

TIU - Numerik (Aljabar, Pangkat Dan Akar)

1. Bentuk Aljabar

- $x, 3y, x + 3y, a+2b, a^2 + b + 3$ disebut bentuk aljabar
- $ax^2 + bx + c = 0$; a, b, c, x dan 0 adalah lambang-lambang aljabar
 - a dan b disebut koefisien ; c disebut konstanta
 - x^2 dan x disebut variabel
- ax dan bx merupakan dua suku sejenis
- ax^2 dan bx merupakan dua suku tidak sejenis
- Unsur-unsur suku sejenis dapat digabung menjadi satu.

Operasi aljabar

- Operasi penjumlahan atau pengurangan pada aljabar hanya dapat dilakukan pada suku yang sejenis saja.

Misalnya,

$$2x + 3x = 5x$$

$$3x + 5x + x^2 = x^2 + 8x$$

- Perkalian 1 suku dengan 1 suku

$$a(b + c) = ab + ac$$

- Perkalian 2 suku dengan 2 suku

$$(a + b)(c + d) = ac + ad + bc + bd$$

- Perkalian Istimewa

$$(a + b)^2 = (a + b)(a + b) = a^2 + 2ab + b^2$$

$$(a - b)^2 = (a - b)(a - b) = a^2 - 2ab + b^2$$

$$a^2 + b^2 = (a - b)^2 + 2ab$$

$$a^2 - b^2 = (a + b)(a - b)$$

$$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

$$(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$$

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

- Bentuk Istimewa Lainnya

$$1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

$$1^3 + 2^3 + 3^3 + \dots + n^3 = (1 + 2 + 3 + \dots + n)^2$$

$$= \left(\frac{n}{2} (n + 1) \right)^2$$

2. Bentuk Berpangkat

Bentuk umum

$$a^n = a \times a \times \dots \times a \text{ (sebanyak } n \text{)}$$

Sifat-sifat :

$$a^m \times a^n = a^{m+n}$$

$$a^m : a^n = a^{m-n}$$

$$(a^m)^n = a^{m \times n}$$

$$(a \times b)^m = a^m \times b^m$$

$$(a : b)^m = a^m : b^m$$

$$a^{-m} = \frac{1}{a^m}$$

$$a^0 = 1$$

$$1^m = 1$$

3. Bentuk Akar

Sifat-sifat :

$$a^{\frac{1}{n}} = \sqrt[n]{a}$$

$$a^{\frac{m}{n}} = (a^m)^{\frac{1}{n}} = \sqrt[n]{a^m}$$

$$a^{\frac{1}{m}} \cdot a^{\frac{1}{n}} = a^{\frac{1}{m} + \frac{1}{n}} = \sqrt[m]{a} \cdot \sqrt[n]{a} = \sqrt[mn]{a^{m+n}}$$

$$a^{\frac{1}{m}} : a^{\frac{1}{n}} = a^{\frac{1}{m} - \frac{1}{n}} = \sqrt[m]{a} : \sqrt[n]{a} = \sqrt[mn]{a^{m-n}}$$

$$(a^{\frac{1}{m}})^{\frac{1}{n}} = a^{\frac{1}{m} \cdot \frac{1}{n}} = \sqrt[mn]{a}$$

$$(ab)^{\frac{1}{n}} = a^{\frac{1}{n}} \cdot b^{\frac{1}{n}} = \sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$$

$$\left(\frac{a}{b}\right)^{\frac{1}{n}} = \frac{a^{\frac{1}{n}}}{b^{\frac{1}{n}}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$$

$$a^{-\frac{1}{n}} = (a^{\frac{1}{n}})^{-1} = \frac{1}{a^{\frac{1}{n}}} = \frac{1}{\sqrt[n]{a}}$$

$$\sqrt[n]{a^n} = (a^n)^{\frac{1}{n}} = a^{\frac{n}{n}} = a^1 = a$$

$$a\sqrt{b} + c\sqrt{b} = (a + c)\sqrt{b}$$

$$a\sqrt{b} - c\sqrt{b} = (a - c)\sqrt{b}$$

$$a\sqrt{b} \times c\sqrt{d} = ac\sqrt{bd}$$

Merasionalkan Bentuk akar

$$\frac{c}{a + \sqrt{b}} = \frac{c}{a + \sqrt{b}} \cdot \frac{a - \sqrt{b}}{a - \sqrt{b}} = \frac{c(a - \sqrt{b})}{a^2 - b}$$

$$\frac{c}{a - \sqrt{b}} = \frac{c}{a - \sqrt{b}} \cdot \frac{a + \sqrt{b}}{a + \sqrt{b}} = \frac{c(a + \sqrt{b})}{a^2 - b}$$

$$\frac{c}{\sqrt{a} + \sqrt{b}} = \frac{c}{\sqrt{a} + \sqrt{b}} \cdot \frac{\sqrt{a} - \sqrt{b}}{\sqrt{a} - \sqrt{b}} = \frac{c(\sqrt{a} - \sqrt{b})}{a - b}$$

$$\frac{c}{\sqrt{a} - \sqrt{b}} = \frac{c}{\sqrt{a} - \sqrt{b}} \cdot \frac{\sqrt{a} + \sqrt{b}}{\sqrt{a} + \sqrt{b}} = \frac{c(\sqrt{a} + \sqrt{b})}{a - b}$$

Contoh Soal :

1. Jika $x + y = 11$ dan $x - y = 3$, maka $x - y = \dots$

- A. 8
- B. 33
- C. 9
- D. 14
- E. 66

Jawab :

$$x - y = (x + y)(x - y)$$

$$x - y = 11 \times 3 = 33$$

$$\therefore \frac{x^2y^2 - 4x^2y + 3x^2}{y^4 - y^3 - 6y^2} = \dots$$

- A. $\frac{x^2}{y^2}$
- B. $\frac{x^2(y+1)}{(y-2)}$
- C. $\frac{x^2(y-1)}{y^2(y+2)}$
- D. $\frac{x^2(y-1)}{(y+2)}$
- E. $\frac{x^2(y+1)}{y^2(y-2)}$

Jawab :

$$\begin{aligned}
&= \frac{x^2(y^2 - 4y + 3)}{y^2(y^2 - y - 6)} \\
&= \frac{x^2(y - 1)(y - 3)}{y^2(y + 2)(y - 3)} \\
&= \frac{x^2(y - 1)}{y^2(y + 2)}
\end{aligned}$$

$$m = \frac{1 + \frac{7}{8} + \frac{3}{4} + \frac{1}{2}}{\frac{1}{8} + \frac{2}{5}}$$

3. Jika diketahui dan $n = (0,6666 + 0,02^2)(0,1^{-1} + 0,1^2)$ maka

1. $m < n$
2. $m = n$
3. hubungan m dan n tidak dapat ditentukan
4. $m, n < 0$
5. $m > n$

Jawab :

Untuk menyederhanakan m , samakan terlebih dahulu penyebutnya.

$$m = \frac{\frac{1 \times 40}{40} + \frac{7 \times 5}{40} + \frac{3 \times 10}{40} + \frac{1 \times 20}{40}}{\frac{1 \times 5}{40} + \frac{2 \times 8}{40}}$$

$$m = \frac{\frac{40 + 35 + 30 + 20}{40}}{\frac{5 + 16}{40}} = \frac{\frac{125}{40}}{\frac{21}{40}}$$

$$m = \frac{125}{40} \times \frac{40}{21} = \frac{125}{21} = 5,95 = 6$$

$$n = (0,6666 + 0,02^2)(0,1^{-1} + 0,1^2)$$

$$n = (0,6666 + 0,0004) \left(\frac{1}{0,1} + 0,01 \right)$$

$$n = 0,6670 (10 + 0,01) = 6,67$$

Maka $m < n$

4. $16^{0,125} - (0,5)^{-0,5} = \dots$

A. $-2\sqrt{2}$

B. $-\sqrt{2}$

C. 0

D. $\sqrt{2}$

E. $2\sqrt{2}$

Jawab :

$$\begin{aligned}
&= (2^4)^{\frac{1}{8}} - \left(\frac{1}{2}\right)^{-\frac{1}{2}} \\
&= 2^{\frac{4}{8}} - (2^{-1})^{-\frac{1}{2}} \\
&= 2^{\frac{1}{2}} - 2^{\frac{1}{2}} \\
&= 0
\end{aligned}$$

5. Diketahui nilai $x = (2^5 \times 2^4 \times 2^3)^{1/2}$ dan $y = \sqrt[3]{8^2}$, pernyataan yang benar adalah ...

- A. $x = y^3$
- B. $x = 3y$
- C. $x = y/3$
- D. $x = 3/y$
- E. $x = y^3 - 3$

Jawab :

$$\begin{aligned}
x &= (2^{5+4+3})^{\frac{1}{2}} = (2^{12})^{\frac{1}{2}} = 2^6 = 64 \\
y &= \sqrt[3]{(2^3)^2} = (2^6)^{\frac{1}{3}} = 2^2 = 4 \\
\text{maka } x &= y^3
\end{aligned}$$

6. Jika $x = 85\% - 25\% + 1,25 - 17/20$ dan $y = \sqrt{16 - 8 \times 5 + 5^2}$ maka ...

- A. $x < y$
- B. $x > y$
- C. $x = y$
- D. $x, y < 0$
- E. hubungan antara x dan y tidak dapat ditentukan

Jawab :

$$\begin{aligned}
x &= \frac{85}{100} - \frac{25}{100} + \frac{125}{100} - \frac{17 \times 5}{100} = \frac{100}{100} = 1 \\
y &= \sqrt{16 - 40 + 25} = \sqrt{1} = 1 \\
\text{maka } x &= y
\end{aligned}$$

7. Jika $A = (-1)^{-1}$, $B = (-1)^1$ dan $C = (1)^{-1}$ maka nilai $A + B - C = \dots$

- A. -3
- B. -1
- C. 0

D. 1

Jawab :

$$A = (-1)^{-1} = (-1/1) = -1$$

$$B = (-1)^1 = -1$$

$$C = (1)^{-1} = (1/1) = 1$$

$$\text{maka } A + B + C = -1 - 1 - 1 = -3$$