

$$V_{1,1} = \bigwedge_{i=1}^3 V_{1i,1}$$

$$V_{1,2} = \bigwedge_{i=1}^3 (V_{1i,2} \vee (V_{1i,1} \oplus V_{1,1}))$$

$$V_{1,3} = \bigwedge_{i=1}^3 (V_{1i,3} \vee (V_{1i,2} \oplus V_{1,2}) \vee (V_{1i,1} \oplus V_{1,1}))$$

$$V_{1i,1} = \bigvee_{j=1}^3 V_{1ij,1}$$

$$V_{1i,2} = \bigvee_{j=1}^3 (V_{1ij,2} \wedge \neg (V_{1ij,1} \oplus V_{1i,1}))$$

$$V_{1i,3} = \bigvee_{j=1}^3 (V_{1ij,3} \wedge \neg (V_{1ij,2} \oplus V_{1i,2}) \wedge \neg (V_{1ij,1} \oplus V_{1i,1}))$$

$$V_{1ij,1} = \bigwedge_{k=1}^3 V_{1ijk,1}$$

$$V_{1ij,2} = \bigwedge_{k=1}^3 (V_{1ijk,2} \vee (V_{1ijk,1} \oplus V_{1ij,1}))$$

$$V_{1ij,3} = \bigwedge_{k=1}^3 (V_{1ijk,3} \vee (V_{1ijk,2} \oplus V_{1ij,2}) \vee (V_{1ijk,1} \oplus V_{1ij,1}))$$

$$V_{1,1} \leftrightarrow \bigwedge_{i=1}^3 V_{1i,1} \wedge$$

$$V_{1,2} \leftrightarrow \bigwedge_{i=1}^3 (V_{1i,2} \vee (V_{1i,1} \oplus V_{1,1})) \wedge$$

$$V_{1,3} \leftrightarrow \bigwedge_{i=1}^3 (V_{1i,3} \vee (V_{1i,2} \oplus V_{1,2}) \vee (V_{1i,1} \oplus V_{1,1})) \wedge$$

$$V_{1i,1} \leftrightarrow \bigvee_{j=1}^3 V_{1ij,1} \wedge$$

$$V_{1i,2} \leftrightarrow \bigvee_{j=1}^3 (V_{1ij,2} \wedge \neg (V_{1ij,1} \oplus V_{1i,1})) \wedge$$

$$V_{1i,3} \leftrightarrow \bigvee_{j=1}^3 (V_{1ij,3} \wedge \neg (V_{1ij,2} \oplus V_{1i,2}) \wedge \neg (V_{1ij,1} \oplus V_{1i,1})) \wedge$$

$$V_{1ij,1} \leftrightarrow \bigwedge_{k=1}^3 V_{1ijk,1} \wedge$$

$$V_{1ij,2} \leftrightarrow \bigwedge_{k=1}^3 (V_{1ijk,2} \vee (V_{1ijk,1} \oplus V_{1ij,1})) \wedge$$

$$V_{1ij,3} \leftrightarrow \bigwedge_{k=1}^3 (V_{1ijk,3} \vee (V_{1ijk,2} \oplus V_{1ij,2}) \vee (V_{1ijk,1} \oplus V_{1ij,1}))$$

$$\begin{aligned}
& \left(\bigwedge_{i=1}^3 (V_{1i,1} \vee \neg V_{1,1}) \right) \wedge \left(V_{1,1} \vee \bigvee_{i=1}^3 \neg V_{1i,1} \right) \wedge \\
& \bigwedge_{i=1}^3 ((\neg V_{1,2} \vee V_{1i,2} \vee V_{1,1} \vee V_{1i,1}) \wedge (\neg V_{1,2} \vee V_{1i,2} \vee \neg V_{1,1} \vee \neg V_{1i,1})) \wedge \\
& \bigwedge_{s \in \{1,2,3,4\}^3} \bigvee_{i=1}^3 t_{s_i} \\
& \text{(where } t_1 := V_{1,2} \vee \neg V_{1i,2}, t_2 := V_{1,2}, t_3 := V_{1,2} \vee V_{1i,1} \vee \neg V_{1,1} \text{ and } t_4 := \\
& V_{1,2} \vee V_{1,1} \vee \neg V_{1i,1}) \\
& \left(\bigwedge_{i=1}^3 (\neg V_{1,2} \vee V_{1i,2} \vee ((V_{1i,1} \vee V_{1,1}) \wedge (\neg V_{1i,1} \vee \neg V_{1,1}))) \right) \wedge \\
& \left(\bigwedge_{i=1}^3 (\neg V_{1,2} \vee V_{1i,2} \vee ((V_{1i,1} \vee V_{1,1}) \wedge (\neg V_{1i,1} \vee \neg V_{1,1}))) \right) \wedge \left(\neg \bigwedge_{i=1}^3 (V_{1i,2} \vee (V_{1i,1} \oplus V_{1,1})) \vee V_{1,2} \right) \wedge \\
& V_{1,3} \leftrightarrow \bigwedge_{i=1}^3 (V_{1i,3} \vee (V_{1i,2} \oplus V_{1,2}) \vee (V_{1i,1} \oplus V_{1,1})) \wedge \\
& V_{1i,1} \leftrightarrow \bigvee_{j=1}^3 V_{1ij,1} \wedge \\
& V_{1i,2} \leftrightarrow \bigvee_{j=1}^3 (V_{1ij,2} \wedge \neg (V_{1ij,1} \oplus V_{1i,1})) \wedge \\
& V_{1i,3} \leftrightarrow \bigvee_{j=1}^3 (V_{1ij,3} \wedge \neg (V_{1ij,2} \oplus V_{1i,2}) \wedge \neg (V_{1ij,1} \oplus V_{1i,1})) \wedge \\
& V_{1ij,1} \leftrightarrow \bigwedge_{k=1}^3 V_{1ijk,1} \wedge \\
& V_{1ij,2} \leftrightarrow \bigwedge_{k=1}^3 (V_{1ijk,2} \vee (V_{1ijk,1} \oplus V_{1ij,1})) \wedge \\
& V_{1ij,3} \leftrightarrow \bigwedge_{k=1}^3 (V_{1ijk,3} \vee (V_{1ijk,2} \oplus V_{1ij,2}) \vee (V_{1ijk,1} \oplus V_{1ij,1})) \wedge
\end{aligned}$$