MATH099: Linear Algebra Homework

Due on October 15, 2024

Prof. Prof. Banana , Fall 2024

John Doe

2024-09-28

Problem 1: *Projecting a Vector*

a) when $\min\left(\|\boldsymbol{x}\|_2\right)$, $\boldsymbol{x} = \boldsymbol{x}^*$ is the solution to the problem, which is $x^* = \begin{pmatrix} \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \end{pmatrix}$

b) We have a matrix $\boldsymbol{A} = \begin{pmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 0 \end{pmatrix}$, the projection operator is

$$\boldsymbol{P} = \boldsymbol{A}(\boldsymbol{A}^T A)^{-1}\boldsymbol{A}^T = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

hence,

$$\boldsymbol{x}^* = \boldsymbol{P}\boldsymbol{v} = \begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \\ 1 \end{pmatrix}.$$

c) We have a matrix $\boldsymbol{A} = \begin{pmatrix} 1 & -1 \\ -1 & 1 \\ 2 & 2 \end{pmatrix}$, the projection operator is

$$\boldsymbol{P} = \boldsymbol{A}(\boldsymbol{A}^T A)^{-1}\boldsymbol{A}^T = \begin{pmatrix} \frac{1}{2} & -\frac{1}{2} & 0 \\ -\frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

hence,

$$\boldsymbol{x}^* = \boldsymbol{P}\boldsymbol{v} = \begin{pmatrix} \frac{1}{2} \\ -\frac{1}{2} \\ 0 \end{pmatrix}.$$

Problem 2: *How many genes for each chromosome?*

a) Codes:
```
zcat annotation.gtf.gz | cut -f 9 | cut -d ";" -f 1 | grep ENSG | cut -d " " -f 2 | cut -d "." -f 1 | uniq | wc -l
```

b)

Chromosome	Counts
chr1	5659
chr2	4344
chr3	3303
chr4	2732

Chromosome	Counts
chr5	3074
chr6	3182
chr7	3147
chr8	2541
chr9	2417
chr10	2428
chr11	3468
chr12	3143
chr13	1457
chr14	2341
chr15	2322
chr16	2672
chr17	3162
chr18	1265
chr19	3076
chr20	1502
chr21	898
chr22	1445
chrX	2484
chrY	601
chrM	37