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An internal Stress Test Model for Liquidity Risk Management in Asset Management

Supervisor:

Professor Michele Bonollo

Candidate:

Luca Persia

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1. INTRODUCTION

The latest financial crises have shown how liquidity risk is inevitably linked to the emergence of market and credit risks, as well as contagious systemic risks. The 2007 crisis is an exemplary instance of so, as liquidity risk first manifested itself in the incorrect valuation of illiquid instruments and then in the so-called fly-to-quality, caused by panic, which followed the massive repricing, leading to the onset of a systemic crisis. However, banks still struggle to build shared models for measuring and managing liquidity risk and implementing appropriate stress tests to assess the prudentiality of their investments. Even in light of the latest financial disruptions caused by the pandemic crisis and the conflict between Russia and Ukraine, some banks struggle to understand their overall liquidity positions and find it impossible to measure liquidity by asset class. This is in particular due to the inconsistency with which liquidity risk is referred to. Strictly following Acerbi and Scandolo (2007), we know that the same term *liquidity risk* is correctly used for at least three distinct phenomena:

1. the risk that our portfolio may run out of money,
2. the risk we run by trading in an illiquid market,
3. the risk of a drain on the liquidity circulating in our economy.

Liquidity risk is actually a more complex occurance, manifesting itself simultaneously through all these interdependent facets. Although the literature often focuses on only one of these facets, it has now become crucial for financial institutions not only to measure liquidity, but also to consider every possible scenario that could impact their overall liquidity profile. In a nutshell, it is necessary to ensure, particularly for the fund and asset management sector in general, a certain degree of resilience and capacity to absorb economic shocks. In this paper, we will analyse liquidity risk under the latter aspect, which is mostly related to the first definition.

1.1 REGULATORY FRAMEWORK

In addition to what has been specified regarding the definition of liquidity risk, there are strictly more practical reasons behind this thesis, particularly regulatory ones. In September 2019, the Financial Stability Board issued recommendations to address structural vulnerabilities in asset management activities, which include provisions on stress tests and liquidity risk estimation models, specifically for fund management companies. The reference framework is the one issued by the European Securities and Markets Authority (ESMA) for stress simulations in the context of liquidity risk management, where the different constituent elements of a stress simulation framework are outlined, along with a menu of options that can be selected by stress testers. In particular, as far as Italian institutions are concerned – on which this paper focuses – this framework has been absorbed by the Bank of Italy directive (Banca d'Italia, 2021), from which the results of a survey carried out in 2020, promoted by the Bank of Italy itself, in agreement with Consob and ESMA, and focused on UCITS liquidity risk management by asset managers, can be deduced. We therefore refer in particular to harmonised open-ended funds (UCITS), which are more suitable also for retail investors, for whom, within certain terms of the law or the fund rules, the timing of entry and exit from the fund is substantially free. The findings of the analysis, conducted at the national level, point to a liquidity situation for UCITS funds that did not reveal any difficulties in the ability to meet redemption requests, and the majority of the managers surveyed showed that they had sufficiently adequate risk management processes in place. However, areas of weakness emerged for the managers as a whole from the 30 countries participating in the initiative, concerning, among other things, the traceability of analyses, the procedures and methodologies adopted, liquidity analyses preceding investment decisions, data quality checks and internal control systems. In this context, the guidelines proposed by ESMA fit in. As already specified, the use of periodic stress tests (Liquidity Stress Tests or LST) is indicated as a liquidity risk control tool, while, for the part of the portfolio to be liquidated, ESMA calls for the following liquidity measures to be used preferentially:

- HQLA *(high quality liquidity assets),* whereby instruments are classified into groups by liquidity classes, which are assigned a percentage weight. The portfolio HQLA is finally obtained as a weighted average, representing an estimated value of the rapidly liquidable portion of the portfolio,
- *Time To Liquidate* (*TTL*), namely the time it takes to liquidate the entire portfolio or a part of it without a significant price impact,
- Finally, some liquidity ratios can be determined, including the *Redempion Coverage Ratio* (RCR), which is equal to the ratio $\frac{\text{Liquid Assets}}{\text{Net Outflows}}$ and the *Funds Liquidity Coverage Ratio* (FLCR), defined as the ratio $\frac{\text{TTL}}{\text{Benchmark Limit}}$.

Consequently, ESMA urges management companies to revise their liquidity risk management models with regard to the quantification of the impact price effect, a topic that we will analyse in detail in Section 2 with the introduction of the calculation models.

A final recommendation of the directive concerns the manager's specification of a liquidation strategy. In particular, the following are considered appropriate:

- the proportional or slicing approach, according to which managers try to keep the portfolio structure constant by selling all securities in the portfolio in the same proportion (Cetorelli et al., 2016). In practice, if the shock is 10% of the NAV, managers will sell 10% of each asset class in the portfolio (ESMA, 2019),
- the waterfall approach, whereby managers are assumed to liquidate the more liquid assets first before using the less liquid securities,
- the near-proportional approach, according to which, if it is not possible to liquidate a percentage of the NAV equal to the shock, the intention is to find a possible size to liquidate while minimising the impact price.

1.2 METHODOLOGY AND LITERATURE

In terms of modelling, we deem it appropriate to report the works and research we have used that have contributed significantly to the understanding of the concepts used and, thus, to the drafting of this thesis.

In the current literature, concerning the application of models for liquidity risk, there are many authors who have dealt with the management of this risk in Asset Management. In general, the models of this type that we will discuss are known as price impact models. Among the main papers for the understanding and definition of liquidity risk itself – in the broader perspective of Risk Management – we should first mention the work of Acerbi and Scandolo (2007), which is of primary importance not only for the theory of liquidity risk itself, but also in the more strictly mathematical formulation of a coherent risk measure. In practical and strictly modelling terms, the work introduced independently by Bouchaud et al. (2004) and Lillo and Farmer (2004), and subsequently extended to continuous time by Gatheral (2010), represents the framework underlying this thesis. These papers introduce the concept of impact surface and, in particular, impact price as a key property characterising market liquidity and important for understanding price dynamics. From a generalisation of this part of the literature comes the model implemented in this analysis, which closely follows the Bloomberg (2018) paper, in which the user is provided with a fully accessible model in which liquidity can be assessed along the dimensions of cost, time and sales volume. The approach in question for calculating the impact price, which we will henceforth refer to as the *Bloomberg model*, is fully probabilistic, is data-driven – the concept of market metaorder is introduced in this regard – and is able to statistically capture the structural relationships between market conditions, available volume and other factors affecting liquidity (Bloomberg, 2018). The model proposed by Bloomberg (2018) uses institutional order execution data from over 600 different clients, executed over a historical period of 8 months. In this respect, the model implemented in this paper instead follows a simulation approach, which we consider to be a significant extension in the case where it is difficult to have a large number of metaorders. We will discuss more about this approach in Section 4. The model is applied to the market data of the portfolios of BCC Risparmio & Previdenza (BCC R&P), the asset manager of the Italian banking group Iccrea. To this end, parameter estimates are obtained through the non-linear least squares method by calibrating the model using both the simulation approach and the impact price obtained from MSCI's LiquidityMetrics, BCC R&P's provider for liquidity risk estimation.

1.3 OUTLINE

The structure of the thesis is outlined as follows. The second section is strictly theoretical: we analyse and report on the essential information for understanding the models in the literature for calculating the impact price, focusing in particular on the Bloomberg model, its calibration and, in brief, the MSCI model. The third section deals with the type of data used, briefly analysing how these were processed and handled, and the sources from which they were extracted. Section four presents the application of what was explained in section two: we implement our impact price model, calculate the liquidation cost and perform the calibration of the parameters, ultimately obtaining the calibrated liquidity surface – both according to the simulation approach and using the LiquidityMetrics impact price – for the funds analysed. The objective of this section is to obtain, in both cases of calibration, parameters that allow an impact price that is not too distant from the impact price calculated by the MSCI provider's model. To this end, we will use performance measures (KPIs) to compute the distance between the two models. In the fifth and final section, we will summarise and analyse the main results in conclusive terms and propose further extensions of the model for possible future research. In the appendix, we report the most important MATLAB codes to reproduce the main results.

2. Impact Price and Liquidity Surface

In this section we will analyse every aspect of the Bloomberg model for the impact price, the variables that come into play and its general workflow. In this regard, we will make extensive use of the paper by Lillo and Farmer (2004) and Zarinelli et al. (2014) for a clear presentation of the concept of impact price, impact surface and for the introduction of all relevant variables for the implementation of the model. We will also introduce the two calibration approaches of the model parameters, the one including the liquidation cost calculation according to the Bloomberg (2018) approach and the one considering the calibration against the impact price obtained from the LiquidityMetrics model of the MSCI provider.

2.1 Impact Price Background

Although introduced by the papers of Lillo and Farmer (2004) and Zarinelli et al. (2014), a comprehensive definition of the market impact of a trade is that proposed by Bloomberg (2018), where it is referred to as the relative price change that can be attributed to the execution of an order and the sign of the order. However, a further clarification needs to be made as to what we mean by an order. In particular, following Zarinelli et al. (2014), we introduce the concept of a market metaorder, i.e. a set of orders of the same sign – buy or sell – defined as a full order, to be distinguished therefore with an individual trade referred to as a child order. In analytical terms, a metaorder can be seen as a tuple of the type

$$O : [(T_1, V_1, S_1), \dots, (T_n, V_n, S_n)]\,, \tag{1}$$

where T is the time of execution – referred to in Bloomberg as the timestamp, V is the size to be executed and S is the relative execution price. Empirical studies have consistently shown that the market impact of a metaorder is a non-linear concave function of its size. Most studies on the subject have also concluded that the market impact of a metaorder is well described by a *square root law* of market impact. Following this approach, a first rigorous representation of market impact is reported by Zarinelli et al. (2014), where the square root law states that

$$\mathcal{I}(Q) = \pm Y \sigma_D \left(\frac{Q}{V_D}\right)^{\delta} \tag{2}$$

In equation (2), market impact $\mathcal{I}$ is defined as the average expected price return between the beginning and end of a metaorder of size Q (Zarinelli et al., 2014), σ_D is the daily volatility of the security, V_D is the daily volume traded and the sign of the metaorder is positive (negative) for buy (sell) trades. The parameter δ is subject to calibration, but in empirical studies on the subject it is between 0.4 and 0.7, more specifically it is very close to 0.5. However, considering time in units of volume traded, then setting V_P as the volume traded by the entire market during the execution of the metaorder, defining the execution duration as $F = \frac{V_P}{V_D}$ and the participation rate $\eta = \frac{Q}{V_D}$ as the ratio of order size Q to market volume during execution, Zarinelli et al. (2014) rewrite the square root law in equation (2) as

$$\pi = \frac{Q}{V_D} = \frac{Q}{V_P}\frac{V_P}{V_D} = F \cdot \eta \,, \tag{3}$$

which implicitly assumes that the market impact only depends on the square root of the product of F and η. However, this assumption is only approximate, and it can be shown that a more complex functional form better describes the empirical data. In this regard, Zarinelli et al. (2014) introduce the concept of *impact surface* (or liquidity surface, as it is referred to by the MSCI provider), a functional form that considers the variation of market impact both as the participation rate changes and as the order execution time changes. Therefore, the temporary market impact curve $\mathcal{I}_{Temp}(\Omega = \{\pi\})$ is defined as the price change conditional on the daily fraction π. The latter expression, when related to the law in equation (2), allows us to state that the temporary market impact curve is described, at least in a first approximation, by a *power-law function* of the type

$$\mathcal{I}_{Temp}(\Omega = \{\pi\}) = Y\pi^{\delta}$$

with δ model parameter, in the same way as in equation (2). As mentioned, however, this is only an approximate form. Zarinelli et al. (2014) in fact analyse how the temporary market impact curve also depends on other conditioning variables. As suggested by Toth et al. (2011), the square root law for market impact seems to be a very robust statistical regularity; it does not seem to depend on the instrument traded (shares, futures, foreign exchange, etc.) or on the period – regarding this last point, Zarinelli et al. (2014) test the robustness of the temporary market impact curve by conditioning it on different time periods (2007, 2008 and 2009). However, if we work in terms of metaorders, then other relevant conditional variables come into play. As expressed, equation (2) depends only on the daily fraction π, which implies that the temporary market impact is independent of both the execution duration F and the participation rate η. In this regard, Zarinelli et al. (2014), conditioning the measurements on the participation rate and the duration of the metaorder, observe that in both cases the temporary market impact curves are locally approximated by a *power-low function*, and even better described by a logarithmic function. The authors therefore propose a law for market impact that goes beyond relation (2), namely a power-law function in both η and F of the following type,

$$\mathcal{I}_{Temp}(\Omega = \{\eta, F\}) = Y\, \eta^{\delta}\, \pi^{\gamma}\ , \tag{4}$$

with Y, δ and γ model parameters to be estimated. From this first formulation, it is still difficult to construct a true model, basically because, since we are working in terms of metaorders, it is necessary to decouple the effect on price of a given order from the effect of other orders occurring at the same time or earlier in the market. Regarding this last point, several studies have proposed a price formation process that describes the effect of each individual transaction on price. This diffusion process, initially introduced by Bouchaud et al. (2004) and Lillo and Farmer (2004), is known as the *transient impact model*, and is intended to account for those empirical observations that the sign of market orders has a persistent correlation while price dynamics is diffusive (Bloomberg, 2018). In relation to the Bloomberg model that we followed for implementation, in this context we will only consider the continuous-time extension of the transient impact model proposed by Gatheral (2010), which assumes that the stock price S_t at time t is given by

$$S_t = S_0 + \int_0^t f(\dot{x}_s)\, G(t-s)\, ds + \int_0^t \sigma\, dW_s\ , \tag{5}$$

where $f(\dot{x}_s)$ represents the impact of trading at time s, $G(t-s)$ is a decay factor and W_s is a Wiener process indexed at time s. Strictly following the work of Gatheral (2010), we will refer to $f(\cdot)$ as the *instantaneous market impact function* and to $G(\cdot)$ as the *decay kernel*. The stock price S_t follows an arithmetic random walk with drift depending on the cumulative impact of previous trades – the latter is assumed implicitly in S_0 and in the noise term. The drift is ignored both because it is a lower order effect and because when estimating the market impact in practice we usually average between purchases and sales (Gatheral, 2010). At this point, considering equation (5) as a proxy for price evolution, but measuring time in units of traded volume, Zarinelli et al. (2014) consider a variation of the transient impact model where $s(t) := \log\left(\frac{S(t)}{\sigma_D}\right)$ evolves in volume time v according to the following process

$$s(v) = s(0) + \int_0^v f(q(s))\, G(v-s)\, ds + \int_0^v dW_s\ , \tag{6}$$

In equation (6), $G(t)$ represents, as in formulation proposed by Gatheral (2010), the decay kernel describing the time dependence of the impact on price, while the function $f(q)$ is an odd function describing its volume dependence. In particular, equation (6) is the starting point for the Bloomberg model, as it shows that in the transient impact model with an impact function $f(q) = q^{\delta}$ of the power-law type and a decay kernel $G(t) = t^{-\lambda}$ of the power-law type, the transient market impact curve is a factorizable power-law function of both the participation rate η and the duration F. In practice, Zarinelli et al. (2014) show that, considering these functional forms, the market impact of a transaction is well described by a market impact surface parameterised by the participation rate η and the order duration F (Bloomberg, 2018). Using this model, Gatheral (2010) also shows that the condition $\delta + \gamma \geq 1$ is necessary to rule out price manipulation – referred to as the *no-dynamic-arbitrage principle* – and observes that if the decay kernel and the impact function have a functional form of the power-law type, then the no-dynamic-arbitrage principle is satisfied.

2.2 PARAMETERS

One of the critical points of the Bloomberg model that we will follow for implementation is the lack of parsimony in the number of parameters. The number of variables that come into play is large and it is therefore appropriate to introduce them one by one to clarify the workflow of the model. From now on we will strictly follow the Bloomberg (2018) paper.

A first important assumption is made about the execution time and concerns the fact that the operation is executed incrementally over a period of time. In this regard, we denote t_α as the start time, t_ω as the end time of the trade, and then $T = t_\omega - t_\alpha$ as the settlement horizon. The variable representing market volumes in the model is denoted as *expected daily volume* (V_E or EDV) and represents the expected number of shares available per day for trading in a given security. The estimate used by Bloomberg (2018) for this quantity is the daily volume calculated on the average of the last 20 trading days; in this case, we will use the average of the last 30 trading days in our model to align with the MSCI model, which uses the 30-day average. The target number of shares to be executed is denoted by V, specifically if $V > 0$ we will have a buy trade, while if $V < 0$ we will have a sell trade. $V_T = V_E(t_\omega) - V_E(t_\alpha)$ represents the expected total number of shares traded in the market during the execution period. The expected volatility of the stock is denoted σ_E and represents the daily volatility calculated as annualized volatility divided by $\sqrt{252}$. Another substantial difference in Bloomberg (2018) on the naming of the variables concerns the participation rate η and order duration F, which are instead denoted as

$$\eta = \pi = \frac{V}{V_T} \,,$$

$$F = \tau = \frac{V_T}{V_E} \,,$$

which represent the average participation rate and execution horizon, respectively. We also introduce the parameter π_x, defined as the trading strategy. For simplicity, the benchmark execution strategy of all the models seen is known as Volume Weighted Average Price, or VWAP. Informally, following the definition of Bilawosky et al. (2005), the VWAP of a stock over a period of time is simply the average price paid per share during that period; in practice it is the sum of the prices of each transaction paid, weighted by its volume. The goal of any trader, who follows the VWAP benchmark, is to find and define ex-ante strategies that ex-post lead to an average trading price as close as possible to the VWAP price. VWAP execution orders account for about 50 percent of all trading by institutional investors (Bilawosky et al., 2005). The price at time t of the traded security is denoted by S_t, and is again expressed in volume-time $S_v \coloneqq S_t(v)$. Specifically, Bloomberg (2018) considers the pricing process in equation (6) – normalized and conditional on the trading strategy π_x – as governed by the equation

$$\frac{S_v - S_\alpha}{S_\alpha} = \sigma_E \int_{v_a}^{v} f(\pi_x)\, G(v - x)\, dx + \int_{v_a}^{v} dW_x \,. \tag{7}$$

Finally, following the observations of Zarinelli et al. (2014), the following functional forms are defined for the introduced model components by Bloomberg (2018)

$$f(\pi_x) = \eta \cdot sgn(\pi_x) \cdot |\pi_x|^\delta \,,$$

$$G(v) = v^{-\gamma} \,,$$

$$sgn(\pi_x) = \begin{cases} 1, & se\ \pi_x > 0\ (buy) \\ -1, & se\ \pi_x < 0\ (sell) \end{cases} \,,$$

where η, δ and γ are the model parameters that will be subject to calibration in the model implementation.

2.3 THE BLOOMBERG MODEL

In practical terms, we analyzed and implemented the Bloomberg (2018) model in two steps:

1. the first part of the model returns us the liquidation cost that we called theoretical $C(V,\tau)$ – does not depend on the market data on metaorders – of which the impact price $I(V,\tau)$ is a component,
2. the second part calculates the liquidation cost $C(O)$, which we defined as *empirical*, calculated using market metaorders and considered as a realization of the function $C(V,\tau)$.

We then report, strictly following Bloomberg (2018), the functions used in the implementation, specifying that in this section the model parameters are not yet subject to calibration. The impact price is derived directly from equation (6), using the VWAP trading strategy as defined in section 2.2. Specifically, the liquidation cost of a metaorder is defined as,

$$C(V,\tau) = \frac{1}{VS_\alpha}\int_{v_\alpha}^{v_\omega} V_v(S_\alpha - S_v)\, d_v =$$

$$= \sigma_E\left(\frac{\eta}{(1-\gamma)(2-\gamma)}\right)\left(\frac{V}{V_E}\right)^{\delta}\tau^{1-\delta-\gamma} + \frac{\tau^{\frac{1}{2}}}{\sqrt{3}}\sigma_E \cdot \varepsilon\,, \tag{8}$$

where $\varepsilon \sim N(0,1)$ and η, δ and γ are still model parameters. Introducing the parameters $\kappa = \frac{\eta}{(1-\gamma)(2-\gamma)}$ and $\beta = 1-\delta-\gamma$, we finally report the impact price function, which represents the deterministic part of equation (8)

$$I(V,\tau) = \sigma_E \cdot \kappa\left(\frac{V}{V_E}\right)^{\delta}\tau^{\beta}\,. \tag{9}$$

The impact price is also generalized as dependent on other market factors, in particular, for developed markets, market capitalization was found to be a significant factor and the effect of this factor is formulated in Bloomberg (2018) with a power law function $M = \left(\frac{C}{\bar{C}}\right)^{\upsilon}$, where C is the market capitalization of the individual stock, $\bar{C}$ is the average market capitalization of stocks in a specific market-in our case, of a specific SGR portfolio, as we will see in Section 3 and υ an additional model parameter subject to calibration. The impact price then becomes,

$$I(V,\tau,M)=\sigma_E\cdot\kappa\left(\frac{V}{V_E}\right)^{\delta}\tau^{\beta}\left(\frac{C}{\bar{\bar{C}}}\right)^{\upsilon}. \tag{10}$$

For emerging markets, in contrast, Bloomberg (2018) reports that market capitalization does not turn out to be a significant factor in influencing market impact, so the model is the same as in equation (9). For simplicity's sake, let us consider that equation (10) can be further extended by including a stochastic component, resulting in what is the final first part of the model,

$$\begin{aligned}&\mathcal{I}(V,\tau,M)=I(V,\tau,M)\varepsilon_I\,,\\&dove\ \varepsilon_I\sim logN\left(-\frac{\sigma^2}{2},\sigma\right)\ e\ E[\varepsilon_I]=1\,.\end{aligned}$$

As for the liquidation cost $C(O)$, in relation to a generic metaorder of the type proposed in equation (1), the calculation is significantly more straightforward. Following the generalization proposed by Bloomberg (2018), a single realization of $C(V,\tau)$ is defined as

$$\begin{aligned}&C(O):=sgn(B/S)\frac{\sum_{i=1}^{n}(V_i*S_i/V)-S_\alpha}{S_\alpha}\,,\\&V=\sum_{i=1}^{n}V_i\,,\\&sgn(B/S)=\begin{cases}1, & if\ buy\\-1, & if\ sell\end{cases},\end{aligned} \tag{11}$$

where S_α is the average price before order execution and n is the number of observations of the metaorder O.

2.4 THE MSCI MODEL

In its implementation in LiquidityMetrics, the MSCI model does not have a disclosure of the analytical tools or equations used to calculate the impact price. However, as the benchmark model for comparison in this paper, it is useful to understand its main components. The LiquidityMetrics methodology is based on the construction of asset-specific liquidity surfaces to capture the time, cost, and size dimensions of market transactions. Through the use of liquidity surfaces, a complete description of asset liquidity can be obtained. Four main characteristics of an asset's liquidity can be observed directly from its liquidity surface in Figure 1:

- Market impact, the dependence of price impact on order size, as we already defined for the Bloomberg model; larger orders usually require higher transaction costs.
- Market elasticity, the speed and extent to which a market regenerates liquidity removed from a transaction and returns to its previous state. This determines how much transaction costs decrease with longer time horizons.
- Market depth, the maximum size thresholds (buying and selling) above which trades are expected to encounter insufficient supply/demand or otherwise distorted and unpredictable prices.
- Bid-ask spread, half of the bid/ask spread is the expected transaction cost for small orders traded over any time horizon.

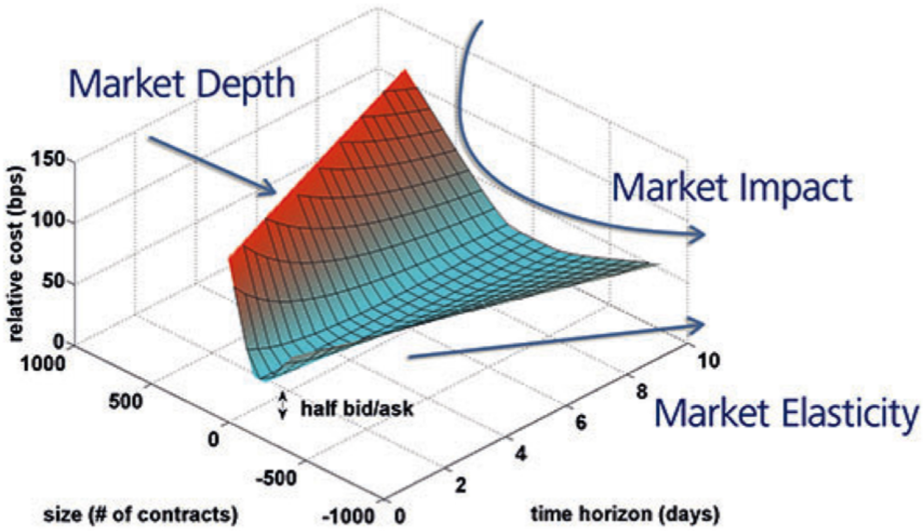

Figure 1. Typical liquidity surface from MSCI (2020) LiquidityMetrics.

In the following sections we will use the concept of a liquidity surface in a different way. We will keep as the main output of the two models and the only component for calibration the impact price, extracted from LiquidityMetrics reports for MSCI and calculated from equation (10) for the Bloomberg model, but, in the end, we will not represent the cost through the half bid-ask – a variable that is not used in the Bloomberg model – but by considering the liquidation cost relative to the present value of a stock/bond position in the portfolio. In practice, we will directly report the dollar cost to liquidate a position.

2.5 CALIBRATION

Calibration of model parameters η, δ, γ and υ is carried out according to the logic of the nonlinear least squares method, resulting in two different estimates depending on the surface against which we calibrate. A first estimate of the parameters is in line with the calibration proposed by Bloomberg (2018), according to which, once the impact price is obtained in equation (10) and the liquidation cost in equation (11), we go to minimize the following distance,

$$\hat{\theta} = [\eta, \delta, \gamma, \upsilon] = \underset{\theta}{\operatorname{argmin}} \sum_{j=1}^{N} \left[I\left(V_j, T_j, M_j; \theta\right) - C\left(O_j\right) \right]^2 , \tag{12}$$

where, in our case, N is the number of stock target scenarios to be executed – as we will see in Section 3, there are six liquidation scenarios considered, equal to 5%, 10%, 30%, 50%, 80% and 100% of the stock/portfolio. We will refer to these scenarios as *stress levels*. A second parameter estimate is instead obtained using the market impacts estimated by the MSCI provider for the BCC R&P portfolios; in this case, the calibration has the sole purpose of having a proxy for comparison with the results of the calibration in equation (12). The calibration function is the same as in equation (12), but with the MSCI impact price in place of $C(O)$,

$$\hat{\theta} = [\eta, \delta, \gamma, \upsilon] = \underset{\theta}{\operatorname{argmin}} \sum_{j=1}^{N} \left[I\left(V_j, T_j, M_j; \theta\right) - I_{MSCI} \right]^2 . \tag{13}$$

3. DATA MANAGEMENT

In this section, we briefly introduce the data collection and data management procedure. The providers used were Bloomberg and MSCI, while the analysis is carried out on data provided by BCC R&P. The sample is represented by the portfolios of BCC R&P, in particular, we have 12 portfolios with the master data reported in Table 1.

Portfolio Code	Portfolio Type
AA	Aureo Equity Pension Fund
AB	Aureo Balanced Pension Fund
AO	Aureo Bond Pension Fund
AS	Aureo Pension Fund Guaranteed Line
EM	Investiper Equity Euro
LQ	Investiper Short-Term Bond
MO	Investiper Global Bond
PA	Investiper Italy PIR 50
PO	Investiper Emerging Countries Bond
RM	Investiper Balanced 50
ZG	Investiper Coupon December 2023
ZL	Investiper Coupon December 2024

Table 1. Master Data for the portfolios of the BCC R&P.

For each portfolio, we did a first order filter by selecting a sample of securities representing at least 30% of the NAV. In the end, we had a structure containing more or less 20 securities for each portfolio, therefore we decided to take exactly 20 securities for each fund. Furthermore, for each security, we downloaded from Bloomberg market data such as opening (O), closing (C) and mid-price (M), minimum (L) and maximum (H) of the day and volumes (Volume), for a time horizon from 30 June 2017 to 30 June 2022 – the stress test analysis on the portfolios is in fact performed for the latter day. From the data provided by BCC R&P, for each security, we selected the following items:

- ISINCODE,
- TIP, the type of instrument – we considered ordinary shares, government and corporate bonds, excluding futures and indices,
- Shares_Value, the number of shares held by the portfolio for that security,
- Price_in_Division, the price at which the security was purchased,
- Market_Value_Euro, the price multiplied by the quantity, and two other items downloaded from Bloomberg:
- 30D_AVG_VOLUME, which is the estimate for the Expected Daily Volume (EAD) of each security,
- Market_CAP, the market capitalisation of each security – in the case of bonds, we used the amount outstanding.

The finale structure we obtained for the entire dataset is depicted in Figure 2.

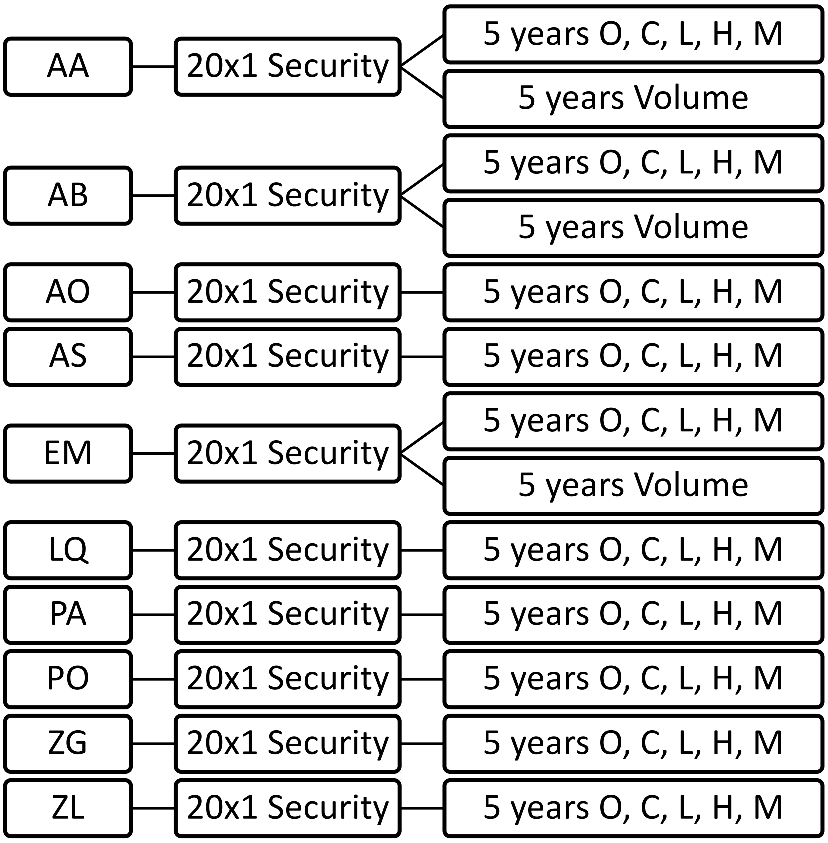

Figure 2. Final Main Structure to access the data for the analysis.

From MSCI, on the other hand, we obtained the parameters of the impact price calculated directly according to their model, on which we perform the first type of calibration in

Section 4. These values are derived for the stress sizes proposed by MSCI in their extractions, i.e. 5%, 10%, 30%, 50%, 80% and 100%. We specify that these values of liquidation cost are expressed in dollars in the extraction, therefore for a direct comparison with the results of the implemented model, we divided the cost calculated by MSCI by the present value of each security, so as to obtain a value of impact price in absolute terms. We also used the data reported by MSCI in order to comply with the difficulties inherent in finding data for the bond market, especially with regard to volumes. Basically, the assumption is that the EDV for a bond is equal to the typical order size, i.e. what is typically traded by medium-sized asset managers such the one in analysis. This is extracted directly from the MSCI reports in order to estimate the liquidation cost.

Regarding some key metrics for the model, we preferred to calculate rather than dump them. In particular, for daily and annualised volatility we used the basic formula in the CESR's guidelines on the methodology for the calculation of the synthetic risk and reward indicator in the Key Investor Information Document (CESR, 2010), while for the Brownian Bridge – that we will see in detail in the simulative approach in Section 4 – as the drift term we used the average of returns over the period of analysis considered as an estimate.

4. RESULTS

In this section, we analyse the results obtained from the implementation of the model. In particular, we report on the entire workflow of the model, thus starting with the uncalibrated results and ending with the results with both types of parameter calibration. Furthermore, we will analyse both the proportional and the near proportional approach for both parameter calibrations. We will briefly highlight some critical issues with the model, which will be further explored in the final section. We specify that the model was used for a rather large data sample, as seen in Section 3 we have 12 portfolios, each containing 20 securities, which means that from a full run of the model we get 240 results for 2 calibration types and 2 settlement methodologies. To allow for smoother reading and more consistent data presentation, we report results for the two most representative portfolio types, namely AA, which in a certain way it represents all other equity portfolios, and MO, which would represent bond portfolios.

4.1 IMPACT PRICE CALCULATION WITHOUT CALIBRATION

Considering the data defined in Section 3, we calculate an initial impact price estimate for all portfolios and for each security using a best estimate on the model parameters. For the results in Table 2, we considered the following parameters values,

$$\gamma = 0.7, \qquad \delta = 1, \qquad \eta = 0.1\,.$$

As specified in Section 2, the parameter υ is set equal to 1 and is not subject to calibration basically because from empirical observations on the results we learned that the effect of the parameter on the ratio market capitalization to average market capitalization can be considered negligible. Since the impact price calculated by the MSCI model is a vector, with six values for the respective six stress levels, in this first estimation we consider a time horizon $\tau = \frac{1}{252}$ to obtain a vector comparable in size with that of MSCI. What we can see from Table 2 are the results of some KPIs[1] computed on the first outputs. It is evident how

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[1] We considered the following measures to make comparisons:

- $KPI\ 1 = IP\ Bloomberg - IP\ Benchmark$,
- $KPI\ 2 = (IP\ Bloomberg - IP\ Benchmark)/IP\ Benchmark$,
- $KPI\ 3 = abs[IP\ Bloomberg - IP\ Benchmark]$,
- $KPI\ 4 = abs[(IP\ Bloomberg - IP\ Benchmark)/IP\ Benchmark]$.

the Bloomberg model, without proper calibration of the parameters, deviates significantly from the benchmark model, in particular we observe way noisier results for the equity segment than for the bond segment, this suggests that the calibration we will obtain on the parameters will be quite different from portfolio to portfolio.

	AA					MO			
Stress Level	**KPI 1**	**KPI 2**	**KPI 3**	**KPI 4**		**KPI 1**	**KPI 2**	**KPI 3**	**KPI 4**
5%	0.00004	179%	0.00004	179%		- 0.000009	- 35.41%	0.000009	35.41%
10%	0.00007	152%	0.00007	152%		- 0.00002	- 35.44%	0.00002	35.44%
30%	0.0002	134%	0.0002	134%		- 0.00006	- 35.46%	0.00006	35.46%
50%	0.0003	122%	0.0003	122%		- 0.00009	- 35.48%	0.00009	35.48%
80%	0.0004	88%	0.0004	88%		- 0.0001	- 35.49%	0.0001	35.49%
100%	0.0005	72%	0.0005	72%		- 0.0002	- 35.62%	0.0002	35.62%

Table 2. Comparison between the internal model which follows the Bloomberg model and the MSCI model (No Calibration).

4.2 PARAMETER CALIBRATION VS MSCI

Next, we calibrate the parameters starting by using equation (13). For this purpose, we used the *fmincon* function of MATLAB's Optimization Toolbox™ (The Mathworks, 2022), imposing as the only constraint that the initial point estimate is equal to the best estimate, i.e. $x_0 = [0.7, 1, 0.1]$ where x is the vector we want to calibrate.

AA					MO			
ISIN	γ	δ	η		**ISIN**	γ	δ	η
US0378331005	0,56	0,93	0,09		DE0001104867	0,82	1,01	0,11
US5949181045	0,51	1,28	0,11		IT0005325946	0,51	1,00	0,13
…	…	…	…		…	…	…	…
US30231G1022	0,56	0,93	0,10		IE00BV8C9418	0,82	1,01	0,12

Table 3. Model parameters calibrated vs. MSCI.

We note that on the best estimate we respect the non-arbitrage constraint, although after that the latter is omitted from the calibration since, as we observe from the results in Table

3, this is respected in any case. Using the parameters obtained in Table 3, we then recalculate the impact price with the Bloomberg model. On the same time horizon, we obtain the KPIs in Table 4.

	AA					MO			
Stress Level	**KPI 1**	**KPI 2**	**KPI 3**	**KPI 4**		**KPI 1**	**KPI 2**	**KPI 3**	**KPI 4**
5%	- 0.000003	- 20.91%	0.000003	20.91%		- 0.0000002	- 0.76%	0.0000002	0.76%
10%	- 0.000006	- 17.04%	0.000006	17.04%		- 0.0000002	- 0.51%	0.0000002	0.51%
30%	- 0.000001	- 1.07%	0.000001	1.07%		- 0.0000001	- 0.11%	0.0000001	0.11%
50%	0.000004	1.97%	0.000004	1.97%		0.0000001	0.06%	0.0000001	0.06%
80%	0.000003	0.66%	0.000003	0.66%		0.0000005	0.10%	0.0000005	0.10%
100%	- 0.000003	- 0.56%	0.000003	0.56%		- 0.0000004	- 0.07%	0.0000004	0.07%

Table 4. Comparison between the internal model which follows the Bloomberg model and the MSCI model (MSCI Calibration).

As can be seen, the results are much better, especially on bond-type portfolios such as MO we have a deviation, in portfolio terms, that is very small. Overall, from this calibration approach the results are quite satisfactory and allow us to replicate a model for estimating liquidation costs that is very reliable when compared to a benchmark model such as MSCI. What we considered so far was the proportional approach, i.e. a constant stress level as specified in Section 1. In order to extend the computation to get more interpretable output, we also compute the case in which we can liquidate the position subject to a stress level over several days, i.e. assuming a time horizon $\tau > 1$.

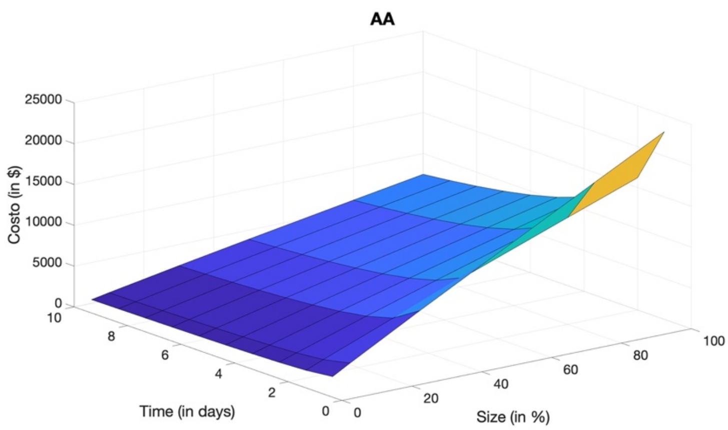

Figure 3. Liquidity Surface for the portfolio AA computed with the parameters calibrated vs MSCI - PROPORTIONAL APPROACH.

By doing so, we can finally construct the liquidity surfaces as we can see from Figure 3 and Figure 4. In our case, we ran the model with $\tau = 10/252$ days.

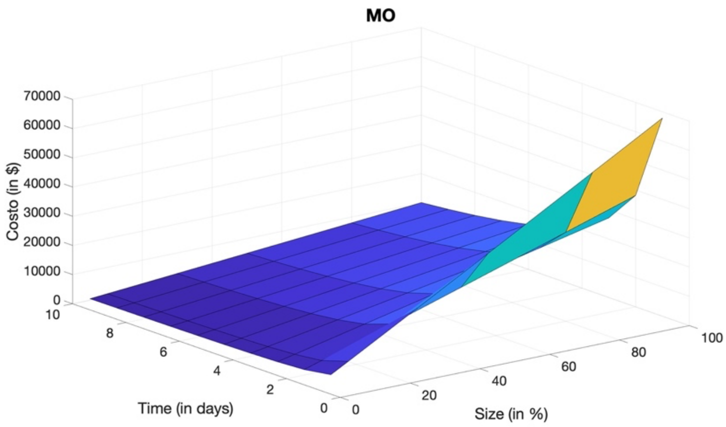

Figure 4. Liquidity Surface for the portfolio MO computed with the parameters calibrated vs MSCI - PROPORTIONAL APPROACH.

As we observe in Figure 3 and Figure 4, compared to the one defined in section 2, we do not report the impact price but directly the dollar cost of liquidating the position. We still obtain the characteristics of a liquidity surface, i.e. the cost is increasing in the stress level and decreasing in the time horizon. The near proportional case, on the other hand, is more complex, requiring a further calibration on the stress value for each security in a portfolio. The calibration is computed according to the following relation,

$$\min_{\psi} \sum_{i=1}^{K} \omega \cdot IP_{\psi,i} \ ,$$

where K is the length of the considered portfolio in terms of titles – in our case, as we had seen before, is 20, ω is the weight of the $i-th$ title and ψ is the optimal near proportional size we want to find for each title. In order to implement it, we still used the function *fmincon*, but in this case with different constraints:

- the starting point x_0 is one of the six stress levels considered by MSCI, e.g. 5%;
- the lower and upper bounds are thus defined

$$lb = x_0 - 0.01 * s, \qquad ub = x_0 - 0.01 * s$$

where s is a number ranging from 1 to the number of stress levels considered. By doing so, we obtain a rather reasonable range, since for a stress of 5%, after the near proportional approach we have a percentage to be liquidated ranging from 4% to 6%, while for a stress of 100% (i.e., the scenario of liquidation of the entire position), we have a range from 94% to 106%.

- we have a linear constraint of the type $Aeq * x = beq$, whereby the sum of the stress levels obtained must be equal to the liquidation level considered - e.g. if we wanted to liquidate 5% of the entire portfolio, once we have obtained the quantity to be liquidated for each security according to the NP, the sum of these quantities must be equal to 5% of the entire portfolio.

For illustrative purposes, in Table 5 we report the results obtained on the AA portfolio only, on a time horizon $\tau = \frac{1}{252}$.

AA							
ISIN	**Stress Level**	**ISIN**	**Stress Level**	**ISIN**	**Stress Level**	**ISIN**	**Stress Level**
US0378331005	4,09%	US91324P1021	5,18%	US75513E1010	5,26%	US8552441094	5,08%
US5949181045	4,44%	US4781601046	5,12%	US0846707026	5,15%	US00287Y1091	5,25%
US02079K3059	4,52%	US92826C8394	5,38%	US0605051046	4,83%	US46625H1005	5,41%
US0231351067	4,37%	US88160R1014	4,73%	US8636671013	5,25%	US79466L3024	5,34%
US67066G1040	5,28%	US30231G1022	5,25%	US8835561023	4,67%	US7170811035	5,39%

Table 5. Near proportional calibrated size to liquidate for a stress level on the entire portfolio equal to 5%.

We then construct the liquidity surface as we did in the proportional case, setting a time horizon $\tau = \frac{10}{252}$ and using the liquidation quantities in Table 5.

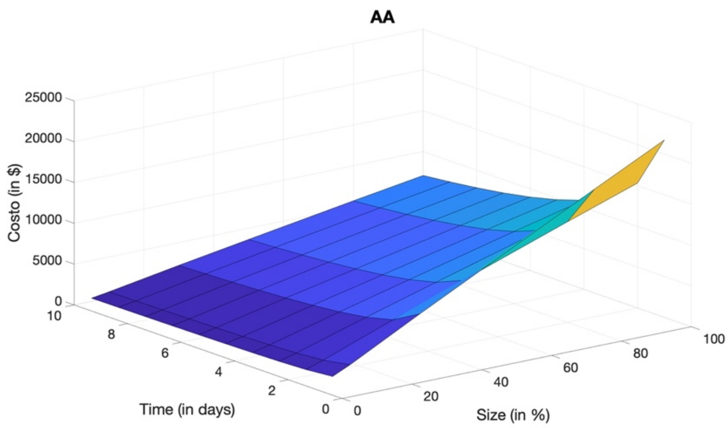

Figure 5. Liquidity Surface for the portfolio AA computed with the parameters calibrated vs MSCI – NEAR PROPORTIONAL APPROACH.

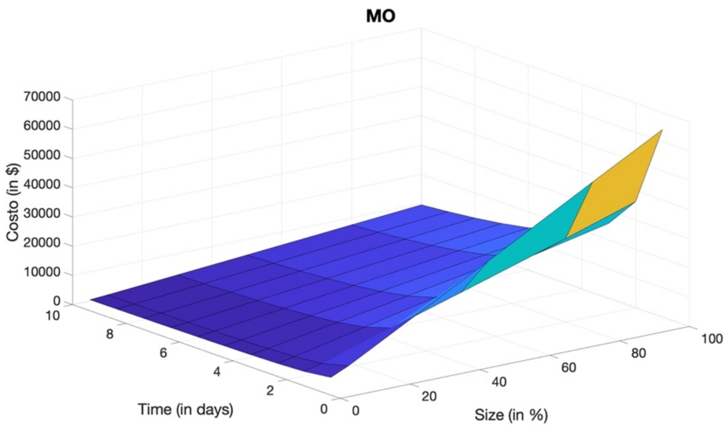

Figure 6. Liquidity Surface for the portfolio MO computed with the parameters calibrated vs MSCI – NEAR PROPORTIONAL APPROACH.

The results in Figure 5 and Figure 6, again shown for the AA and MO fund, respect the logic of the near proportional approach – as the cost is generally lower than in the proportional case in Figure 3 and 4 – and the proper characteristics of a liquidity surface.

4.3 PARAMETER CALIBRATION VS SIMULATED METAORDER

As specified in Section 2, the objective of the paper is also to define a more analytical calibration linked to market data, that is, using metaorders, objects defined in equation (1). The main problem with this type of calibration is the need for clean volume data, both in the case of equities and bonds. In the case of BCC R&P, the sample derivable from their order book was found to be too small and, given the results obtained, we also considered the order of magnitude to be too small relative to market size. The inability to obtain market data led us to the idea of populating metaorders following a simulative approach.

As stated, for prices simulation we used a Brownian bridge, which is a Wiener process B on the time horizon $[t, T]$ that is pinned on both sides. Therefore, at time t it is equal to a known value B_t and at time T it is equal to a known value B_T. If we consider B_s as the Brownian bridge at time s in (t, T), it can be shown that B_s is normal with mean

$$a + \frac{s-t}{T-t}(b-a)\,,$$

and variance

$$\frac{(T-s)(s-t)}{T-t}.$$

For the simulation exercise, we then considered these characteristics of the Brownian bridge distribution, with the starting point at t equal to the day's open (O) of the price and the ending point T equal to the day's close (C).

For quantities simulation we used a function called *randfixedsum* implemented by Stafford (2016), which basically generates an n by m array x, each of whose m columns contain n random values lying in the interval $[a, b]$, but subject to the condition that their sum be equal to s. The scalar value s must accordingly satisfy the following condition,

$$n \cdot a \leq s \leq n \cdot b .$$

The distribution of values is uniform in the sense that it has the conditional probability distribution of a uniform distribution over the whole n-cube, given that the sum of all x values is s. In Table 6, we can observe a simulated metaorder for a 5% stress level of the Apple stock in the AA portfolio.

Execution Day	Execution Time	Traded Quantity	Execution Price
30-Jun-2022	09:30:05	5189	136.78
30-Jun-2022	09:30:05	2363	136.78
30-Jun-2022	10:18:25	11978	136.72
30-Jun-2022	10:18:25	1859	136.72
30-Jun-2022	10:46:08	9225	137.31
30-Jun-2022	10:46:08	4277	137.31
30-Jun-2022	10:46:08	12561	137.31
…	…	…	…
…	…	…	…
30-Jun-2022	16:00:14	9845	136.83
30-Jun-2022	16:00:14	4310	136.83
30-Jun-2022	16:00:14	3198	136.83
30-Jun-2022	16:00:14	3175	136.83

Table 6. Sample of a simulated metaorder.

Once we obtained the simulated metaorders for each stock in each portfolio, we implemented equation (11) in the model, thus obtaining the estimated liquidation cost using the metaorders. At this point, we launched the calibration, this time using equation (12) defined in Section 2, that is, minimizing the squared distances between the two vectors. At the application level, we use *lsqnonlin*, an additional function of MATLAB's Optimization Toolbox^{TM} (The Mathworks, 2022), in this case with the no arbitrage constraint specified Section 2 and the initial point equal to the best estimate on the parameters – as in the calibration against MSCI. The application of the model then follows the same steps as in the previous calibration; in this regard, we avoid reporting the calibrated parameters since the format of the output is the same.

We then proceed with the calculation of the calibrated impact price vector, again running the model implementing equation (10) with a $\tau = \frac{1}{252}$. In Table 7 we can observe the results obtained in terms of the KPIs used and for the six stress levels considered. Here we report results only for the MO fund, because for the AA fund we did not obtain satisfactory estimates and the distance in percentage terms from the MSCI model was too large.

	MO			
Stress Level	**KPI 1**	**KPI 2**	**KPI 3**	**KPI 4**
5%	- 0.00009	- 69.71%	0.00009	69.71%
10%	- 0.0001	- 64.96%	0.0001	64.96%
30%	- 0.0004	- 55.55%	0.0004	55.55%
50%	- 0.0007	- 53.50%	0.0007	53.50%
80%	- 0.001	- 56.01%	0.001	56.01%
100%	- 0.002	- 58.52%	0.002	58.52%

Table 7. Comparison between the internal model which follows the Bloomberg model and the MSCI model (Simulative Approach).

In this case, the noise on the results is significantly greater, and we are unable to obtain satisfactory results for all portfolios. However, for example for the MO portfolio or for other bond portfolios, such as PA or PO, we still get decent results and in line with effective calibration, as we can observe from the column KPI 4 in Table 7. In general, given the degree of experimentation with the simulated methodology, we are still satisfied with the results obtained, although in Section 5 we consider some possible corrections to the model. We then construct the liquidity surface as we did in the previous calibration, setting a time horizon $\tau = \frac{10}{252}$ and using the new parameters obtained from this type of calibration, both for the proportional and for the Near Proportional approach. We can have a look at the results in Figure 7.

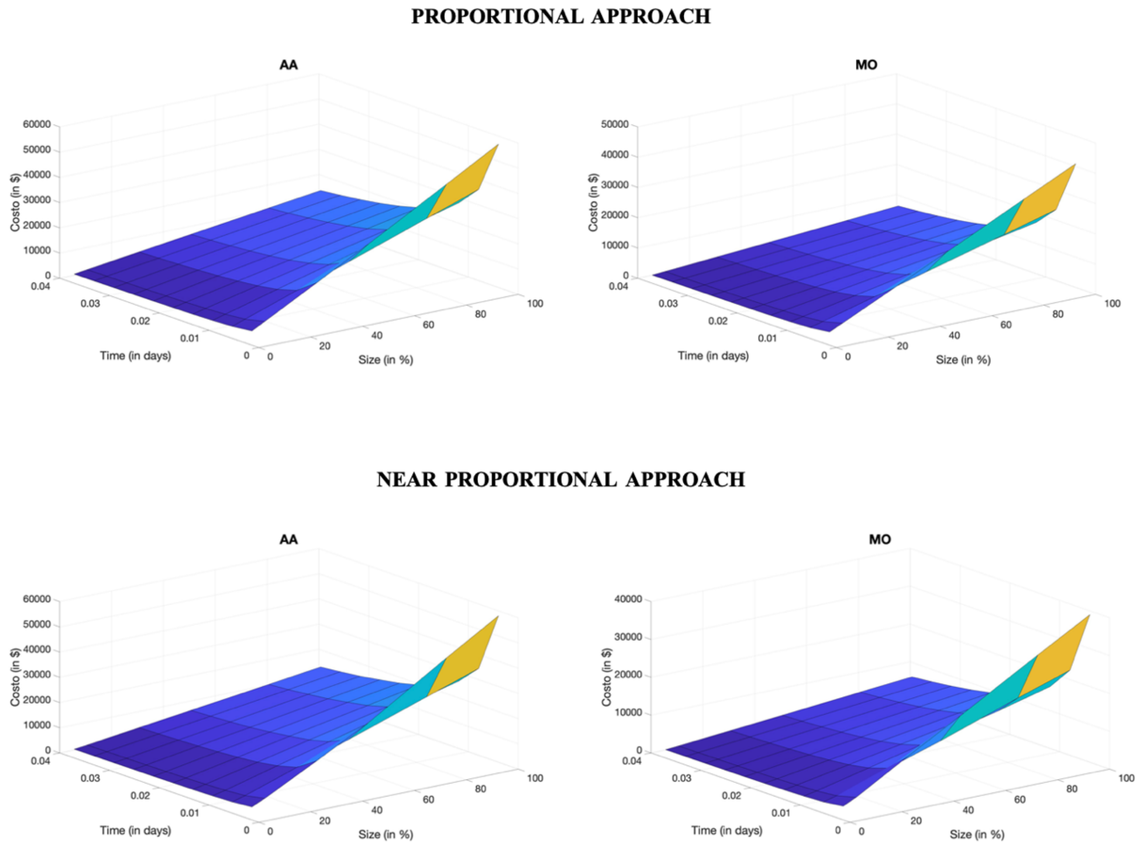

Figure 7. Liquidity Surfaces for the portfolio AA and MO computed with the parameters calibrated with the Simulative method.

5. CONCLUSIONS

In the presented thesis, we analysed stress testing issues in the context of liquidity risk for medium-sized Italian fund managers – specifically for the BCC R&P, the asset manager of Banca Iccrea. Initially, we gave an exhaustive summary of the existing literature, defining the main components and the procedures that allow to understand and analytically derive the model formula used, initially introduced by Bloomberg (2018). Given this theoretical premise, the aim of the thesis was to obtain a more or less effective calibration of the model parameters, for which Bloomberg (2018) provides no guidance. In this regard, we introduced, again from a more theoretical aspect, two calibration methodologies, both defined for six levels of stress on the sizes to be liquidated. The first uses the surface derived from MSCI – provider of the BCC R&P for liquidity risk computation – minimising its differences with the impact price surface calculated with the Bloomberg model using parameters not calibrated. The other methodology uses objects that in Zarinelli et al. (2014) are called metaorder, first calculating the market liquidation cost given the trades made on a particular security, and then minimising the squared distances between the obtained liquidation cost vector and the Bloomberg vector with uncalibrated parameters. In the latter type of calibration, given the lack of historical order data, we proposed a more experimental approach, simulating metaorders using a Brownian Bridge for prices (where the final and initial observations are respectively the opening and closing price of the day) and a function implemented by Stafford (2016) for quantities. Furthermore, we replicated the calculation of the impact price in the case of proportional and near proportional settlement, concepts introduced in Section 1.

In terms of actual results, we obtain a definitely functioning model for the first type of calibration, for which the KPIs considered indicate a good degree of replication of the MSCI model. We believe, in conclusion, that the parameters derived from this calibration are definitely usable in an operational context to have efficient estimates of the cost of liquidation. In the second case, the results are more divergent; in particular, we have non-comparable KPIs levels for the two equity funds and for balanced funds in general, while more or less good levels for the rest of the bond funds. In particular, there are bond funds such as MO, PA, and PO where we find a distance between the surfaces found with the Bloomberg model in its simulated version and the surface given by MSCI that is not too large, although we do have background noise on the results if we pool the model for a set number of trials.

5.1 Future Research

What is applied in this thesis allows us to replicate a model for calculating liquidity risk in the context of Italian asset managers. From a strictly operational point of view, we can in fact say that the results obtained are more than satisfactory, since we have a set of parameters that the asset manager with whom we carried out the analysis can use to calculate the liquidation cost of its exposures. However, as specified, although we used a different model, we are still dependent on the MSCI model, as we only obtain correctly calibrated parameters if we minimise the difference between the implemented model and the surface extracted from MSCI. The limitation, therefore, is that we are not able to use the Bloomberg model entirely, i.e. we were not able – or at least, not completely – to obtain efficiently calibrated parameters using market metaorders. The problem is therefore strictly related to the lack of data – the so-called Trade & Quotes (TAQ), which are available through various providers for a fee. Future research on the topic should therefore first of all start with obtaining a clean dataset, to actually see whether the Bloomberg model can actually be used as a substitute for the MSCI model in the case of the BCC R&P. However, it could also be that the Bloomberg model is actually more aggressive than the MSCI model in general, and that it estimates higher impact prices, and thus higher liquidation costs. This could be related to the fact that the model from Bloomberg (2018) does not give full disclosure, omitting some scaling parameters on impact prices. Although, a further study could be related to an improvement of the simulation approach, in particular to the use of a function or methodology to simulate the traded quantities that is more performative and less noisy than the one implemented with the use of the Stafford (2016) function. Similarly, the price vector could also be simulated more efficiently by implementing more sophisticated techniques than Brownian Bridge – for example, the typical Heston (1993) model could already improve the simulative approach.

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