ASSIGNMENT 5

3/19/25

① $T(n) = 3T(n/4) + 4n$

Repeated Substitutions

$T(n) = 3T(n/4) + 4n$ ← 1st

$T(n/4) = 3T(n/16) + 4(n/4)$ ← 2nd

$T(n) = 3(3T(n/16) + 4(n/4)) + 4n$

$= 9T(n/16) + 3 \cdot 4(n/4) + 4n$

$= 9T(n/16) + 3n + 4n$

$= 9T(n/16) + 7n$

$T(n/16) = 3T(n/64) + 4(n/16)$ ← 3rd

$T(n) = 9(3T(n/64) + 4(n/16)) + 7n$

$= 27T(n/64) + 9 \cdot 4(n/16) + 7n$

$= 27T(n/64) + 9(n/4) + 7n$

$= 27T(n/64) + \frac{9}{4}n + 7n$

$= 27T(n/64) + \frac{37}{4}n$

Pattern: $T(n) = 3^k T(n/4^k) + \sum_{i=0}^{k-1} 4n \cdot 3^i/4^i = 3^k T(n/4^k) + 4n \sum_{i=0}^{k-1} (3/4)^i$

$\sum_{i=0}^{k-1} (3/4)^i = \frac{1-(3/4)^k}{1-3/4} = \frac{1-(3/4)^k}{1/4} = 4(1-(3/4)^k)$.

$T(n) = 3^k T(n/4^k) + 16n(1-(3/4)^k)$

Recursion stops when $k = \log_4 n \rightarrow T(1) = \theta(1) \rightarrow T(n) = 3^{\log_4 n} T(1) + 16n(1-(3/4)^{\log_4 n})$

$T(n) = \Theta(n^{\log_4 3}) + 16n$

$T(n) = \Theta(n^{\log_4 3}) \approx T(n) = \Theta(n^{0.792})$ ← result

Master Thm: $T(n) = 3T(n/4) + 4n$

- $a = 3$
- $b = 4$
- $f(n) = 4n = \theta(n^1)$
- $f(n) = \theta(n^d) \rightarrow n^{\log_b a}$

$\log_4 3 \approx 0.792$ $d = 1 > 0.792$

$T(n) = \Theta(n^1) = \Theta(n)$ ← result