, 4, 1, 5, 6, 7, 8)$$

and express it in the cyclic form:

$$\sigma = (1\ 4\ 3\ 2)\ (5)\ (6)\ (7)\ (8) = (1\ 4\ 3\ 2)$$



Rubik’s Cube

- As a result of function composition:
  - $(1\ 4\ 3\ 2)=(14)(13)(12)$
- We will find later that for every n, its n-cycle can be written as either an even or an odd number of permutations

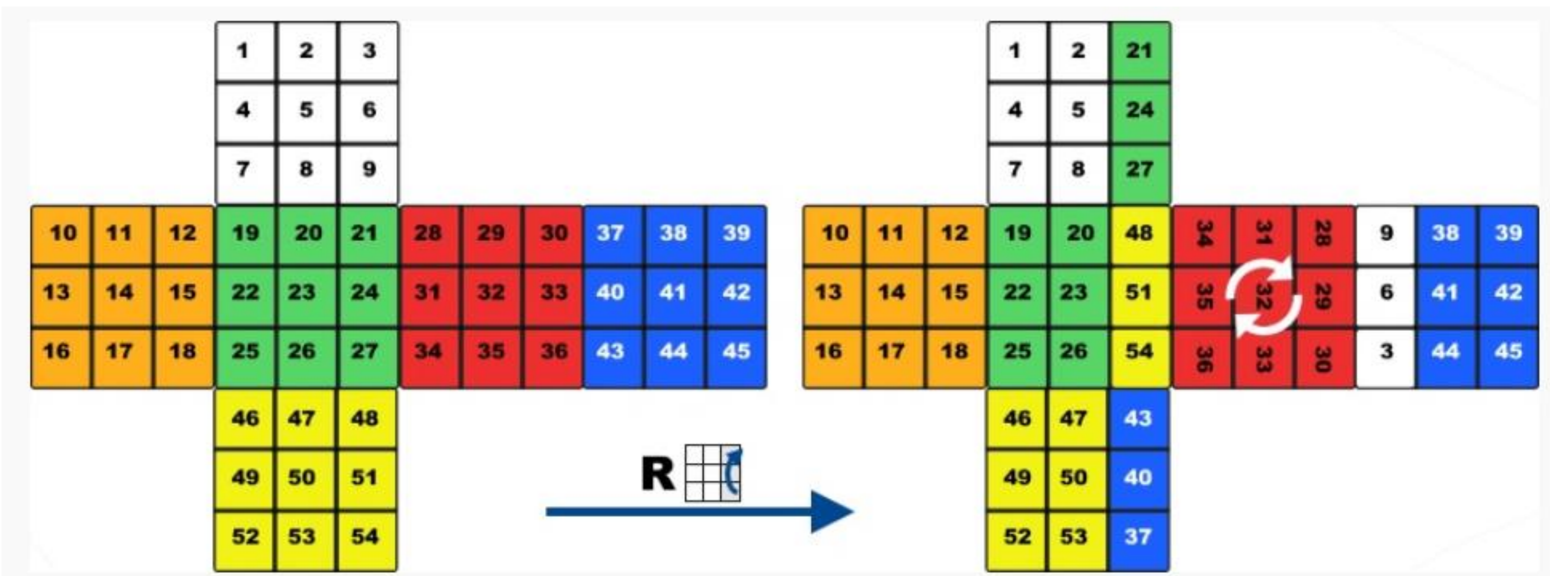
Rubik’s Cube

U: $\sigma = (1\ 4\ 3\ 2)$

U^3: $\sigma = (1\ 2\ 3\ 4)$

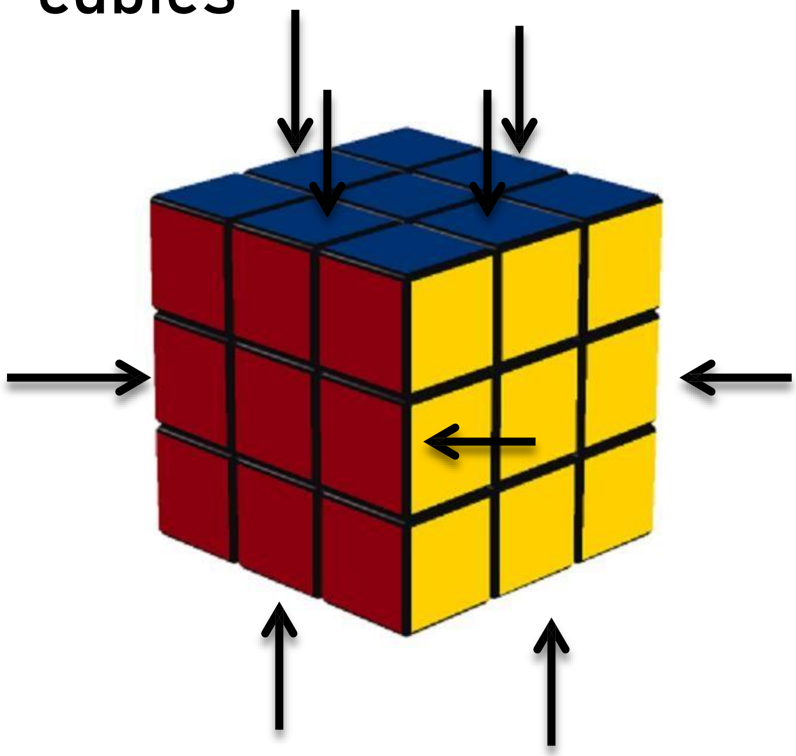
U^2: $\sigma = (13)\ (24)$

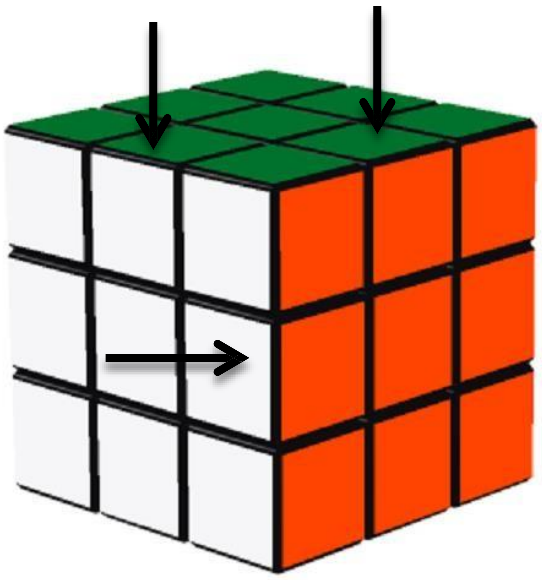
U^4: $\sigma = (1)(2)(3)(4) = 1$



Rubik's Cube

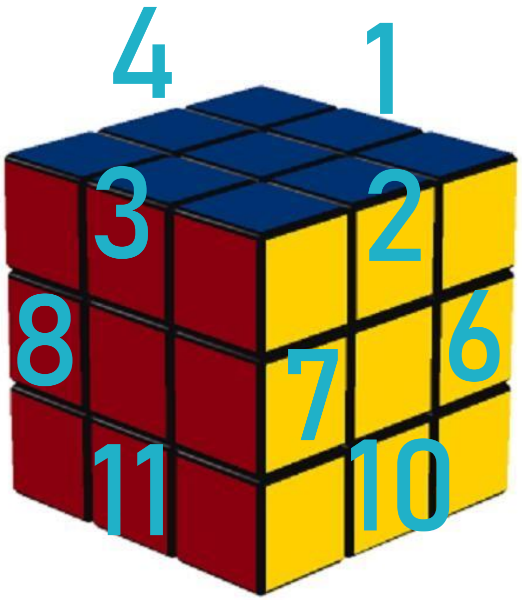
- There are 12 edge cubies

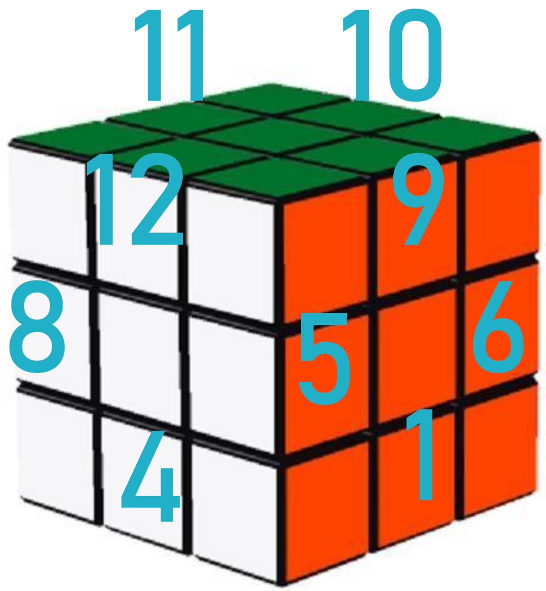




Rubik's Cube

- We number the edge cubies as follows:





Rubik's Cube

- There are 12! ways to rearrange the edge cubies.

- Each corresponds to a permutation of the form:
  (1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12)

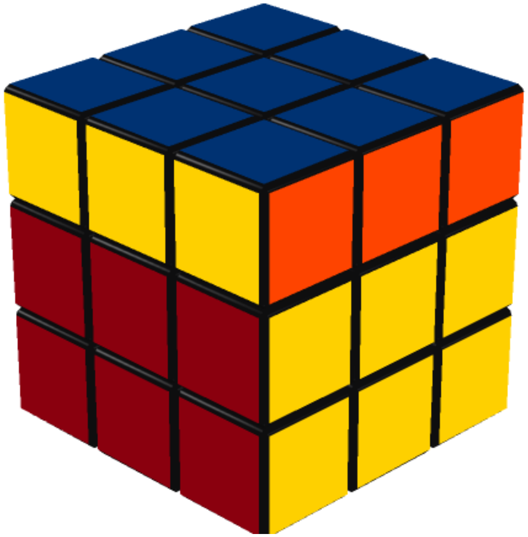
Rubik's Cube

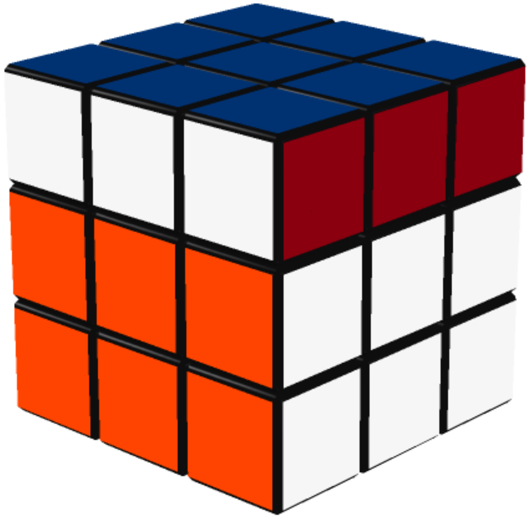
The images below correspond to an upper or U twist on the Rubik's cube
The U twist permutation has the form:

(1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12)

↓

(4, 1, 2, 3, 5, 6, 7, 8, 9, 10 , 11,12)





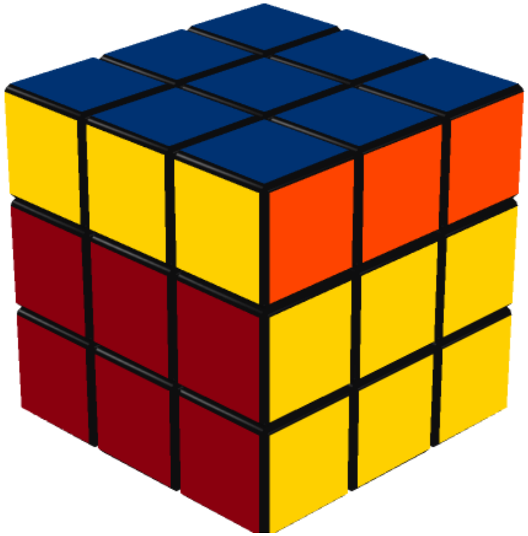
Rubik’s Cube

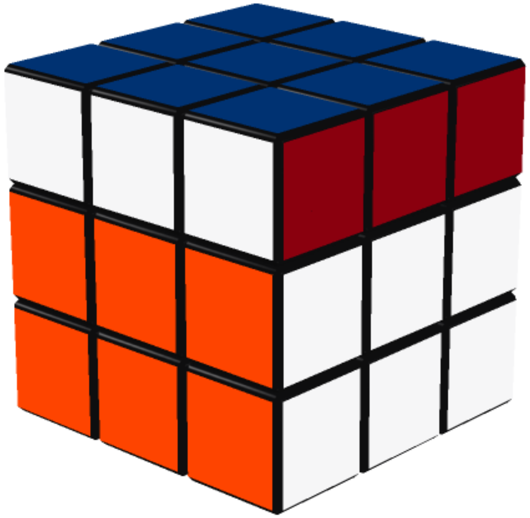
We then take the permutation

$$(4, 1, 2, 3, 5, 6, 7, 8, 9, 10 , 11, 12)$$

and express it in the cyclic form:

$$\tau = (1\ 2\ 3\ \ 4)=(12)(13)(14)$$





Rubik’s Cube – Related to group Structure

- Closure: If a and b are two components in the group, G, then the product “$a * b$” is also in “G”. Let’s Suppose two move form the group structure or the rubik’s cube move < R,L,U,D,F,B >,if we took any two sequence or any two individual move if we perform them we will find the complete sequence in the closed group $a * b$ which belongs to G a = RURUR and b =URURU
- Associativity: If a and b are two components in the group, G, then the product “$a * b$” is also in “G”. The operation is associative; that is, $(a * b)c = a(b * c)$ for all a,b,c in G. a = RU b = R ′U′ c = RU
- Identity: There is an element e (called the identity) in G such that ae = ea = a for all a in G
- Inverses:For each element a in G, there is an element b in G (called an inverse of a) such that ab = ba = e. Any move from < R,L,U,D,F,B > there exist an inverse element like as for R its R ′

Permutation and Orientation

- The puzzle was originally advertised as having "over 3,000,000,000 (three billion) combinations but only one solution".
- The possibilities come from:
  - 8! From permuting the corner cubies
  - 12! From permuting the edge cubies
  - 3^8 From orienting the corner cubies
  - 2^{12} From orienting the edge cubies
- So that's
  - $(8! \times 12! \times 3^8 \times 2^{12}) \approx 519$ quintillion or 5.19×10^{20}

Permutation and Orientation

- But this number represents all of the valid and invalid configurations.
- To find the number of valid configurations:
  - Each corner has three possible orientations, although only seven (of eight) can be oriented independently; the orientation of the eighth (final) corner depends on the preceding seven, giving 37(2,187) possibilities.
  - Eleven edges can be flipped independently, with the flip of the twelfth depending on the preceding ones, giving 211(2,048) possibilities.

Permutation and Orientation

- So this number becomes
  - $(8! \times 12! \times 2^{12} \times 3^{8}) / 3*2*2$ = 43252003274489856000
  - which is approximately 43 quintillion.

Permutation and Orientation

Figure : Feliks Zemdegs

Permutation and Orientation

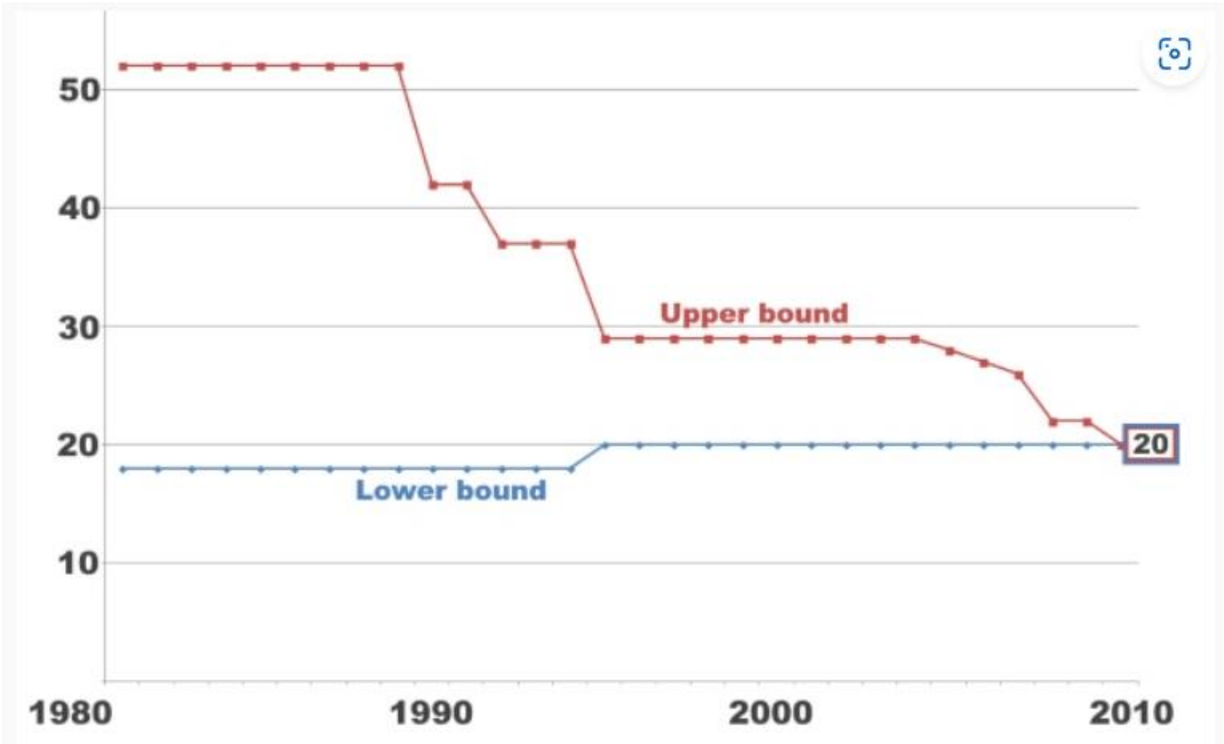
- If the earth has a only one rubik's cube solver
- If he or she averages 30 seconds to solve and 15 seconds to scramble and 15 seconds to solve 1 minute per permutation and he or she practices 24/7 and no breaks at all ever that means she can solve 1440 solve per day as$24*60$min = 1440 min.
- That's a lot for a average cuber

Permutation and Orientation

- Then the total number of scramble is 43,252,003,274,489,856,0000 then the number of days he or she need is
- 43252003274489856000/1440 .
- 30036113850624000 number of days.
- Basically it is impossible for one of us to solve all the scramble.

God's Number

- Every position of Rubik's Cube can be solved in twenty moves or less.



Conclusion

We attempted to demonstrate the algebraic combinatorics of the Rubik's cube.We have tried to find the relation between rubik's cube and mathematics more precisely with Group theory.The Rubik's cube contains many mysteries.We only find the god's number for 2*2*2 and 3*3*3 but not for large cubes.Hopefully, one day we will discover.

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