MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Department of Electrical Engineering and Computer Science

6.341 DISCRETE TIME SIGNAL PROCESSING

Fall 2005

BACKGROUND EXAM

September 22, 2005.

Full Name:

Note:

This exam is closed book. No notes or calculators permitted.

There are 13 problems on the exam. The exam will be graded on the basis of the answer booklet only. Only turn in the answer booklet. Please don't put any explanations or work in the answer booklet.

Five pages of scratch paper are attached at the end of the exam. These are not to be handed in. Let us know if you need additional scratch paper.
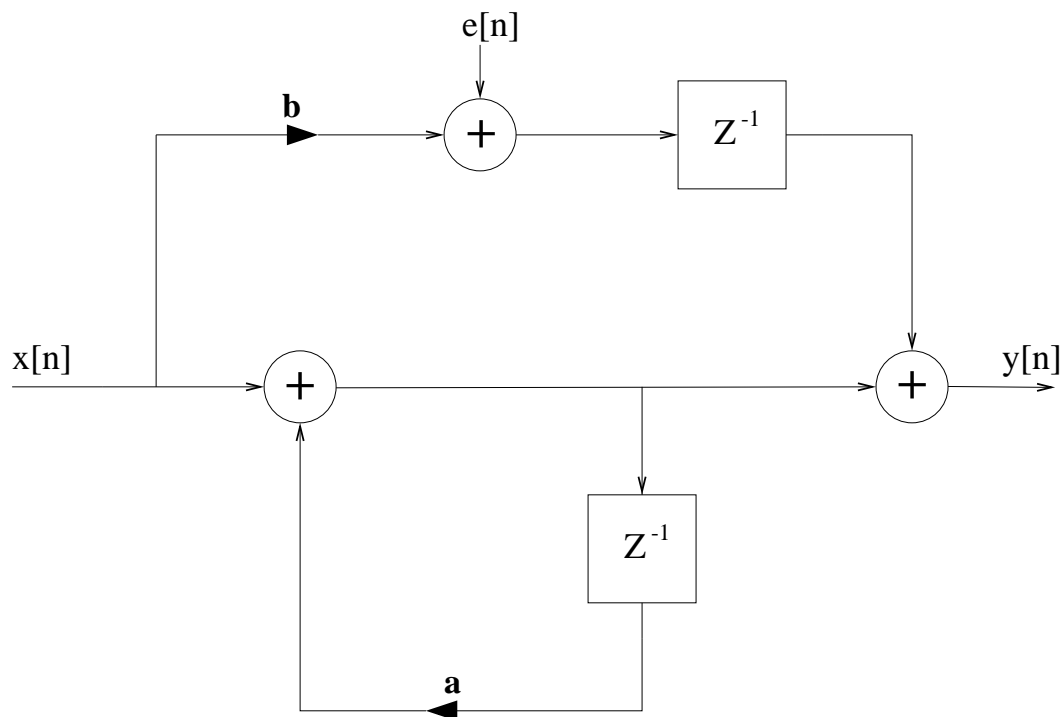
Problem 1 [12%] For each of the three systems below specify whether or not the system is (1) linear, (2) time-invariant, (3) causal, (4) stable. The system input is $x[n]$ and the system output is $y[n]$.

4% (a) $y[n] = x[2n]$

4% (b) $y[n] = x[n] + x[n-1]$

4% (c) $y[n] = (x[-|n|])^2$

Problem 2 [6%] Determine the transfer function $H_{xy}(z)$ from $x[n]$ to $y[n]$ and $H_{ey}(z)$ from $e[n]$ to $y[n]$ for the following system:

Problem 3 [7%]

3% (a) Determine the difference equation relating the input and output of the LTI system described by the following frequency response:

$$H(e^{j\omega}) = \frac{1 + \frac{1}{2}e^{-j\omega}}{1 - 2e^{-j\omega}}$$

2% (b) Is the system in (a) stable?

2% (c) Is the system in (a) causal?

Problem 4 [8%] $x[n]$, a white random sequence with zero mean and unit variance, is processed with an LTI system that satisfies the following difference equation: $y[n] = x[n+1] + x[n-1]$

2% (a) Determine the impulse response $h[n]$ and the transfer function $H(z)$ of the LTI system.

3% (b) What is the autocorrelation $\phi_{yy}[m]$ of the process $y[n]$?

3% (c) What is the power spectral density $P_{yy}(\omega)$ of the process $y[n]$?

Problem 5 [10%] We process a zero-mean, unit variance, white, discrete-time signal $x[n]$ through the stable LTI system with transfer function

$$H_1(z) = \frac{1 - 2z^{-1}}{1 - \frac{1}{3}z^{-1}}$$

to get an output $y_1[n]$.

4% (a) Find a causal minimum-phase system $H_2(z)$ (i.e. one with all its poles and zeroes inside the unit circle) such that if we pass $x[n]$ through $H_2(z)$, the output $y_2[n]$ has the same autocorrelation as $y_1[n]$ (i.e. $\phi_{y_1 y_1}[m] = \phi_{y_2 y_2}[m]$)

3% (b) Is your answer to (a) unique?

3% (c) Find a stable, causal system $H_w(z)$ such that if we pass $y_1[n]$ through $H_w(z)$, the output is white.

Problem 6 [6%] The continuous-time signal

$$x_c(t) = \cos(2000\pi t)$$

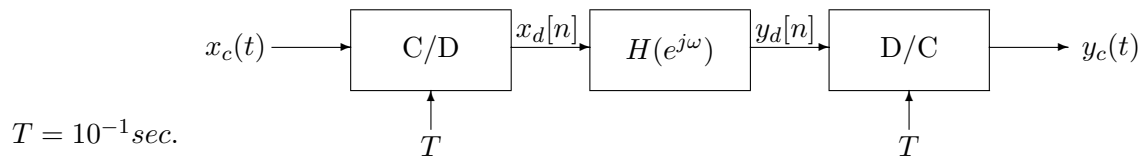is sampled with a sampling period T to obtain a discrete-time signal

$$x[n] = \cos(\frac{\pi n}{3})$$

3% (a) Determine a choice for T consistent with this information.

3% (b) Is your choice for T in part (a) unique? If not, specify another choice of T consistent with the information given.

Problem 7 [9%]

In the system below, the discrete time filter has frequency response $H(e^{j\omega}) = \frac{j\omega}{T}$ for $|\omega| < \pi$.

$$x_c(t) \longrightarrow \boxed{\text{C/D}} \xrightarrow{x_d[n]} \boxed{H(e^{j\omega})} \xrightarrow{y_d[n]} \boxed{\text{D/C}} \longrightarrow y_c(t)$$

$T = 10^{-1} sec.$

with T into C/D and T into D/C.

For each of the following cases determine $y_c(t)$

4% (a) $x_c(t) = \cos(6\pi t)$

5% (b) $x_c(t) = \cos(14\pi t)$

Problem 8 [8%] For the pole-zero plot given below answer the following questions:

3% (a) Specify a transfer function $H(z)$ of the system consistent with this pole-zero plot

2% (b) Can a system with this pole-zero plot be both stable and causal?

3% (c) If the system is stable, is there a finite integer m such that the impulse response of the system $h[n]$ is 0 for all $n > m$, or for all $n < m$?
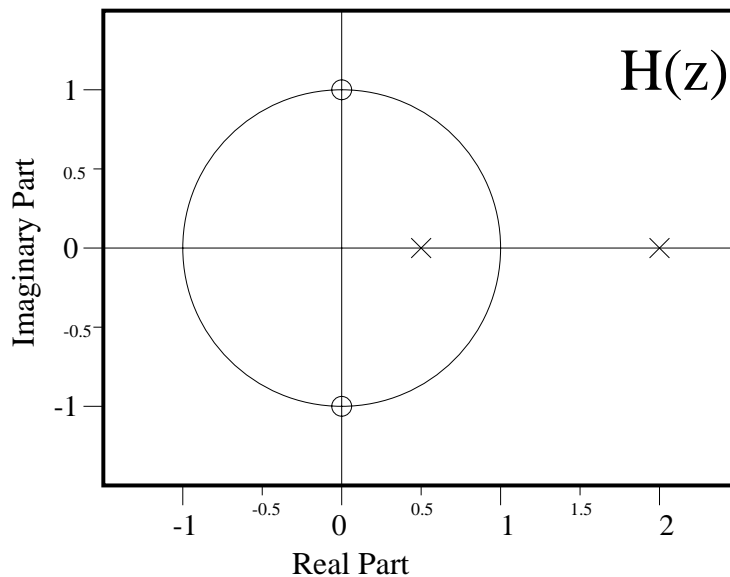

Figure 8-1: Pole-Zero Diagram for Problem 8

Problem 9 [10%] Consider a discrete-time LTI system for which the frequency response $H(e^{j\omega})$ is described by:

$$H(e^{j\omega}) = -j, \quad 0 < \omega < \pi$$
$$H(e^{j\omega}) = j, \quad -\pi < \omega < 0$$

2% (a) Is the impulse response of the system $h[n]$ real-valued? (i.e. is $h[n] = h^*[n]$ for all n)

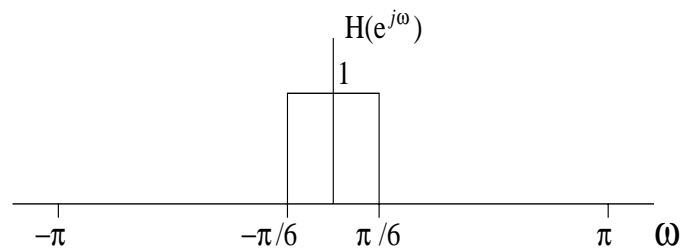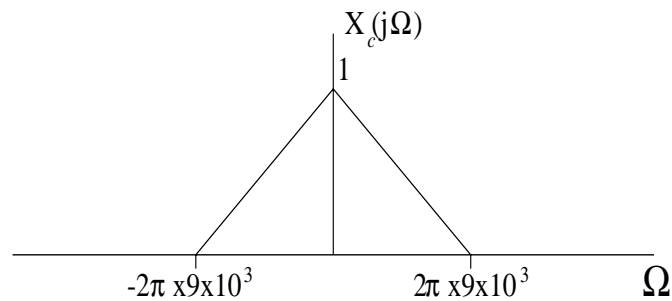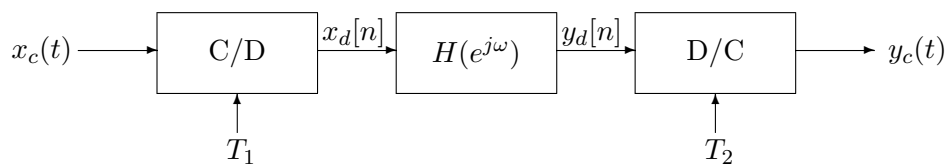2% (b) Calculate the following:

$$\sum_{n=-\infty}^{\infty} |h[n]|^2$$

6% (c) Determine the response of the system to the input $x[n] = s[n]\cos(\omega_c n)$, where $0 < \omega_c < \frac{\pi}{2}$ and $S(e^{j\omega}) = 0$ for $\frac{\omega_c}{3} \le |\omega| \le \pi$.

Problem 10 [7%] In the system below, $X_c(j\Omega)$ and $H(e^{j\omega})$ are as shown, and $\frac{1}{T_1} = 3 \times 10^4$, $\frac{1}{T_2} = 10^4$
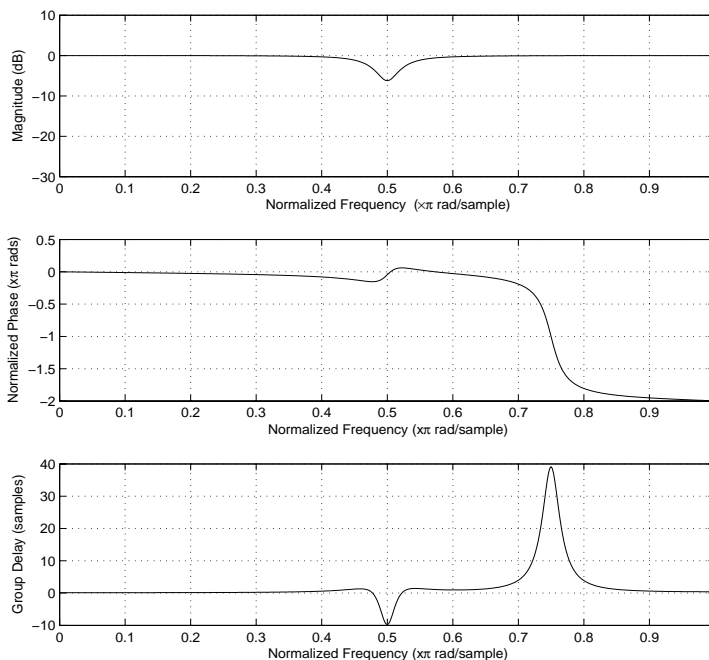
5% (a) Sketch and label the Fourier transforms of $y_d[n]$ and $y_c(t)$.

2% (b) Calculate the following: $\displaystyle\sum_{n=-\infty}^{\infty} y_d[n]$

Problem 11 [5%] Consider a system with the following frequency response characteristics:

(Note that the x-axis is **normalized frequency**, i.e., 1 on the x-axis corresponds to π. Also note that the frequency response for this system is conjugate-symmetric)
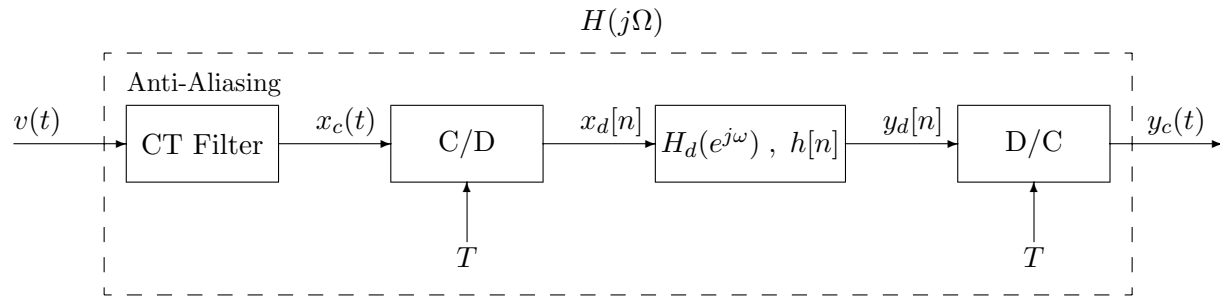

Write an approximate expression for the output of the system if the input is $x[n] = s_1[n]\cos(\frac{3\pi}{4}n)$, where $s_1[n]$ contains only very low frequencies compared to its carrier.
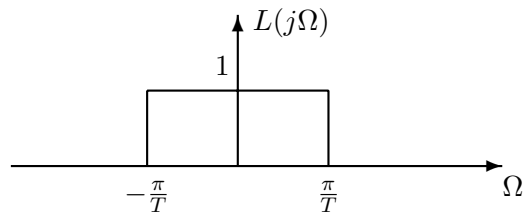
Problem 12 [3%]

We process the signal $x[n] = \cos(\frac{\pi}{3}n)$ with a unity-gain all-pass LTI system, with frequency response $H(e^{j\omega})$ and a group delay of 4 samples at frequency $\frac{\pi}{3}$, to get the output $y[n]$. We also know that $\angle H(e^{j\frac{\pi}{3}}) = \theta$ and $\angle H(e^{-j\frac{\pi}{3}}) = -\theta$. Choose the most accurate statement:

A. $y[n] = \cos(\frac{\pi}{3}n + \theta)$

B. $y[n] = \cos(\frac{\pi}{3}(n-4) + \theta)$

C. $y[n] = \cos(\frac{\pi}{3}(n-4-\theta))$

D. $y[n] = \cos(\frac{\pi}{3}(n-4))$

E. $y[n] = \cos(\frac{\pi}{3}(n-4+\theta))$

Problem 13 [9%] Consider the following system:

$$H(j\Omega)$$

The anti-aliasing filter is a continuous-time filter with the frequency response $L(j\Omega)$ shown below:

The frequency response of the LTI discrete-time system between the converters is given by:

$$H_d(e^{j\omega}) = e^{-j\frac{\omega}{3}}, \quad |\omega| < \pi$$

2% (a) What is the effective continuous-time frequency response of the overall system, $H(j\Omega)$?

2% (b) Choose the most accurate statement:

A. $y_c(t) = \frac{d}{dt}x_c(3t)$
B. $y_c(t) = x_c(t - \frac{T}{3})$
C. $y_c(t) = \frac{d}{dt}x_c(t - 3T)$
D. $y_c(t) = x_c(t - \frac{1}{3})$

2% (c) Express $y_d[n]$ in terms of $y_c(t)$.

3% (d) Determine the impulse response $h[n]$ of the discrete-time LTI system.

Problem 14 [0%] What is your best estimate of your grade on this exam?