

Firstly, we write expressions for the values of the bits of each node in terms of the values of those nodes' children.

$$\begin{aligned} V_{1,1} &= \bigwedge_{i=1}^3 V_{1i,1} \\ V_{1,2} &= \bigwedge_{i=1}^3 (V_{1i,2} \vee (V_{1i,1} \oplus V_{1,1})) \\ V_{1,3} &= \bigwedge_{i=1}^3 (V_{1i,3} \vee (V_{1i,2} \oplus V_{1,2}) \vee (V_{1i,1} \oplus V_{1,1})) \end{aligned}$$

$$\begin{aligned} V_{1i,1} &= \bigvee_{j=1}^3 V_{1ij,1} \\ V_{1i,2} &= \bigvee_{j=1}^3 (V_{1ij,2} \wedge \neg(V_{1ij,1} \oplus V_{1i,1})) \\ V_{1i,3} &= \bigvee_{j=1}^3 (V_{1ij,3} \wedge \neg(V_{1ij,2} \oplus V_{1i,2}) \wedge \neg(V_{1ij,1} \oplus V_{1i,1})) \\ V_{1ij,1} &= \bigwedge_{k=1}^3 V_{1ijk,1} \\ V_{1ij,2} &= \bigwedge_{k=1}^3 (V_{1ijk,2} \vee (V_{1ijk,1} \oplus V_{1ij,1})) \\ V_{1ij,3} &= \bigwedge_{k=1}^3 (V_{1ijk,3} \vee (V_{1ijk,2} \oplus V_{1ij,2}) \vee (V_{1ijk,1} \oplus V_{1ij,1})) \end{aligned}$$

Next, we change our equations to propositional logic biconditionals, and conjunct all of our terms together. This gives us a single true expression to convert to CNF.

$$\begin{aligned} V_{1,1} &\leftrightarrow \bigwedge_{i=1}^3 V_{1i,1} \wedge \\ V_{1,2} &\leftrightarrow \bigwedge_{i=1}^3 (V_{1i,2} \vee (V_{1i,1} \oplus V_{1,1})) \wedge \\ V_{1,3} &\leftrightarrow \bigwedge_{i=1}^3 (V_{1i,3} \vee (V_{1i,2} \oplus V_{1,2}) \vee (V_{1i,1} \oplus V_{1,1})) \wedge \\ V_{1i,1} &\leftrightarrow \bigvee_{j=1}^3 V_{1ij,1} \wedge \\ V_{1i,2} &\leftrightarrow \bigvee_{j=1}^3 (V_{1ij,2} \wedge \neg(V_{1ij,1} \oplus V_{1i,1})) \wedge \\ V_{1i,3} &\leftrightarrow \bigvee_{j=1}^3 (V_{1ij,3} \wedge \neg(V_{1ij,2} \oplus V_{1i,2}) \wedge \neg(V_{1ij,1} \oplus V_{1i,1})) \wedge \\ V_{1ij,1} &\leftrightarrow \bigwedge_{k=1}^3 V_{1ijk,1} \wedge \\ V_{1ij,2} &\leftrightarrow \bigwedge_{k=1}^3 (V_{1ijk,2} \vee (V_{1ijk,1} \oplus V_{1ij,1})) \wedge \\ V_{1ij,3} &\leftrightarrow \bigwedge_{k=1}^3 (V_{1ijk,3} \vee (V_{1ijk,2} \oplus V_{1ij,2}) \vee (V_{1ijk,1} \oplus V_{1ij,1})) \end{aligned}$$

Now, we begin simplifying each term of our massive conjunction to CNF, so that our whole expression will end up in CNF.

We do this by writing everything in terms of conjunctions, disjunctions, and negations, then distribute as necessary until we find ourselves in CNF.

$$\begin{aligned}
& \left( \bigwedge_{i=1}^3 (V_{1i,1} \vee \neg V_{1i,1}) \right) \wedge \left( V_{1i,1} \vee \bigvee_{i=1}^3 \neg V_{1i,1} \right) \wedge \\
& \bigwedge_{i=1}^3 ((\neg V_{1i,2} \vee V_{1i,2} \vee V_{1i,1} \vee \neg V_{1i,1}) \wedge (\neg V_{1i,2} \vee V_{1i,2} \vee \neg V_{1i,1} \vee \neg V_{1i,1})) \wedge \\
& \bigwedge_{s \in \{1,2,3,4\}^3} \bigvee_{i=1}^3 t_{s_i}
\end{aligned}$$

(where  $t_1 := V_{1i,2} \vee \neg V_{1i,2}$ ,  $t_2 := V_{1i,2}$ ,  $t_3 := V_{1i,2} \vee V_{1i,1} \vee \neg V_{1i,1}$  and  $t_4 := V_{1i,2} \vee V_{1i,1} \vee \neg V_{1i,1}$ )

$$\wedge$$

At this point, we gave up on manually expanding everything to CNF, as about half of the expansions cause the creation of  $2^n$  new terms, and we grew tired of inventing notations in order to fit everything in a sensible amount of space. It should be evident that the distributive law can properly yield the rest of these subexpressions into Conjunctive Normal form.

$$\begin{aligned}
& \neg V_{1i,3} \vee \bigwedge_{i=1}^3 (V_{1i,3} \vee ((V_{1i,2} \vee V_{1i,2}) \wedge (\neg V_{1i,2} \vee \neg V_{1i,2})) \vee ((V_{1i,1} \vee V_{1i,1}) \wedge (\neg V_{1i,1} \vee \neg V_{1i,1}))) \wedge \\
& V_{1i,3} \vee \neg \bigwedge_{i=1}^3 (V_{1i,3} \vee ((V_{1i,2} \vee V_{1i,2}) \wedge (\neg V_{1i,2} \vee \neg V_{1i,2})) \vee ((V_{1i,1} \vee V_{1i,1}) \wedge (\neg V_{1i,1} \vee \neg V_{1i,1}))) \wedge \\
& V_{1i,1} \vee \neg \bigvee_{j=1}^3 V_{1ij,1} \wedge \\
& \neg V_{1i,1} \vee \bigvee_{j=1}^3 V_{1ij,1} \wedge \\
& V_{1i,2} \vee \neg \bigvee_{j=1}^3 (V_{1ij,2} \wedge ((V_{1ij,1} \wedge V_{1i,1}) \vee (\neg V_{1ij,1} \wedge \neg V_{1i,1}))) \wedge \\
& \neg V_{1i,2} \vee \bigvee_{j=1}^3 (V_{1ij,2} \wedge ((V_{1ij,1} \wedge V_{1i,1}) \vee (\neg V_{1ij,1} \wedge \neg V_{1i,1}))) \wedge \\
& V_{1i,3} \vee \neg \bigvee_{j=1}^3 (V_{1ij,3} \wedge ((V_{1ij,2} \wedge V_{1i,2}) \vee (\neg V_{1ij,2} \wedge \neg V_{1i,2})) \wedge ((V_{1ij,1} \wedge V_{1i,1}) \vee (\neg V_{1ij,1} \wedge \neg V_{1i,1}))) \wedge \\
& \neg V_{1i,3} \vee \bigvee_{j=1}^3 (V_{1ij,3} \wedge ((V_{1ij,2} \wedge V_{1i,2}) \vee (\neg V_{1ij,2} \wedge \neg V_{1i,2})) \wedge ((V_{1ij,1} \wedge V_{1i,1}) \vee (\neg V_{1ij,1} \wedge \neg V_{1i,1}))) \wedge \\
& \neg V_{1ij,1} \wedge \bigwedge_{k=1}^3 V_{1ijk,1} \wedge \\
& V_{1ij,1} \wedge \neg \bigwedge_{k=1}^3 V_{1ijk,1} \wedge \\
& V_{1ij,2} \vee \neg \bigwedge_{k=1}^3 (V_{1ijk,2} \vee ((V_{1ijk,1} \vee V_{1ij,1}) \wedge (\neg V_{1ijk,1} \vee \neg V_{1ij,1}))) \wedge \\
& \neg V_{1ij,2} \vee \bigwedge_{k=1}^3 (V_{1ijk,2} \vee ((V_{1ijk,1} \vee V_{1ij,1}) \wedge (\neg V_{1ijk,1} \vee \neg V_{1ij,1})))
\end{aligned}$$

$$\begin{aligned}
& V_{1ij,3} \vee \neg \bigwedge_{k=1}^3 (V_{1ijk,3} \vee ((V_{1ijk,2} \vee V_{1ij,2}) \wedge (\neg V_{1ijk,2} \vee \neg V_{1ij,2})) \vee ((V_{1ijk,1} \vee V_{1ij,1}) \wedge (\neg V_{1ijk,1} \vee \neg V_{1ij,1}))) \\
& \wedge \\
& \neg V_{1ij,3} \vee \bigwedge_{k=1}^3 (V_{1ijk,3} \vee ((V_{1ijk,2} \vee V_{1ij,2}) \wedge (\neg V_{1ijk,2} \vee \neg V_{1ij,2})) \vee ((V_{1ijk,1} \vee V_{1ij,1}) \wedge (\neg V_{1ijk,1} \vee \neg V_{1ij,1})))
\end{aligned}$$