

$$\text{In}[1]:= \text{SecondOrderAllPass} = \frac{s^2 - \frac{W}{Q} s + W^2}{s^2 + \frac{W}{Q} s + W^2}$$

$$\text{Out}[1]= \frac{s^2 - \frac{s W}{Q} + W^2}{s^2 + \frac{s W}{Q} + W^2}$$

$$\text{In}[2]:= s = P \frac{z - 1}{z + 1}$$

$$\text{Out}[2]= \frac{P (-1 + z)}{1 + z}$$

$$\text{In}[3]:= \text{SecondOrderAllPass}$$

$$\text{Out}[3]= \frac{W^2 + \frac{P^2 (-1+z)^2}{(1+z)^2} - \frac{P W (-1+z)}{Q (1+z)}}{W^2 + \frac{P^2 (-1+z)^2}{(1+z)^2} + \frac{P W (-1+z)}{Q (1+z)}}$$

$$\text{In}[4]:= \text{Together}[\text{ExpandAll}[\%]]$$

$$\text{Out}[4]= \frac{P^2 Q + P W + Q W^2 - 2 P^2 Q z + 2 Q W^2 z + P^2 Q z^2 - P W z^2 + Q W^2 z^2}{P^2 Q - P W + Q W^2 - 2 P^2 Q z + 2 Q W^2 z + P^2 Q z^2 + P W z^2 + Q W^2 z^2}$$

$$\text{In}[5]:= \text{Collect}[\text{Numerator}[\%4], z]$$

$$\text{Out}[5]= P^2 Q + P W + Q W^2 + (-2 P^2 Q + 2 Q W^2) z + (P^2 Q - P W + Q W^2) z^2$$

$$\text{In}[10]:= \text{Collect}[\text{Denominator}[\%4], z]$$

$$\text{Out}[10]= P^2 Q - P W + Q W^2 + (-2 P^2 Q + 2 Q W^2) z + (P^2 Q + P W + Q W^2) z^2$$

$$\text{In}[11]:= \text{Collect}[\%5 / (P^2 Q + P W + Q W^2), z]$$

$$\text{Out}[11]= 1 + \frac{(-2 P^2 Q + 2 Q W^2) z}{P^2 Q + P W + Q W^2} + \frac{(P^2 Q - P W + Q W^2) z^2}{P^2 Q + P W + Q W^2}$$

$$\text{In}[12]:= \text{Collect}[\%10 / (P^2 Q + P W + Q W^2), z]$$

$$\text{Out}[12]= \frac{P^2 Q - P W + Q W^2}{P^2 Q + P W + Q W^2} + \frac{(-2 P^2 Q + 2 Q W^2) z}{P^2 Q + P W + Q W^2} + z^2$$