MATHEMATICS FOR PROGRAMMING

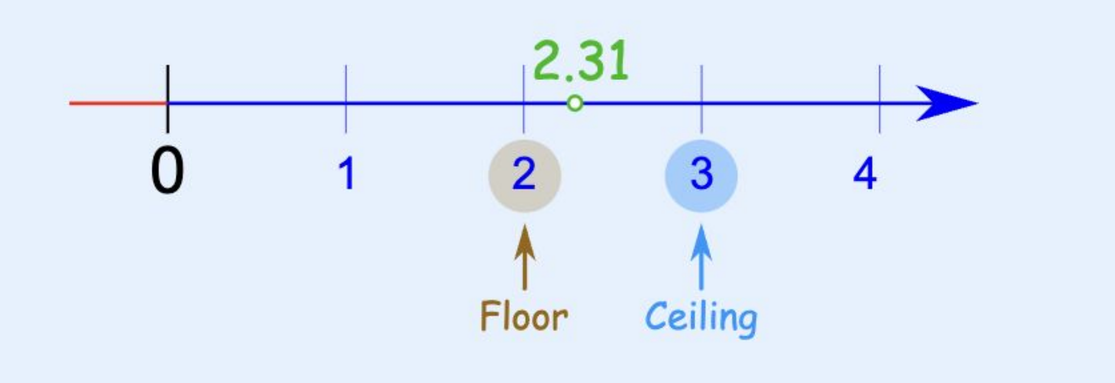
Phitron

NECESSARY RECAP

- Real number, Integer, Natural number and more
- Divisor, dividend, quotient, remainder
- Factor
- Prime number
- Even vs odd

FLOOR, CEILING, ROUND!

- Floor: Nearest integer below
- Ceiling: Nearest integer above
- Round: Nearest integer



- 12.65 → Floor: 12, Ceiling: 13, Round: 13
- 9.21 → Floor: 9, Ceiling: 10, Round: 9
- 24 → Floor: 24, Ceiling: 24, Round: 24

DIVISOR COUNTING

- **Problem statement**: Given an integer n. How many divisors are there for n?

- You know the answer if n is a prime, right?
  - Example: $3, 11, 73$ etc

- What about if it is not a prime number? How do you count divisors?
  - Pause the video and think
  - Example: $8, 15, 27$ etc

DIVISOR COUNTING – NAIVE

- Take all the integers (1 to n) and test if remainder is 0.
  - Example: Let's take 12

1	12%1=0	√
2	12%2=0	√
3	12%3=0	√
4	12%4=0	√
5	12%5=2	x
6	12%6=0	√

7	12%7=5	x
8	12%8=4	x
9	12%9=3	x
10	12%10=2	x
11	12%11=1	x
12	12%12=0	√

  - The divisors: 1, 2, 3, 4, 6, 12.
  - Number of divisors: 6

DIVISOR COUNTING – BETTER?

- We already know 1 and n will be there!
- So, 2 to $(n-1)$ need to be tested.
- But … look at the second half!

1	`12%1=0`	√
2	`12%2=0`	√
3	`12%3=0`	√
4	`12%4=0`	√
5	`12%5=2`	x
6	`12%6=0`	√

7	`12%7=5`	x
8	`12%8=4`	x
9	`12%9=3`	x
10	`12%10=2`	x
11	`12%11=1`	x
12	`12%12=0`	√

- We only need to check for 2 to $n/2$
- What about odd n?

DIVISOR COUNTING – EVEN BETTER?

1	12%1=0	√		7	12%7=5	x
2	12%2=0	√		8	12%8=4	x
3	12%3=0	√		9	12%9=3	x
4	12%4=0	√		10	12%10=2	x
5	12%5=2	x		11	12%11=1	x
6	12%6=0	√		12	12%12=0	√

- Could you stop earlier than 6?
  - Yes!
  - $12\%2 = 0$ means $12/2 = 6$ also divides 12
  - Same goes for 3 and 4
  - So, we could stop at 3 and still identify all the divisors!

DIVISOR COUNTING – EVEN BETTER?

- Then where do I stop?
- For 12, we stopped at 3.
- Let's look at 16

1 x 16 = 16

2 x 8 = 16

$\boxed{4 \times 4 = 16}$

8 x 2 = 16

16 x 1 = 16

- So, we do not need to test any number greater than $\sqrt{n}$
- $\sqrt{12} = 3.464\ldots, \sqrt{16} = 4$

PRIMALITY TEST

- **Problem statement:** Given an integer n. Determine if n is a prime number or not?

- You already know divisor counting
- Take numbers from 2 to $\sqrt{n}$
  - If any of them divide n, NOT prime

PRIMALITY TEST

- Is 77 a prime number?
- $\sqrt{77} = 8.77 \ldots$
- Check 2 to 8
  - $77 \,\%\, 2 \;=\; 1$
  - $77 \,\%\, 3 \;= 2$
  - $77 \,\%\, 4 \;=\; 1$
  - $77 \,\%\, 5 \;= 2$
  - $77 \,\%\, 6 \;= 5$
  - $\mathbf{77 \,\%\, 7 \;= 0} \rightarrow$ **Factors: 7 and** $\mathbf{77/7 \;=\; 11}$
- **NOT Prime!**

PRIMALITY TEST

- But … do you see the extra calculations we did?
  - $77 \% 2 = 1$
  - $77 \% 3 = 2$
  - $77 \% 4 = 1$
  - $77 \% 5 = 2$
  - $77 \% 6 = 5$
  - $\mathbf{77 \% 7 = 0}$
- Can we get a better method?
  - Yes. Let's talk about **Sieve of Eratosthenes**

SIEVE OF ERATOSTHENES

- Identifies all the prime numbers in a given range (very fast)
- Idea:
  - Step 1: Take a number (start with 2) and find its multiples in the range
  - Step 2: Those multiples must be composite, cross them out
  - Step 3: Proceed to the next number
  - Step 4: If it's prime, go to step 1, otherwise step 3

SIEVE OF ERATOSTHENES

- Find out all the prime numbers between 1 *to* 100

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

SIEVE OF ERATOSTHENES

- FIND OUT ALL THE PRIME NUMBERS BETWEEN 1 *to* 120

	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100
101	102	103	104	105	106	107	108	109	110
111	112	113	114	115	116	117	118	119	120

Prime numbers

EXTRACT DIGITS FROM AN INTEGER

- You can tell the digits just by looking at the number
- Then why do we solve this problem?
  - Think of computers
  - If you store a number and want to get a certain digit, you need to get it yourself!
  - Formulate a set of operations on the number that can return individual digits
- Division is all you need!
- Let's go through an example with $n = 237$
  - How do you get the last digit?
  - $237 \% 10 = 7$

EXTRACT DIGITS FROM AN INTEGER

- $$237 \,\%\, 10 \;=\; 7$$
- So, we know the rightmost digit is 7
- After knowing this, we do not have anything to do with 7 anymore
- Now we can focus on 23. But how do you get 23 separated?
  - Divide 237 with 10 and get the quotient
  - $Floor(237 \div 10) = 23$
- Repeat the same process with 23

EXTRACT DIGITS FROM AN INTEGER

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- Find the i-th digit of a number (Say $\mathrm{n} = 91408$) from right. [$i = 0,1,2,3,4$]
- $i = 0 \rightarrow n\%10 = 8$
- $i = 1 \rightarrow Floor(n/10)\%10 = 0$
- $i = 2 \rightarrow Floor(n/100)\%10 = 4$
- $i = 3 \rightarrow Floor(n/1000)\%10 = 1$
- $i = 4 \rightarrow Floor(n/10000)\%10 = 9$

- General Formula: $Floor\left(\frac{n}{10^i}\right)\%10 = \left\lfloor\frac{n}{10^i}\right\rfloor\%10$

SUMMARY

- Floor vs Ceiling
- Counting number of divisors
- Improving our method
- Prime number check
- Sieve of Eratosthenes
- Digit extraction

PRACTICE DAY