Homework 4

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Problem 9.10

The riddle can be expressed in first-order logic as:
$Male(m) \wedge$
$(\neg \exists s : Brother(s, I) \vee Sister(s, I)) \wedge (\exists f : Son(f, Father(I)) \wedge f = Father(m))$
Where I represents the speaker in the riddle, and m represents "that man". From this, we can use the axioms of the kinship domain to deduce the identity of "that man":

$$\begin{aligned} Male(m)\wedge \\ (\neg\exists s : Brother(s, I) \vee Sister(s, I))\wedge \\ (\exists f : Son(f, Father(I)) \wedge f = Father(m)) \end{aligned} \tag{1}$$

$$\begin{aligned} Male(m)\wedge \\ (\neg\exists s : (Sibling(s, I) \wedge Male(s)) \vee (Sibling(s, I) \wedge Female(s))\wedge \\ (\exists f : Son(f, Father(I)) \wedge f = Father(m)) \end{aligned} \tag{2}$$

$$\begin{aligned} Male(m)\wedge \\ (\neg\exists s : Sibling(s, I) \wedge (Male(s) \vee Female(s)))\wedge \\ (\exists f : Son(f, Father(I)) \wedge f = Father(m)) \end{aligned} \tag{3}$$

$$\begin{aligned} Male(m)\wedge \\ (\neg\exists s : Sibling(s, I))\wedge \\ (\exists f : Son(f, Father(I)) \wedge f = Father(m)) \end{aligned} \tag{4}$$

$$\begin{aligned} Male(m)\wedge \\ (\neg\exists s : Sibling(s, I))\wedge \\ (\exists f : Son(f, Father(I)) \wedge Parent(f, m) \wedge Male(f)) \end{aligned} \tag{5}$$

$$
\begin{aligned}
&Male(m)\wedge \\
&(\neg\exists s : Sibling(s, I))\wedge \\
&(\exists f : Child(f, Father(I)) \wedge Parent(f, m) \wedge Male(f))
\end{aligned} \tag{6}
$$

$$
\begin{aligned}
&Male(m)\wedge \\
&(\neg\exists s : s \neq I \wedge (\exists p : Parent(p, s) \wedge Parent(p, I)))\wedge \\
&(\exists f : Child(f, Father(I)) \wedge Parent(f, m) \wedge Male(f))
\end{aligned} \tag{7}
$$

$$
\begin{aligned}
&Male(m)\wedge \\
&(\forall s : s = I \vee \neg(\exists p : Parent(p, s) \wedge Parent(p, I)))\wedge \\
&(\exists f : Child(f, Father(I)) \wedge Parent(f, m) \wedge Male(f))
\end{aligned} \tag{8}
$$

$$
\begin{aligned}
&Male(m)\wedge \\
&(\forall s : s = I \vee \neg(\exists p : Parent(p, s) \wedge Parent(p, I)))\wedge \\
&(\exists f, g : g = Father(I) \wedge Child(f, g) \wedge Parent(f, m) \wedge Male(f))
\end{aligned} \tag{9}
$$

$$
\begin{aligned}
&Male(m)\wedge \\
&(\forall s : s = I \vee \neg(\exists p : Parent(p, s) \wedge Parent(p, I)))\wedge \\
&(\exists f, g : Parent(g, I) \wedge Child(f, g) \wedge Parent(f, m) \wedge Male(f) \wedge Male(g))
\end{aligned} \tag{10}
$$

$$
\begin{aligned}
&Male(m)\wedge \\
&(\forall s : s = I \vee \neg(\exists p : Parent(p, s) \wedge Parent(p, I)))\wedge \\
&(\exists f, g : Parent(g, I) \wedge Parent(g, f) \wedge Parent(f, m) \wedge Male(f) \wedge Male(g))
\end{aligned} \tag{11}
$$

$$
\begin{aligned}
&Male(m)\wedge \\
&(\forall s : (\exists p : Parent(p, s) \wedge Parent(p, I)) \rightarrow s = I)\wedge \\
&(\exists f, g : Parent(g, I) \wedge Parent(g, f) \wedge Parent(f, m) \wedge Male(f) \wedge Male(g))
\end{aligned} \tag{12}
$$

$$
\begin{aligned}
&Male(m)\wedge \\
(\forall s : (\exists p : Parent(p, s) \wedge Parent(p, I)) \rightarrow s = I)\wedge \\
(\exists f : f = I \wedge Parent(f, m) \wedge Male(f))
\end{aligned} \tag{13}
$$

$$Male(m) \wedge Parent(I, m) \wedge Male(I) \tag{14}$$

$$Male(m) \wedge Child(m, I) \tag{15}$$

$$Son(m, I) \tag{16}$$

From this, we can conclude that "that man" is, in fact, the speaker's son.

In prenex normal form, the original riddle can be expressed as follows:

$\exists f : \forall s : (\neg Brother(s, I) \vee \neg Son(f, Father(I)) \vee \neg f = Father(m) \vee \neg Male(m)) \wedge (\neg Sister(s, I) \vee \neg Son(f, Father(I)) \vee \neg f = Father(m) \vee \neg Male(m))$

Problem 9.23

a. Premise: $\forall h : Horse(h) \rightarrow Animal(h)$
Conclusion: $\forall c, q : (HeadOf(c, q) \wedge Horse(q)) \rightarrow \exists a : (HeadOf(c, a) \wedge Animal(a))$

b. Premise: $\neg Horse(h) \vee Animal(h)$
Conclusion: $\exists c, q : \forall a : (HeadOf(c, q) \wedge Horse(q)) \wedge (\neg HeadOf(c, a) \vee \neg Animal(a))$