Measuring the Performance of Low Earth Orbit Regional Positioning Satellite Constellations

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State-of-the-art navigation systems, which provide continuous global coverage, are located in medium or geosynchronous Earth orbits, and require high production, launch, operation, and sustainment costs. There are systems that do not require continuous global coverage and can use commercial off-the-shelf products implemented in low Earth orbit constellations to provide regional coverage at reduced costs. The objective of this paper is to analyze the performance of several low Earth orbit constellations that provide regional positioning and navigation coverage to the University of Maryland. The performance quantities analyzed include average revisit time, average coverage time, total coverage time, position dilution of precision, and total constellation change in velocity.

Nomenclature

(Nomenclature entries should have the units identified)

$\vec{a}_d$	=	acceleration due to atmospheric drag (km/s^2)
A	=	matrix of receiver position unit vectors
A_r	=	ram area (km^2)
C_Δ	=	covariance matrix
c_D	=	drag coefficient
m	=	mass of satellite (kg)
$PDOP$	=	position dilution of precision
$R_\oplus$	=	mean equatorial radius of the Earth (km)
r_a	=	apogee of orbit (km)
r_p	=	perigee of orbit (km)
$\vec{r}_s$	=	position vector of a satellite (km)
$\vec{r}_u$	=	position vector of ground station(km)
$\vec{v}_r$	=	satellite velocity relative to the atmosphere (km/s)
ΔV	=	change in velocity (km/s)

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ΔV_t = total constellation change in velocity (km/s)

μ = standard gravitational parameter of Earth (km^3/s^2)

ρ = atmospheric density (kg/km^3)

σ_{11},σ_{22}, and σ_{33} = diagonal elements of the covariance matrix

τ_m = minimum line-of-sight parametric value

$\omega_\oplus$ = rotational velocity of the Earth (rad/s)

Subscripts

$-$ = before impulse burn

$\oplus$ = Earth

$+$ = after impulse burn

I. Introduction

There are several advantages to having a specialized regional positioning and navigation constellation in low Earth orbit (LEO). Nano-satellites are made up of commercial off-the-shelf components, and therefore their signal transmission strength is restricted to use in LEO. The size and cost of nano-satellites drives down production and launch costs. Lastly, the number of satellites in the constellation can be chosen to maximize the particular performance metrics, including coverage time and precision. High accuracy services, which were originally primarily offered to professional users, are now propagating the mass market in entry level internet of things devices [1]. Demand for these systems will continue to grow. Regional coverage specification, cost effectiveness, and performance customization of these systems makes them an attractive option for prototype system design verification testing, research teams that require an exclusively controlled system, and hobbyists.

This paper is aimed at comparing the performance of several low Earth orbit constellations that provide regional positioning and navigation coverage to a preselected latitude and longitude. The performance figures analyzed include average revisit time, average coverage time, total coverage time, position dilution of precision (PDOP), and total constellation change in velocity required to combat atmospheric drag. The framework provided may be used for future research or applied by any user that wishes to model the performance of their constellation. The user may input the latitude and longitudinal coordinates of their desired coverage region, input the size and initial conditions of their constellation, and then analyze the system's performance.

II. Description of Investigation

The following section describes the methods used to initialize the simulation, propagate the orbits, and measure constellation performance.

A. Initialization

The ground station is located at the geocentric longitude and latitude of Martin Hall at the University of Maryland. In this investigation, it is assumed that the Earth is perfectly spherical. Therefore, the Earth's eccentricity is ignored in the ground station location calculations. Earth-Centered Inertial (ECI) frame is used in this analysis, so the ground station location is rotated about true north at $\omega_\oplus$. The coordinates are then converted from spherical to Cartesian using $R_\oplus$ as the magnitude of the radius. All of the satellites are initialized at an altitude of 880 km at the same geocentric longitude as the ground station. The constellations are distributed at random latitudes around the globe. None of the latitudes are within 5 degrees of each other, to ensure the network geometry is dispersed. Each constellation is made up of the previous constellation's orbits, with the addition of a newly generated orbit. This construction technique ensures that the performance variation is not due to unique geometry, but rather constellation size.

B. Propagation

The constellation's system state vector containing initial conditions is fed into a 2-body problem numerical integrator. Next to the oblateness of the Earth, atmospheric drag most strongly influences the motion of a satellite near Earth [2]. Zonal harmonic perturbations can be ignored due to the spherical Earth assumption; consequently, this propagation will only include perturbations due to atmospheric drag. The quantities used for the drag acceleration are based on cube-satellite standards: A_r=1*10^{-8} km^2, m = 1.3 kg. The flat plate model used in Ref. [2] dictates that c_D=2.2. Sliding and lifting forces due to atmospheric drag are ignored. The atmospheric density is considered a constant in this model, ρ= 7.67*10^{-6} kg/km^3 [3]. The ram area is also considered a constant in this model, as tumbling and satellite attitude are ignored. The following equation is modeled in the integration function to describe acceleration due to atmospheric drag[2]:

$$\vec{a}_d = -(c_D A_r \rho)(v_r^2)(\vec{v}_r)/2m|\vec{v}_r| \tag{1}$$

The relative velocity is the satellites velocity vector relative to the Earth's atmosphere, and the Cartesian coordinates are those of the satellite being influenced:

$$\vec{v}_r = [(dx/dt + (y)\omega_\oplus); (dy/dt - (x)\omega_\oplus); (dz/dt)]^T \tag{2}$$

Simulations were also run without any perturbing forces, in order to measure the effect of the perturbations on the constellation performance.

C. Orbital Modifications

The Keplerian orbital elements are then computed for each satellite at every time step. The minimum and maximum orbital radii of each satellite is compared to the bounds of LEO. The upper bound is defined as an altitude of 2000 km, and the lower is the surface of the Earth [4]. Initial conditions are modified until an orbit that meets LEO limitations is generated. After the constellation's initial conditions are set, the apoapsis at initialization and after 1 period are compared to each other for each respective orbit. The change in velocity required to perform a general orbital modification to return the apoapsis back to its original state is computed using the following equation:

$$\Delta V = |\sqrt{(2\mu)(r_{a+})/(r_p)(r_p + r_{a+})} - \sqrt{(2\mu)(r_{a-})/(r_p)(r_p + r_{a-})}| \tag{3}$$

This computation is done for each satellite, and then summed for the total constellation change in velocity, ΔV_t.

D. Line of Sight

The line-of-sight (LOS) between the ground station and the individual satellites is computed using the sight algorithm featured in Ref. [2]. The following check is performed for each satellite in the constellation at every time step. If τ_m is between 0 or 1, then there is sight between the ground station and the satellite:

$$\tau_m = (|\vec{r}_u|^2 - \vec{r}_u.\vec{r}_s)/(|\vec{r}_u|^2 + |\vec{r}_s|^2 - 2\vec{r}_u.\vec{r}_s) \tag{4}$$

If the condition in Eq. (4) is not satisfied, then another condition may be checked. If the following is true, there is LOS:

$$(1 - \tau_m)|\vec{r}_u|^2 + (\vec{r}_u.\vec{r}_s)\tau_m >= R_\oplus^2 \tag{5}$$

E. Position Dilution of Precision

Dilution of precision (DOP) is a quantification of a satellite system's accuracy. PDOP is the only precision metric considered because it depends solely on the geometry of the constellations, and not the error in the transmission equipment that is factored into other DOP variations. The PDOP is only measured when there is LOS for the total constellation, and only those satellites which have LOS are included in A. If the PDOP of the constellation is larger than 6, the measurement is considered too imprecise, and therefore the constellation is considered out-of-sight at that moment [5]. Assuming that there is no model error, the following equations are used to compute the PDOP:

$$C_\Delta = (A^T A)^{-1} \tag{6}$$

$$PDOP = \sqrt{\sigma_{11}^2 + \sigma_{22}^2 + \sigma_{33}^2} \tag{7}$$

F. Coverage

The constellation has coverage of the ground station when 4 or more satellites have LOS with the ground station [5]. This restriction guarantees that enough mutually uncorrelated measurements are available, and that the precision is sufficient for an accurate measurement. In this model, there are no topographical obstructions to LOS, and there is not a minimum elevation mask angle. The coverage and revisit times are computed by measuring the amount of time in between a change in LOS. If the LOS changes from out-of-sight, to in-sight, coverage is recorded until LOS changes back to out-of-sight, and vice versa for the revisit time.

III. Results

Table 1 Constellation coverage

Constellation Size	Average Coverage Time, s	Average Revisit Time, s	Percent Coverage
4	NaN	NaN	NaN
5	193.2559	274878	0.0703
6	193.1401	274878	0.0702
7	172.4065	38284	0.1564
8	179.8511	9248	0.2192
9	158.4278	3266	0.3151
10	158.4073	2839	0.3755

The constellation that contains 4 satellites was not robust enough to log any coverage over 1 week. This constellation would be insufficient for any application that required navigation or positioning data in that period of time. It is possible that a 4 satellite constellation with different orbits may satisfy the conditions for coverage.

A. Coverage

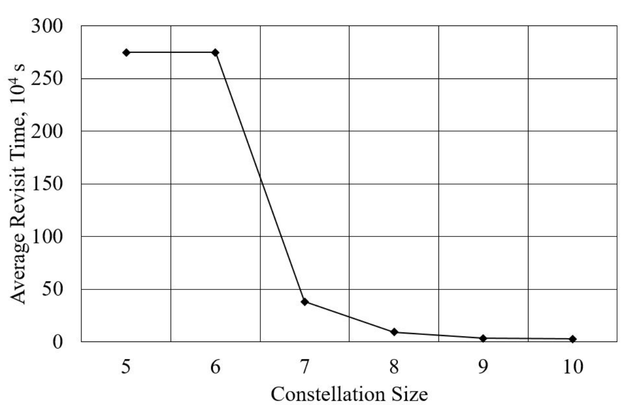

Fig. 1 Average time between coverage as a function of constellation size

The average revisit time decreases rapidly as the constellation size grows. However, the inverse is not true of the average coverage time. Table. (1) shows that increasing the size of a constellation will not yield a greater average coverage time. If the end user of a navigation and positioning constellation is concerned only with long blocks of coverage, and not the time between blocks, they may implement a constellation with a small size. Conversely, a large constellation size should be implemented if a minimal average revisit time is desired.

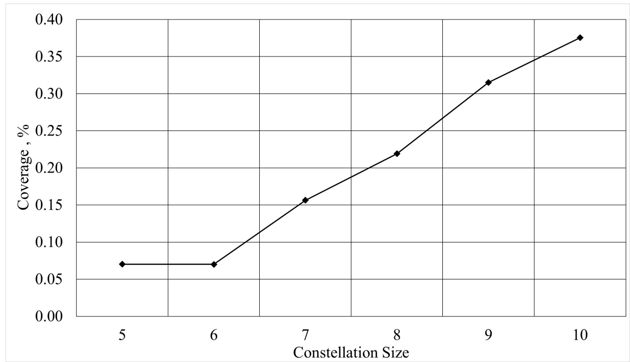

Fig. 2 Percent of time the ground station is covered by the constellation as a function of constellation size

Although the average coverage time does not increase with the size of the constellation, the total percent of time with coverage does. The larger constellations make more frequent contact with the ground station which adds up to greater overall coverage.

B. Position Dilution of Precision

Table 2 Effects of atmospheric drag

Constellation Size	PDOP	PDOP (no drag)	ΔV_t,$10^{-11} km/s$	ΔV_t per Satellite,$10^{-12} km/s$
4	NaN	NaN	0.174	1.74
5	3.4025	3.3991	0.879	1.76
6	3.4019	3.4032	1.064	1.76
7	3.7362	3.7344	1.24	1.77
8	3.8810	3.8815	1.41	1.77
9	3.6211	3.6204	1.59	1.77
10	3.5778	3.5786	1.76	1.76

PDOP does not get better as the constellation size grows. PDOP is worsened by a close proximity of satellites. As the constellation size grows, excess satellites in the PDOP computations reduce the distance between satellites. These excess contributions crowd out the 4 satellites that would offer the best PDOP. There is no discernible difference in the performance when atmospheric drag is included in the simulation. The percent difference in the PDOP values with and without perturbations in Table. (2) are all less than one tenth of one percent. Future simulations with the goal of measuring a constellation's PDOP which contain 10 or fewer satellites, may ignore atmospheric drag. The ΔV_t for the system scales linearly with how many satellites are included in the constellation. This is expected, as satellites with similar Keplerian orbital parameters will have to perform modifications that require similar ΔV. If this is combined with the fact that we know maintenance costs are scaled linearly with the number of satellites, we can say that operation and sustainment costs scale linearly [6].

IV. Conclusion

This paper introduced the method for modelling constellations of satellites in LEO taking into account the perturbing force for atmospheric drag, and demonstrated their performance. The results cover 8 constellations, made up of between 4 and 10 satellites. As the constellation size grows, the total coverage grows, and the average revisit time rapidly decreases. The average coverage time do not grow with the constellation size. The PDOP does not improve with the constellation size, however that may be due to an ineffective computation. The penalty of increasing the size of the constellation is a linear increase in the fuel required to maintain an orbit being perturbed by atmospheric drag. Therefore, there are limitations to the value added by implementing more satellites in a constellation. A designer should take into account which performance metrics need to be optimized, and act accordingly.

This model could be made more accurate by including the following: zonal harmonic perturbations, geometric model error in the computation of PDOP, atmospheric density that varies with altitude, and an algorithm that produces the best PDOP at each time step by factoring in the 4 most spread out satellites.

References

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