Evacuation Traffic Queuing: Local vs. Global Disciplines On a Tree Evacuation Route

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Abstract— **In this work, we analyze small-scale emergency evacuations, addressing a fundamental coordination problem: extremely high traffic demand must be routed through capacity-constrained road networks under time pressure during time-sensitive emergencies. In practice, evacuation routes often consist of hierarchical arterial networks in which local intersections independently control traffic signals, frequently resulting in gridlock and inefficient use of available capacity. This paper develops a queue-network model of evacuation traffic on a directed tree and studies the performance of decentralized versus coordinated traffic signal policies. Intersections are modeled as decision-making agents that control access for incoming roads, while vehicles are generated stochastically at tributary roads and traverse the network toward a common safe node. We evaluate four intersection-level and system-level cost functions, including blind cycling, prioritizing maximum intersection wait time, and prioritizing total travel time, and analyze these signaling policies through simulation. Using tools from queuing theory and game theory, we examine whether evacuation signal control can be cast as a minimization valid utility game and assess the potential price of anarchy arising from decentralized control. Simulation results reveal sharp qualitative differences between local and global policies near and beyond critical traffic generation rates, including highly skewed delay distributions and policy-dependent fairness tradeoffs. While no policy uniformly dominates across all vehicle utilities, the results illustrate structural challenges in achieving socially optimal evacuation performance using purely local signal objectives and highlight the need for coordination mechanisms tailored to each emergency conditions.**

I. Introduction

On February 7, 2017, damage to the emergency spillway of the Oroville Dam in Northern California resulted in rapid erosion and raised concerns regarding the structural integrity of the dam [1]. Subsequent uncontrolled flow over the emergency spillway, used for the first time since the dam's construction, led authorities to issue evacuation orders on February 12 for approximately 180,000 residents across three counties. The evacuation caused extreme traffic congestion along primary evacuation routes, which consisted mostly of a handful of two-lane highways impeded by traffic lights in a few mid-sized cities.

Large-scale evacuations pose a fundamental traffic management problem: how to efficiently route high vehicle demand through capacity-constrained road networks while accounting for safety, delay, and evolving hazard conditions. Traffic signal control plays a central role in this setting. Fixed or poorly coordinated signal timing can induce gridlock at intersections, particularly when inflows from tributary roads interfere with throughput along arterial routes. Such congestion increases total evacuation time and may elevate the likelihood of traffic incidents as evacuees remain within hazardous regions for longer periods of time.

This paper develops an analytical framework for studying traffic signal coordination in evacuation scenarios, as well as presenting some initial insight from simulated evacuation policies. We consider a road network consisting of a directed tree, with several roads merging at some number of intersections. Within this model, we evaluate traffic signal policies under multiple intersection-level cost functions, including total travel time and time spent at a standstill. At the system level, we analyze social welfare objectives such as minimizing worst-case evacuation time across all vehicles.

The proposed formulation draws on queuing and network theory and traffic flow modeling and enables comparison between coordinated and decentralized signal control policies. In particular, we quantify inefficiencies arising from locally optimized or noncooperative control strategies via the price of anarchy. This analysis yields insight into the potential performance gains achievable through centralized or coordinated signal timing during evacuations and provides guidance for the design of traffic control policies under emergency conditions.

The main contributions of this paper include:

- A queue-based model for evacuation traffic along a directed road network with variable signaling policies at each intersection.
- A comparative analysis of coordinated and decentralized traffic signal policies under intersection- and system-level costs.
- Quantification of the price of anarchy for evacuation signal control and characterization of conditions under which coordination yields substantial performance gains.

II. Background: What Is Game Theory?

Game theory is, most generally, the study of optimization problems. It was first heavily studied in the context of economics and the study of rational agent decisions [2]. We define various aspects of games below.

1) Players and Actions: When multiple objective functions (or *players*, defined by their cost or utility function J_i) interact, the nuances of *simultaneous optimization* emerge. These objective functions or players share a common set of arguments γ (called *actions*) which live in some set

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of possible assignments Γ. It is common to assume that the players have control over non-overlapping subsets of the decision variables, such that each of the n players (or optimization functions) optimizes over their disjoint decision variables or actions $\gamma_i \in \Gamma_i$, $\Gamma = \bigotimes_i \Gamma_i$, subject to the actions of the other players, i.e.,

$$\begin{aligned} \forall i,\ \min_{\gamma_i}\ & J_i(\gamma_i, \gamma_{-i}) \\ \text{s.t. } & g(\gamma_i, \gamma_{-i}), \end{aligned} \tag{1}$$

where $\gamma_{-i} \in \bigotimes_{j \neq i} \Gamma_j$ indicates the decision variables or actions of players other than player i, and g is a relation encoding any additional constraints of the problem.

Zero-player games are often considered in predictive modeling, physics, and simulation. One-player games are found in the discipline of optimal control and other traditional optimization, such as machine learning and convex optimization. The presence of two or more players makes up traditional game theory, with some relation to robust control when nature or noise is modeled as an adversarial decision process.

The degenerate case where each J_i is only a function of γ_i is known as *decoupled games* (as they become uncorrelated optimization problems, only coordinating via the constraints, which may also be decoupled), while the opposite case J_i only depending on γ_{-i} are called *dummy games* (as no optimization can be done by any player acting in a non-cooperative way). Simultaneous optimization problems, which are a simple linear combination of a decoupled and dummy component, are called *potential games*. Games where all players share the same utility or objective function are called *identical interests* games, and games where the objective functions are only equivalent up to permuting the players are called *symmetric* games.

2) Distinctness of Players: Non-cooperative games are the main focus of modern game theory [3]. This is the assumption that players act to optimize their own objective function J_i without consideration of other players' objectives. This arises from the assumption that individual objective functions are not comparable, for if there were a measurable trade-off between these values, they would be a single optimization problem. This is why, when comparing the value profiles during simultaneous optimization, one can only speak of individual preferences or Pareto dominance of subsets of the optimization processes. *Cooperative games* can be encoded as non-cooperative games by appropriate design of correlated actions and by combining coordinating players appropriately (such ideas as correlated equilibria emerge here, where joint actions are no longer independent).

3) Scheduling: Static games or *simultaneous games* happen once and simultaneously, whereas *dynamic games* or *sequential games* involve a time component, and available/optimal actions may depend on the state of the game, which evolves according to previous actions. This allows for interesting extensions, including player self-coordination with its past and future selves (such ideas as subgame-perfect equilibria emerge here). This is essentially a difference in the scheduling of the individual optimization processes (i.e., whether they commit their variable assignments simultaneously, in some order, or round-robin until convergence).

4) Shared Process Knowledge: Complete information is characterized by all players knowing the game constraints, non-decision-variable states, and the objectives and decision variables available to all other players, as opposed to *incomplete information*. This is equivalent to each optimization process containing a model of the other optimization processes, as opposed to only knowing the constraints. An interesting extension is for players to be aware of subsets of other players and subsets of the game features listed above.

5) Shared Execution State: Perfect information is characterized by every player knowing all previous actions taken by all other players, as opposed to *imperfect information*. This is equivalent to the individual optimization processes sharing some subset of their decision variables with other players during execution to use in their own decisions.

6) Nash and Mixed Strategies: The most common solution concept in game theory is the *Nash Equilibrium* [4], where players have no unilateral incentive to change their decision variables given the actions selected by the other players. In other words, each optimization process can do no better than its current decision variable assignments, assuming that the other decision processes do not also deviate, i.e.:

$$J_i(\gamma_i^*, \gamma_{-i}) \leq J_i(\bar{\gamma}_i, \gamma_{-i}) \quad \forall i, \bar{\gamma}_i \in \Gamma_i \tag{2}$$

For any game, it is possible to define a meta-game where the players sample their action from a probability distribution, then play the original game using the sampled action. This concept is ubiquitous in game theory, and is called a *mixed strategy*. This construction allows us to extend finite and countable games in such a way as to recover guarantees provided by games of continuous and complete (in the real analysis sense) action spaces, such as the sure existence of a mixed Nash equilibrium [4].

7) Resource Games: We will consider a subset of such games sharing a common distinction as *resource games*. These are games where additional structure arises due to the presence of a set $\mathcal{R}$ of distinct objects within the problem, called resources $r \in \mathcal{R}$, which players choose to include or exclude in their decision variables (i.e. $\Gamma_i \subseteq 2^{\mathcal{R}}$, so $\gamma_i \subseteq \mathcal{R}$, $\forall i$). This introduces the interpretation of a dual problem from the perspective of individual resources, as opposed to individual players. A common approach to solving resource games is to define a global or *social welfare function* $W : \Gamma \to \mathbb{R}$ over the resources, derived using the actions of each player. The marginal contribution to this welfare function as each player selects a resource is one measure that translates global objectives into individual player decisions, and Shapley values provide even stricter guarantees at the cost of more computation [5]. We define the set-function difference operator Δ:

$$\begin{aligned} \Delta_B W(A) := W(A \cup B) - W(A), \\ \forall A, B \subseteq \Gamma, \end{aligned} \tag{3}$$

which is interpreted as the marginal contribution of adding the elements of B to A, so the marginal contribution of player i participating in the game is $\Delta_{\gamma_i} W(\gamma_{-i})$. We also define a second-order difference function by noting the linearity of Δ, i.e., $\Delta_C \Delta_B W(A)$ is evaluated by first expanding $\Delta_B W(A)$, then distributing and evaluating the Δ_C operator as defined. Furthermore, the difference operators commute for any sets B, C. Various useful restrictions of resource games can be defined in terms of this difference operator, including (for maximizing-utility problems, reversing the inequalities for minimizing-cost games):

$\forall A, B, C, \gamma_i, \gamma_{-i} \subseteq \Gamma$, $\gamma = \gamma_i \cup \gamma_{-i}, \forall i$:

1) Submodular welfare utility: $\Delta_C \Delta_B W(A) \leq 0$
2) Supermarginal player utilities: $\Delta_{\gamma_i} W(\gamma_{-i}) \leq J_i(\gamma)$
3) Superadditive welfare utility: $\sum_i J_i(\gamma) \leq W(\gamma)$
4) Nondecreasing welfare utility: $0 \leq \Delta_B W(A)$
5) Normalized welfare utility: $W(\varnothing) = 0$

The first three properties characterize *valid utility games*, and all five properties restrict the set of resource games so much as to provide a worst-case *price of anarchy* (the ratio between the optimal social welfare and the worst Nash equilibrium's social welfare, given Γ, or the reciprocal of that for minimization games) of $1/2$ [6]. The proof of this bound on the price of anarchy for such games utilizes the fact that opposite-signed first- and second-order conditions constrain the second and higher orders to approach zero in the limit of increasing argument. The proof also employs the following telescoping identity:

$$\Delta_B W\Big(\bigcup_i A_i\Big) = \sum_i \Delta_{\Big(\bigcup_{k=1}^{i-1} A_k \cup B\Big)} W(A_i) \tag{4}$$

III. Formulation

Let us model a road network as a directed tri-modal graph $(\mathcal{G}, \mathcal{I}, \mathcal{V}, \mathcal{Q})$. Let our traffic network have a set of nodes $\mathcal{N} = \mathcal{G} \cup \mathcal{I} \cup \mathcal{V}$ with labels g_k, n_i, v_ℓ, where $k, i, \ell \in \{-1, ..., -|\mathcal{G}|\}, \{1, ..., |\mathcal{I}|\}, \{0\}$, respectively. We will make a simplifying assumption about the family of networks under consideration in the current work; let our network be an acyclic tree, meaning that every node has a unique path to the root node. In this tree, the root node is the safe node v_0, all leaves are generators g_k, and all intermediate nodes are intersections n_i, as defined below.

1) Generator Nodes: g_k are generator nodes, forming a node subset we denote $\mathcal{G}$. They are distinguished by a negative index $k < 0$. These nodes have out-degree one and in-degree zero, and they produce cars at some rate $\lambda_k \in \lambda$, adding them to the queue of their single out-edge (as described below).

Each car has an attribute $\tau_{\text{car has existed}}(t)$ keeping track of how long it has been on the road since being generated, or the time it took to reach a safe node if it has already reached one. We make the additional simplifying assumption that all cars are equivalent modulo what generator node k they originate from, so properly τ_k is the maximum total travel times over all cars originating from generator $g_k \in \mathcal{G}$ over the course of the simulation.

2) Intersection Nodes: n_i are intersections, which node subset we denote $\mathcal{I}$. Each intersection has a state $x_i \in \mathcal{X}_i$ at every time t encoding which in-edges are open and to which out-edges they flow, and a policy $\pi_i : \mathcal{X}_i \times \mathcal{Q}_i \to \mathcal{X}_i$, which determines the scheduled decision (for each time t and neighboring queue states $\mathcal{Q}_i \subseteq \mathcal{Q}$, as described below) of that node to move cars from the queue of some in-edge of node i to the queue of some out-edge of node i. The intersections move cars by changing their state x_i to open traffic from some number of their in-queues, subject to constraints on which inlets can be open simultaneously.

3) Safe Nodes: $\mathcal{V} = \{v_0\}$, the safe region. This safe region does not need to be topologically consistent with the road layout; all safe exits can W.L.O.G. feed into the same safe node V. This node has zero out-degree.

4) Directed Edge Queues: Let there be a set of directed edges $\mathcal{Q}$ with labels $(m, n) \in \{\mathcal{N}\} \times \{\mathcal{N}\}$, representing an edge from node n to node m. Let this set be partitioned into subsets $\mathcal{Q}_i \subseteq \mathcal{Q}$ such that the edges $q_{ij} \in \mathcal{Q}$ have out-edges into node $n_i \in \mathcal{N}$ (or $v_0 \in \mathcal{V}$). Note that, according to the degree constraints of the nodes, the following are true:

- $\nexists j \in \mathbb{Z} : q_{j0} \in \mathcal{Q}$ (the safe node has out-degree zero)
- $\nexists j \in \mathbb{Z}, k < 0 : q_{kj} \in \mathcal{Q}$ (the generator nodes have in-degree zero)
- $\forall k < 0,\ |\{j : j \geq 0, q_{jk} \in \mathcal{Q}\}| = 1$ (the generator nodes have out-degree one)

Every edge $q_{mn} \in \mathcal{Q}$ is a queue. Each queue has a current traffic volume S_{mn} and a maximum capacity C_{mn}, as well as a minimum holding time for each car $T_{mn}(S_{mn})$, which is a function of the current volume of traffic in that queue.

Let every queue have an attribute $\tau_{mn}(t)$ which measures the maximum time over all cars in the queue q_{mn} at time t that the cars have been waiting in that particular queue ($\tau_{\text{car in queue}}(t)$, distinct from but never greater than $\tau_{\text{car has existed}}(t)$).

5) Welfare Cost: Let us now define the welfare cost as a sum over all maximum travel times from each generator g_k, weighted by some parameter α_k (either $= \frac{1}{|\mathcal{G}|}$ or $= \lambda_k$, or possibly some weight accounting for proximity to danger or safety, depending on our model):

$$\begin{aligned} W(\lambda, \pi) &= \sum_{g_k \in \mathcal{G}} \alpha_k \mathbb{E}_\lambda \big[\tau_k\big] \qquad (5) \\ &= \sum_{g_k \in \mathcal{G}} \alpha_k \mathbb{E}_\lambda \big[\max_t \max_{\text{car } \in g_k} \tau_{\text{car has existed}}(t)\big] \end{aligned}$$

where π is the set of all intersection policies $\pi_i \in \pi$, and λ is the set of all generator parameters $\lambda_k \in \lambda$.

6) Player Costs: We also define individual cost functions for the intersections $n_i \in \mathcal{I}$ in our traffic model; these are objectives that minimize the maximum wait time over all incoming roads at that intersection during the evacuation:

$$\begin{aligned} J_i(\lambda, \pi_i, \pi_{-i}) &= \mathbb{E}_\lambda \big[\max_t \max_{q_{ij} \in \mathcal{Q}_i} \tau_{ij}(t)\big] \qquad (6) \\ &= \mathbb{E}_\lambda \big[\max_t \max_{q_{ij} \in \mathcal{Q}_i} \max_{\text{car } \in q_{ij}} \tau_{\text{car in queue}}(t)\big] \end{aligned}$$

where π_{-i} is the set of all policies of intersection other than intersection i. In the most general case of an arbitrary network, these policies constitute selections of resources, where the resources in question are all paths leading from generators to the safe node, some of which go through each intersections.

A. As a Markov Decision Process

We could model the evolution of the network state as a Markov Decision process (MDP), where different possible configurations of the queue volumes (including the variables of each car in the queues) represent different states of the network, and transitions between states are defined for incrementing time, generating cars, and executing intersection policies.

We construct a 4-tensor $X_{q_{mn},\tau_{mn},k,\tau_{\text{car}}}(t)$ (which, when flattened, is the state vector), where the four dimensions of this array represent the queue, the time spent in that queue, the associated generator, and the time since generation. The entries of this array count the number of vehicles at time t having the configuration specified by the indices of X. For example, if at time step $t = 42$, two cars in queue $q_{2,3}$ have been in that queue for 2 time steps, and were both generated at node -1 and have existed for 5 time steps total, then the entry $X_{q_{2,3},2,-1,5}(42) = 2$. The MDP is, in general, nonlinear due to the variable minimum wait time of the queue (varying with total traffic on that queue). In the simplified case where $T_{mn}(S_{mn})$ is a constant, the MDP can be expressed as follows:

$$\begin{aligned} x(t+1) &= Ax(t) + B_g u_g(t) + B_n u_n(t), \\ y(t) &= f(x(t)), \end{aligned} \tag{7}$$

where x is the flattened state 4-tensor described above, u_g is the input from the generators g_k, which adds cars to their assigned queues probabilistically, u_n is the movement of cars between queues according to the policy of each intersection n_i (encoded as adding a negative car to the old state entry, and adding a car to the new entry), and y is the measurement, which is a vector listing the player and welfare cost functions.

The construction of A, B_g, B_n, f are described:

- A simply increments the timers of each vehicle currently in X by moving all nonzero entries to one index farther in the τ_{mn} and τ_{car} directions, similar to a Jordan block with eigenvalue zero. We assume the state vector is large enough that for a simulation of fixed length, there is always a time-index that the cars can occupy. The timers of cars already in the safe node's queue are not incremented.
- B_g simply maps each of the $|\mathcal{G}|$ generators to the queue each car is added to, so that on a time step a car is added by g_k, the k entry of $u_k = 1$.
- B_n maps instructions from u_n to move cars between queues (including to the safe queue). This is done similar to the moving of cars from one time state to another in the same queue in A, but the queue timer is reset by putting the single car's value into an index $\tau_{\text{car}} = 0$.
- f has one output for each intersection cost, plus another output for the welfare cost. The player costs can be modified to be linear combinations of the queue states, or a nonlinear f can be employed to better represent (6). The welfare function depends only on the whole-path-time axis for the entries in the safe queues.

Even with a nonlinear MDP to encompass the full model, one could derive a Fokker-Planck equation that is linear in the expectations of the values for each entry, which might better encompass the expected behavior of the network under each policy $u_n = \pi(x)$.

We did not program this MDP explicitly as a nonlinear system of equations, but instead constructed the model using object-oriented programming in the Python language. The code repository can be found in [7]. The state of the network is tracked over time, and we analyze the results of the simulation of a particular road network under four different policies.

B. As a Minimization Valid Utility Game

We model the evacuation traffic queuing on a directed tree network as a resource game, where the players are the intersections n_i and they select resources constituting their incoming queues q_{ij}, or paths $\mathcal{P}_{ik}$ from generators g_k that feed into those intersections, or the generators g_k themselves upstream of n_i (all are equivalent in a tree network). These resources are selected for each time step, such that a single resource is one green light for one time step, with non-selection of that resource meaning the light is red at that incoming road at that time step. While an intersection interprets these resources in terms of their local traffic light, the social welfare sees the full paths from generator to safety as the resource, so that a path is open to a vehicle if all intersections on that path choose that path or resource, and are greenlit. There is some nuance concerning time; as vehicles take certain time to complete their path, the intersections must select that resource in a temporal order conducive to that car moving, such that the longer it takes for the intersections to select a full path, the larger the social cost component τ_{car} will be. We recall the modification of valid utility game conditions for minimization games:

$\forall A, B, C, \gamma_i, \gamma_{-i} \subseteq \mathcal{G}$, $\gamma = \gamma_i \cup \gamma_{-i}, \forall i$:

1) Supermodular welfare cost: $\Delta_C \Delta_B W(A) \geq 0$
2) Submarginal player costs: $\Delta_{\gamma_i} W(\gamma_{-i}) \geq J_i(\gamma)$
3) Subadditive welfare cost: $\sum_i J_i(\gamma) \geq W(\gamma)$
4) Nonincreasing welfare cost: $0 \geq \Delta_B W(A)$
5) Normalized welfare utility: $W(\varnothing) = 0$

We show that modeling greenlit times as resources almost satisfies these axioms. Note that this discussion is in expectation over the random vehicle generation processes (and the costs are defined in this way); it may be possible for some combinations of vehicle generation events to create enough temporary conflict to make that particular run look like it violates any of these properties.

1) Supermodular Welfare Cost: In short, the convexity of the $\max$ function should provide the convexity, or positive second-order difference, condition for W. An equivalent definition of supermodularity is:

$$W(A \cup B \cup C) \geq W(A) + \Delta_B W(A) + \Delta_C W(A) \quad (8)$$

This means that the longest-travel-time-car over the evacuation with network generator sets A, B, C stems from at least the congestion we would have with only the generators in A, plus the marginal improvement of adding only the green lights in B to A, plus the marginal improvement of adding only the green lights in C to A, plus some extra congestion that might occur due to interaction between the two new groups B and C. This guarantee that adding more vehicles on the road cannot decrease travel times is provided by the monotone increasing minimum queue time T_{mn}, so more cars cannot decrease travel times.

2) Submarginal Player Costs: The property states that the change in the social welfare cost when green-lighting the generator nodes upstream of node n_i for a new time step must be no less than the individual cost to intersection n_i. As n_i is a bottle neck for all cars passing through it (all cars generated upstream of it), the marginal welfare cost contribution of green-lighting the roads upstream of n_i is lower-bounded by the maximum wait time at the new node n_i (unless it is on a path that did not get close to congested before, then it doesn't really increase the welfare cost).

This property is unfortunately trivially untrue for our resource set, as an intersection not participating would contribute infinite cost; moreover, with positive intersection costs, it is impossible to satisfy this condition and non-increasing (4) simultaneously.

3) Subadditive Welfare Cost: The car that took the longest to get from g_k to v_0 over the entire simulation could not have taken longer than if they were also the maximum-waiting car at every intersection along the path $\mathcal{P}_{0k}$ from g_k to v_0:

$$\begin{aligned} \max_t \max_{\text{car } \in g_k} \tau_{\text{car}}(t) &\leq \sum_{i \in \mathcal{P}_{0k}} \max_t \max_{q_{ij} \in \mathcal{Q}_i} \tau_{ij}(t) \quad \forall g_k \in \mathcal{G} \\ &\leq \sum_{i \in \mathcal{I}} \max_t \max_{q_{ij} \in \mathcal{Q}_i} \tau_{ij}(t). \end{aligned} \quad (9)$$

Now, summing over all generators:

$$\sum_{g_k} \max_t \max_{\text{car } \in g_k} \tau_{\text{car}}(t) \leq |\mathcal{G}| \cdot \sum_{i \in \mathcal{I}} \max_t \max_{q_{ij} \in \mathcal{Q}_i} \tau_{ij}(t). \quad (10)$$

Dividing by $|\mathcal{G}|$ and taking the expectation over the generation processes λ (a linear operation) produces the subadditive welfare cost condition, with $\alpha_k = \frac{1}{|\mathcal{G}|} \; \forall k$ (3 for the conditions in Sec. II-.7):

$$W(\gamma) \leq \sum_{i \in \mathcal{I}} J_i(\gamma). \quad (11)$$

4) Nonincreasing Welfare Cost: This condition probably holds for sub-critical generation rates. Under these conditions, the expected traffic volume over all time does not exceed the capacity of the roadways, so opening a green light can only improve the travel times of vehicles that would have waited at the red light, assuming that no more than one car moves through an intersection no matter how many lights are green.

5) Normalized: This condition holds in a different sense, as the action of no intersection allowing any cars through for any time step leads to infinite travel times. We might consider this infinite cost as the zero-point in our model; this might suggest, however, that using the negative reciprocal or some other transform of our costs would recover some these conditions.

IV. Results and Criticisms

If cars are generated faster than the road can evacuate them, the wait times grow unbounded with longer simulations, as expected, due to the in-roads becoming backed up. The critical point is, of course, when the expected number of cars generated upstream of an intersection exceeds one per time step, as the intersection can only shunt one car per time step, so the number of cars waiting grows unbounded in expectation. In a supercritical evacuation network, the policies take turns prioritizing certain paths, which is seen as waves in the distribution of recorded wait times. During blank expected times, that road cleared while others waited, and cars were not recorded on that route for that wait time, until some time passed and the expected wait time increased, leaving a blank band at that expected wait time.

Local, greedy policies tend to favor cars that generate close to safety, while prioritizing the cars' total travel time favors cars from farther generators. See Fig. 5; the generator nodes farthest from safety (g_{-6}, g_{-12}), experiencing more non-trivial intersections and merging with more tributary roads, so performed poorly under in local policy. The generator nodes closer to safety did much better, and did not even become backed up under the supercritical generation rate (see Fig. 6). In comparison, the global policy improved the travel times of the farthest cars marginally, at the expense of increasing the travel times of other vehicles by more than an order of magnitude. The expected maximum travel times at each generator was approximately equal under the global FIFO policy, which makes sense as the bottleneck intersection would always give priority to the highest-time car, equalizing the routes accordingly.

The social welfare was not improved by using the tested globally coordinated policy for any rate cars were introduced into the network. The distributions of travel times were comparable at best, and less skewed at worst (so the expected time increased) until the critical generation rate. We also did not prove that any of these policies are optimal for either the welfare or individual intersection costs.

We were also unable to prove any price-of-anarchy equivalent for this queue network, but only provide some heuristics for future investigation of such a property. In addition to a socially optimal policy, we attempted to compare a Nash equilibrium strategy to the "all lights are green all the time" policy, but without the proper inequality conditions, we found counterexamples to many of our suspected "theorems." If some formulation exists (consisting of cost functions and a resource set from which the intersections select) that satisfies the valid utility conditions, we would be able to state that that particular set of cost functions has a price of anarchy no greater than two (for minimization games), meaning that any Nash equilibrium of the distributed system achieves no worse than twice the social welfare of a globally optimal strategy. We suspect that information sharing might not be an issue [8], as intersection costs depend indirectly on the policies of other intersections due to the flow of vehicles being influenced thereby.

We do not know how the results discussed here extend to general directed networks, or even to directed acyclic graphs, where multiple paths exist between the generator nodes and the safe nodes. This model is also too simplified to be directly applicable in real-world evacuation procedures, as it is missing such essential features as the structure of standard four-way intersection, other constraints to minimize minor accidents during evacuation, and analyses of general-weighted cost functions accounting for a (possibly mobile) positioned danger, such as a wildfire or flood front.

V. Conclusion and Future Work

This paper introduced a queue-network model of evacuation traffic, restricted to a directed tree, and used it to compare decentralized and coordinated traffic signal policies under multiple performance metrics. By representing intersections as agents controlling access to incoming queues and vehicles as stochastic flows from generators to a safe node, the model captures essential features of evacuation traffic, including bottleneck formation, merging conflicts, and highly skewed (even uniform unbounded, in some cases) delay distributions. Simulation results across subcritical, critical, and supercritical traffic generation rates demonstrate that local signal policies can induce substantial inequity across routes and that globally coordinated policies do not necessarily reduce worst-case delays enough to warrant their use. While global policies do seem to improve evacuation times for certain vehicles, possibly vehicles closest to danger, they do so at the expense of significantly increasing average travel times for other vehicles. Notably, no tested policy consistently minimized social welfare across all regimes, underscoring the intrinsic trade-offs between fairness, throughput, and robustness during evacuations.

From a game-theoretic perspective, we investigated whether evacuation signal control can be formalized as a minimization valid utility game, which would enable nontrivial price-of-anarchy guarantees. While several structural properties—such as subadditivity of welfare cost and monotonicity under subcritical flow—hold under reasonable assumptions, other required conditions seem to fail in this setting. In particular, the incompatibility between nonincreasing welfare cost and finite intersection-level costs prevents direct application of existing bounds, and must be reconciled via a better choice of cost function, possibly a Shapley cost. As a result, we do not establish a formal price-of-anarchy guarantee, but instead provide counterexamples and heuristics that delineate the limits of standard resource-game abstractions for evacuation traffic.

There are several natural directions for future work. First, the current analysis is restricted to directed tree networks, which exclude cycles and alternative routing paths. Extending the framework to directed acyclic graphs or general road networks would allow vehicles to reroute dynamically and would more closely reflect real evacuation scenarios. Second, the signaling policies considered here are heuristic and memoryless; incorporating predictive or state-augmented policies—such as limited look-ahead scheduling or queue-length forecasting—may yield improved trade-offs between local and global objectives. Third, the welfare formulation could be enriched to account explicitly for spatially varying hazard intensity, incident risk, or heterogeneous vehicle priorities, allowing evacuation urgency to be modeled directly rather than indirectly through travel time. Finally, we should investigate computing Shapley values for each green light, at the cost of much computation time, though a Monte Carlo (or random rollout) approximation may suffice, provided measurable expected error.

Overall, this work highlights that evacuation traffic control differs qualitatively from standard traffic optimization problems: extreme demand, safety considerations, and fairness objectives fundamentally alter the structure of optimal policies. Ultimately, there may not be a perfect evacuation policy, especially in extremely hazardous conditions, where the trade-offs of different policies and cost measurements must be evaluated as an ethical argument beyond the scope of policy optimization. The math will always be the easy part, and deciding the cost of exposing any one vehicle to danger will always remain the intractable problem.

References

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Appendix

The figures referenced in the text, and other similar figures, are located here to summarize the data collected from our simulations. The code is found in [7].

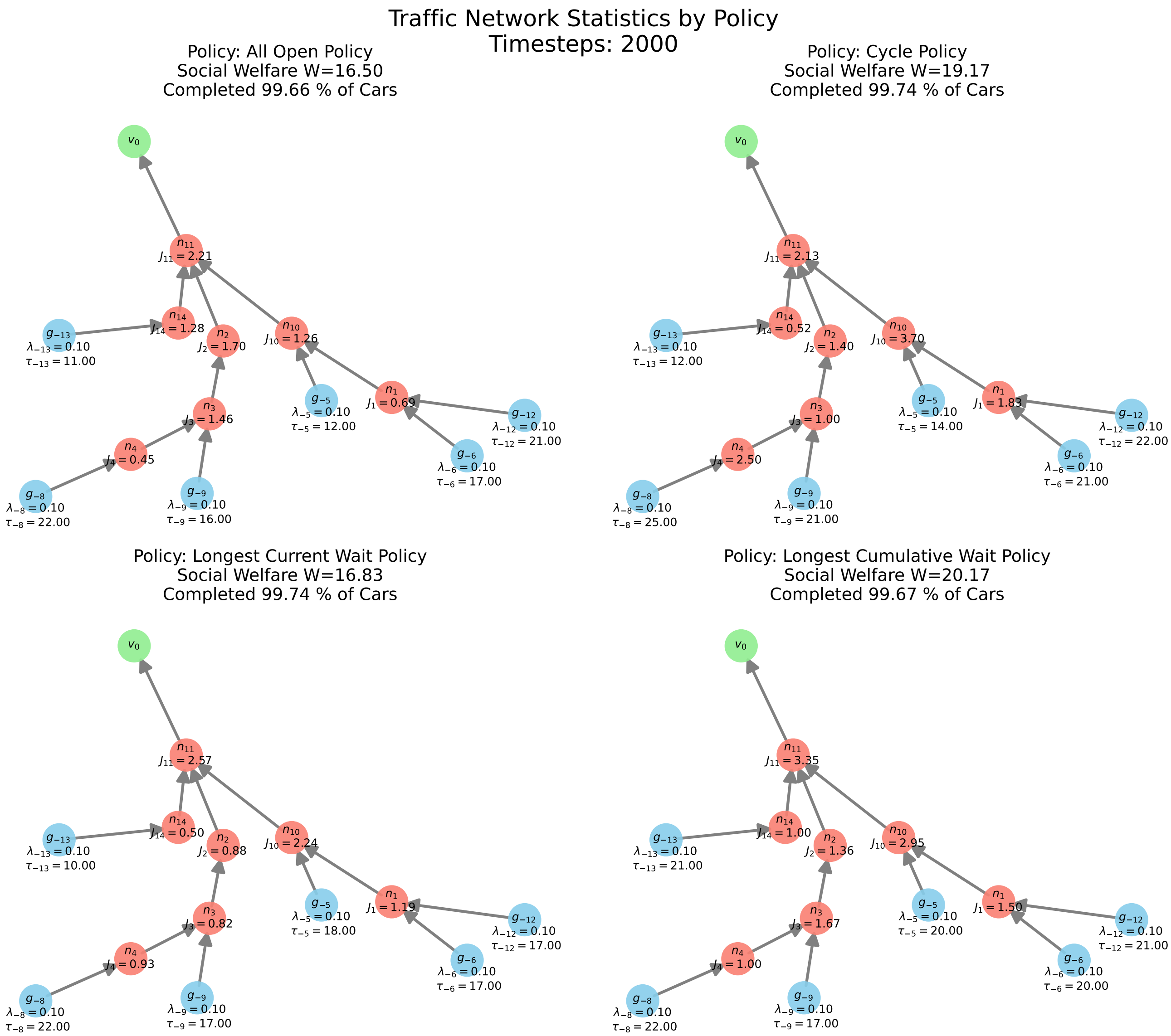

Fig. 1. A subcritical generation rate on the network model used in these simulations. Each edge has capacity 5, and minimum wait time before dequeuing $\tau_{mn}(t) \geq 2 + 2\ln(S_{mn})$. Blue nodes or leaves are generators (in this run, a subcritical rate is used, so $\lambda_k = 0.1 < \frac{1}{|\mathcal{G}|}\ \forall k$), red or internal nodes are intersections employing the policies in the title of each sub-figure, and the green root node is the safe node v_0. The numerical labels on the generators are the generation rates λ_k (constant) and the maximum times any car from that node took to reach the safe node $\approx \tau_k$, and the costs over the intersection nodes are the maximum time a car spent at that intersection $\approx J_i$, averaged over the 2000 time steps of this simulation. The social welfare W was computed by weighing all generator nodes equally, i.e. $\alpha_k = \frac{1}{|\mathcal{G}|}\ \forall k$.

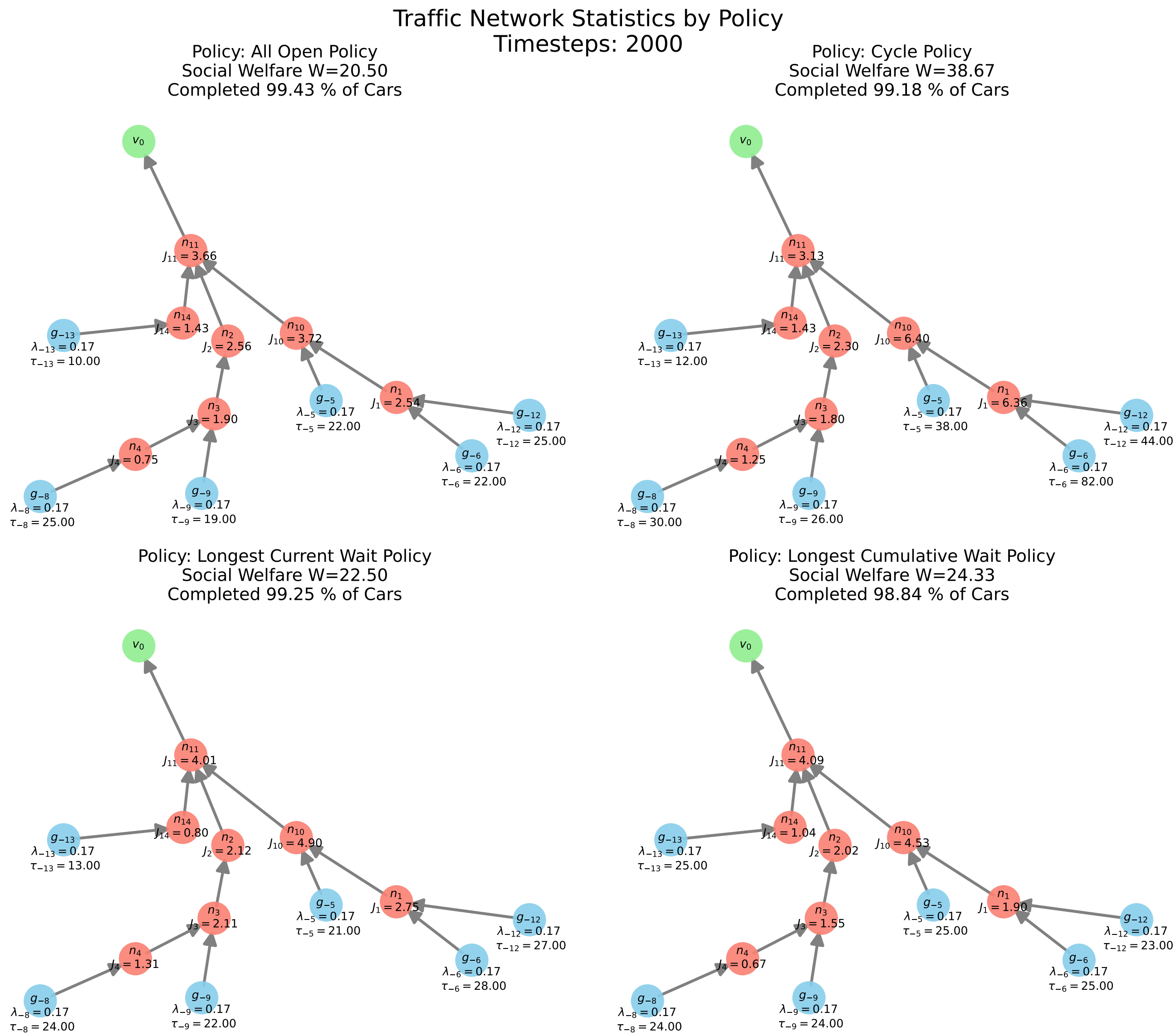

Fig. 2. A critical generation rate on the network model used in these simulations. Each edge has capacity 5, and minimum wait time before dequeuing $\tau_{mn}(t) \geq 2 + 2\ln(S_{mn})$. Blue nodes or leaves are generators (in this run, the critical rate is used, so $\lambda_k = \frac{1}{|\mathcal{G}|} \approx 0.167\ \forall k$), red or internal nodes are intersections employing the policies in the title of each sub-figure, and the green root node is the safe node v_0. The numerical labels on the generators are the generation rates λ_k (constant) and the maximum times any car from that node took to reach the safe node $\approx \tau_k$, and the costs over the intersection nodes are the maximum time a car spent at that intersection $\approx J_i$, averaged over the 2000 time steps of this simulation. The social welfare W was computed by weighing all generator nodes equally, i.e. $\alpha_k = \frac{1}{|\mathcal{G}|}\ \forall k$.

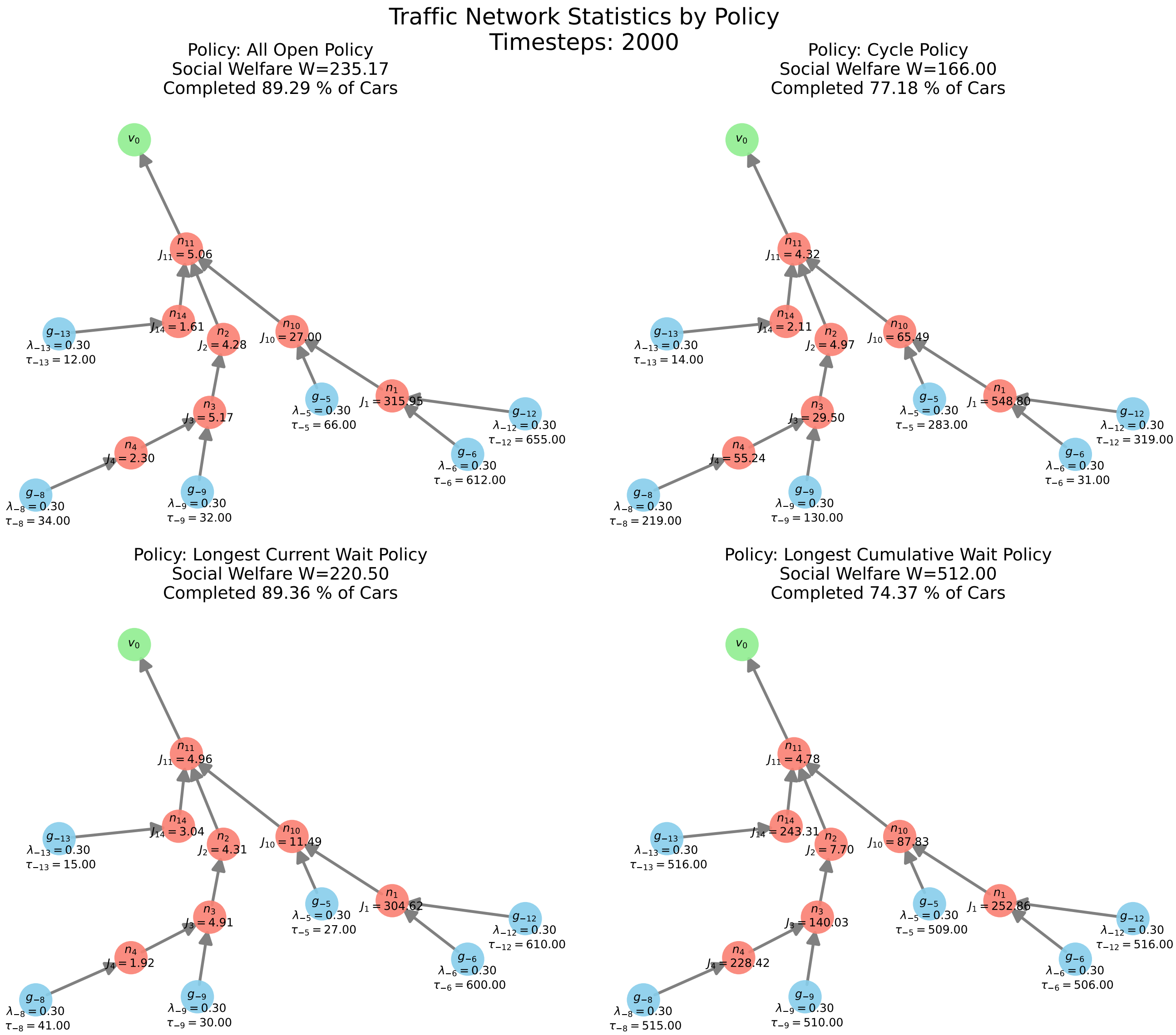

Fig. 3. A supercritical generation rate on the network model used in these simulations. Each edge has capacity 5, and minimum wait time before dequeuing $\tau_{mn}(t) \geq 2 + 2\ln(S_{mn})$. Blue nodes or leaves are generators (in this run, the critical rate is used, so $\lambda_k = 0.3 > \frac{1}{|\mathcal{G}|}\ \forall k$), red or internal nodes are intersections employing the policies in the title of each sub-figure, and the green root node is the safe node v_0. The numerical labels on the generators are the generation rates λ_k (constant) and the maximum times any car from that node took to reach the safe node $\approx \tau_k$, and the costs over the intersection nodes are the maximum time a car spent at that intersection $\approx J_i$, averaged over the 2000 time steps of this simulation. The social welfare W was computed by weighing all generator nodes equally, i.e. $\alpha_k = \frac{1}{|\mathcal{G}|}\ \forall k$.

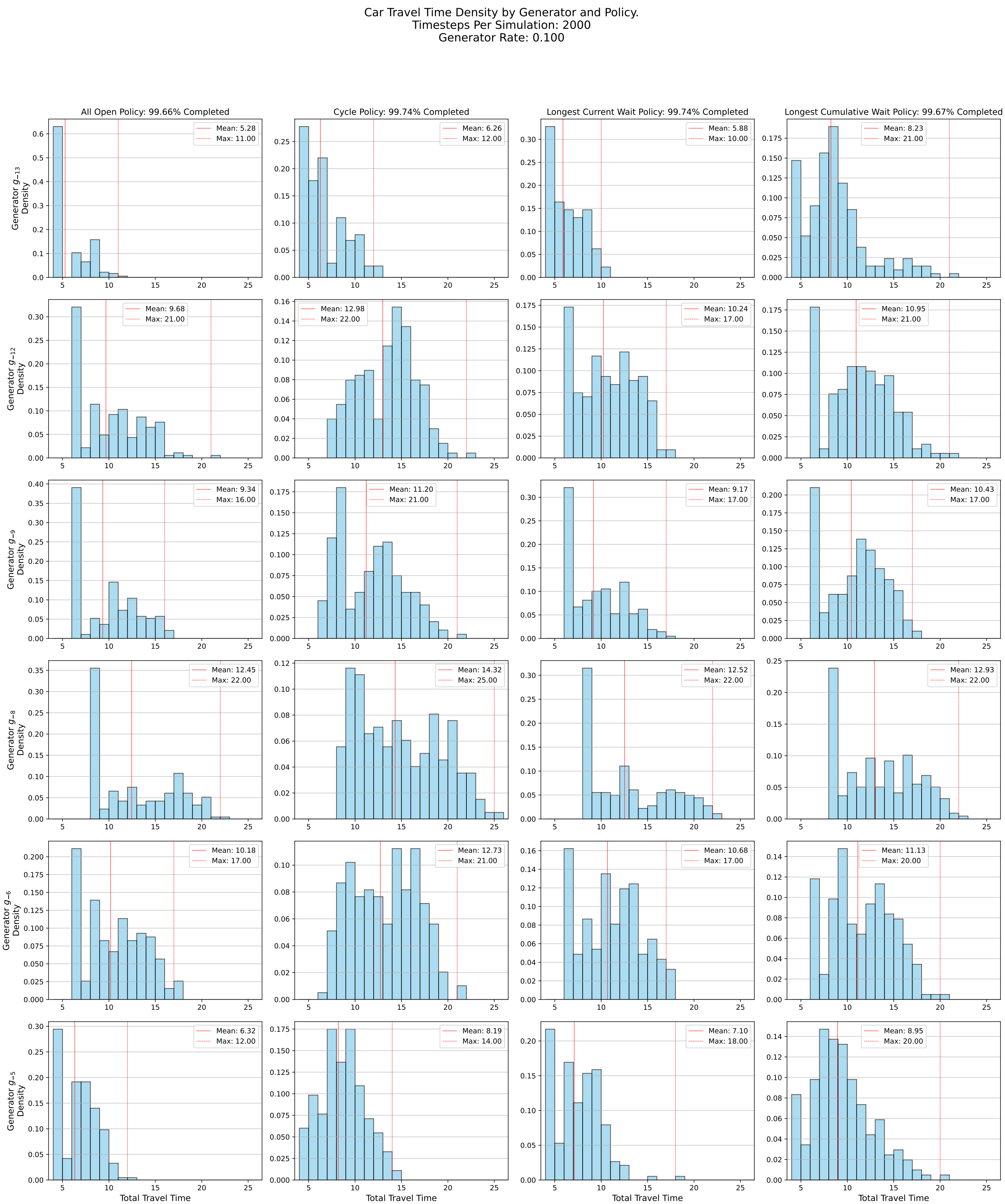

Fig. 4. Histograms of the route completion times for vehicles originating from each generator (row) when the intersections engage in four possible policies (column). These settings are below the critical generation rate, where the expected flow into the network does not exceed the maximum flow out. Note the positive skew of the distributions.

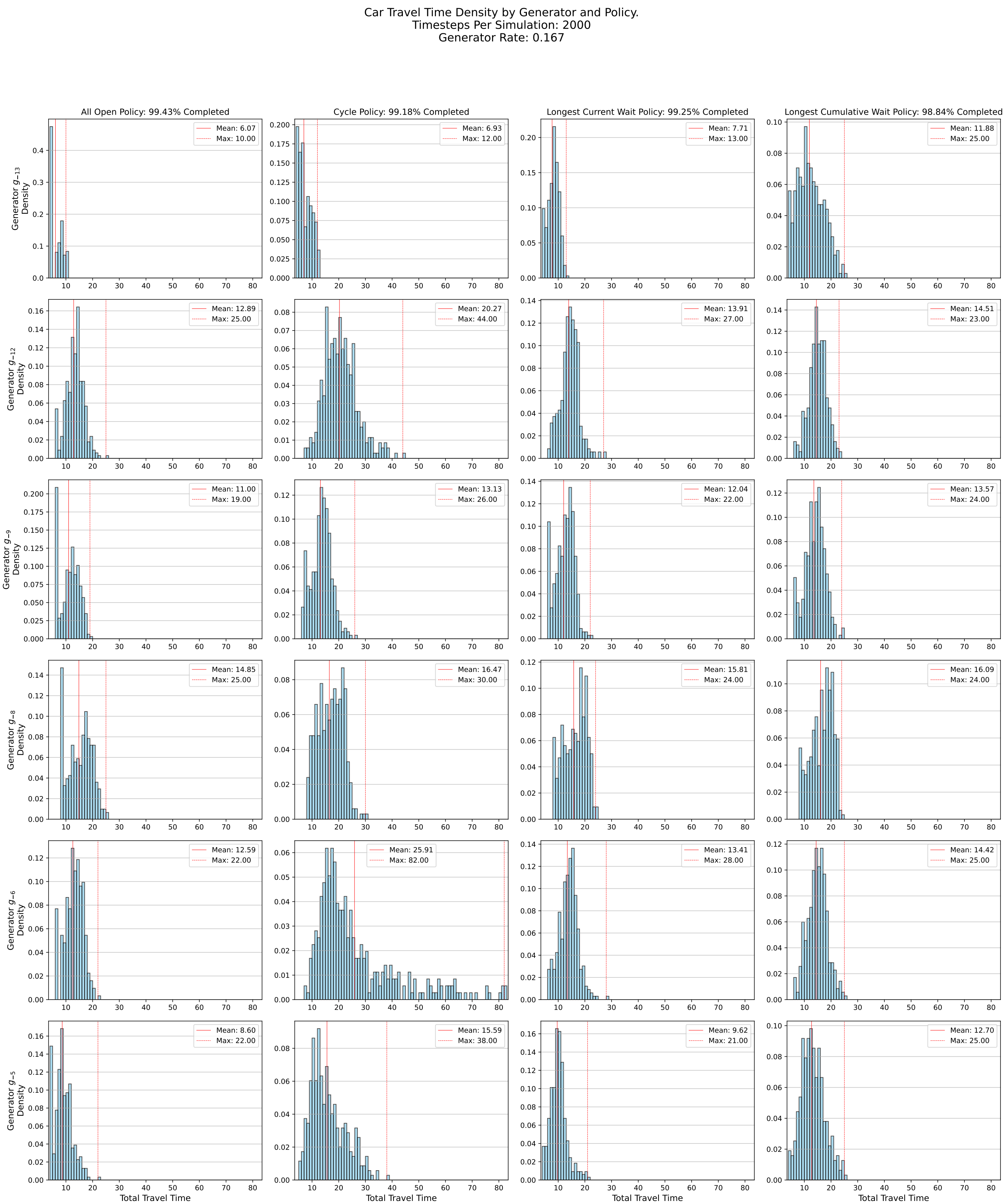

Fig. 5. Histograms of the route completion times for vehicles originating from each generator (row) when the intersections engage in four possible policies (column). These settings are at the critical generation rate where the expected flow into the network equals the maximum flow out. Note that some generators' distributions begin to exhibit a thick tail.

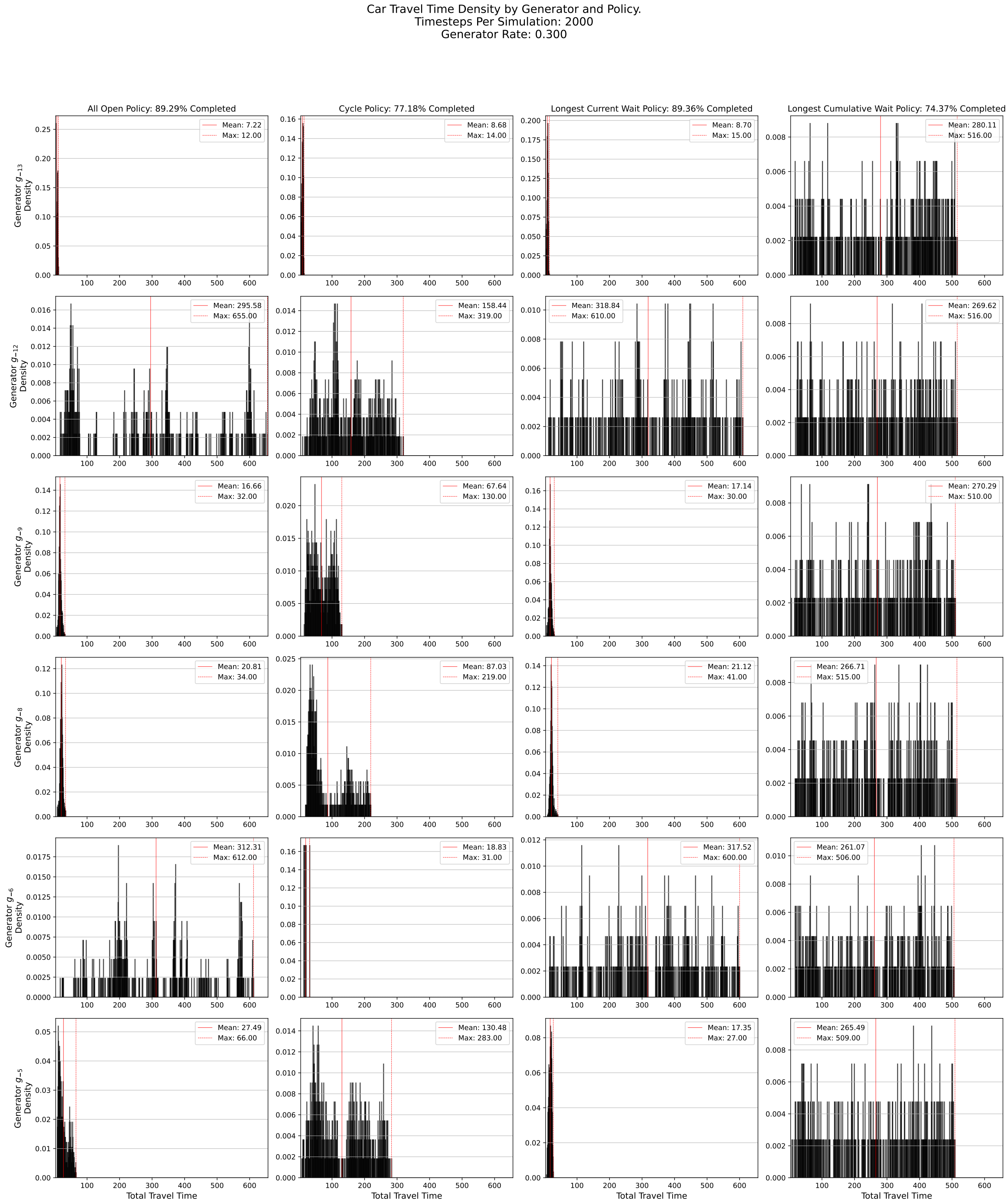

Fig. 6. Histograms of the route completion times for vehicles originating from each generator (row) when the intersections engage in four possible policies (column). These settings are beyond the critical generation rate where the expected flow into the network exceeds the maximum flow out. Note that some generators produce cars that flow quickly to safety regardless of other roads backing up, while some generators become unboundedly backed up, and the distributions of backed-up roads show repetitive patterns, due to taking longer turns under the prioritizing policies than a blind cycle. Also note that many more cars ended the simulation stuck in the network compared to the other two configurations.