MATHEMATICS FOR PROGRAMMING

RECAP

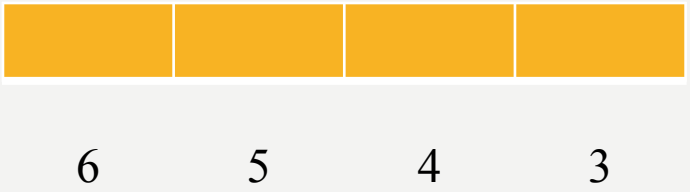
- Unitary method and percentage
  - Solving problems
- Interest rate and related problems
- Mean and Median

COMBINATORICS

- Number of ways to choose or arrange objects
- Arranging some objects in all possible orders
  - Remember factorial?
- Choose some objects out of a collection of objects
  - Combination problem
  - How many ways are there to choose r objects from a collection of n objects?
    - ${}^{n}C_{r} = ?$
- Choose and order some objects out of a collection of objects
  - Permutation problem
  - How many ways are there to arrange r objects from a collection of n objects?
    - ${}^{n}P_{r} = ?$

${}^{n}P_{r}$

- You have n objects in hand (all different)
- You have r slots to put them ($r \leq n$) *in order*
- Example:
  - Batting order of a 11-player cricket match from a squad of 15 players
  - Answering order in an exam to answer 4 out of 6 questions
  - Number of ways: 6 x 5 x 4 x 3 = 360
  - But could we write this?
    - $\frac{(6\times5\times4\times3\times2\times1)}{(2\times1)} \Rightarrow \frac{6!}{2!}$



${}^{n}P_{r}$

- You have n objects in hand (all different)
- You have r slots to put them ($r \leq n$) *in order*
- Generalize the formula

1	2	…	r
n	n-1	…	n-r+1

- Number of permutation: $n \times (n-1) \times (n-2) \times \cdots \times (n-r+1)$
- $\frac{n \times (n-1) \times (n-2) \times \cdots \times (n-r+1) \times (n-r) \times \cdots \times 1}{(n-r) \times \cdots \times 1} \Rightarrow \frac{n!}{(n-r)!}$
- Batting order of a 11-player cricket match from a squad of 15 players?

${}^{n}C_{r}$

- You have n objects in hand (all different)
- You have r slots to put them ($r \leq n$) [*order does not matter]*
- *Choosing objects instead of arranging them*
- Example:
  - Choosing 11 players from a squad of 15 players
  - Answering combinations in an exam to answer 4 out of 6 questions
  - We already know how many ways are there if the order is considered too (${}^{n}P_{r} = {}^{6}P_{4}$)
  - Here: $2 -> 3 -> 1 -> 6$ is same as $1 -> 2 -> 6 -> 3$ as we only need the choice (without order)
  - How many orders are possible with a fixed set of 4 questions?
    - 4!
    - All of them should be count as one combination
    - So, we can just divide the permutation by 4!

${}^{n}C_{r}$

- You have n objects in hand (all different)
- You have r slots to put them ($r \leq n$) [*order does not matter]*
- Generalize the formula

1	2	…	r
n	n-1	…	n-r+1

- Number of permutation: ${}^{n}P_{r} = n \times (n-1) \times (n-2) \times \cdots \times (n-r+1)$

- Each $r!$ corresponds to the same *choice*
- ${}^{n}C_{r} = \frac{{}^{n}P_{r}}{r!} = \frac{n!}{r!(n-r)!}$
- Batting order of a 11-player cricket match from a squad of 15 players?

PROBABILITY

- Probability is a **number** to indicate the chance or likelihood of a particular event
- Event: outcome of an experiment or process
- For example-
  - Tossing a coin is an experiment
  - Event would be the appearance of Head or Tail
- The number is between always between 0 to 1
  - Impossible event: 0
  - Certain event: 1
  - Everything in between
- P(event) = $\frac{Number\ of\ ways\ it\ can\ happen}{Number\ of\ all\ possbile\ outcomes}$

PROBABILITY

- What is the probability of getting a prime number if you throw a 6-faced dice?

- All possible outcomes: 1,2,3,4,5,6
- Target outcomes / prime numbers: 2, 3, 5
- P = 3/6 = 0.5

PROBABILITY

- What is the probability of getting exactly two heads if you throw three coins?

- Outcomes of single coin: H, T
- Two coins: HH, HT, TH, TT
- What about three coins?
- P = 3/8
- Can you do it without building the table?
- Number of possible outcomes: 2 x 2 x 2 = 8
- Outcomes with exactly 2 heads out of 3 trials
  - Choose 2 positions out of 3 for H
  - $^{3}C_{2} = \frac{3!}{1!\times 2!} = 3$
- P = 3/8

	H	**T**
H	HH	HT
T	TH	TT

	HH	**HT**	**TH**	**TT**
H	HHH	HHT	HTH	HTT
T	THH	THT	TTH	TTT

BINARY <-> OCTAL

- Remember binary to decimal?
- Easier than that
- 8 and 16 are powers of 2, 10 is not!
  - $8 = 2^3 \; and \; 16 = 2^4$
- 000 to 111 => 0 to 7 (Octal)
- 0000 to 1111 => 0 to F (Hexadecimal)
- Binary 10111011 to octal?
  - 010 111 011 => 273
- Octal 314 to binary?
  - 314 => 011 001 100

SUMMARY

- Combinatorial problems
  - Choice / Combination / nCr
  - Arrangement / Permutation / nPr
- Probability
- Number system revisited

PRACTICE DAY

- Convert this binary number to hexadecimal number
    - 01110100111011
- Convert this octal number to hexadecimal number
    - 127416
- If you throw a dice and toss a coin at the same time, what is the probability that you will get Head (coin) and value less than 3 (dice)?
- 10 people came to party. Everyone shook hands with everyone. How many handshakes took place?
- Section A and B have 25 students and 20 students, respectively. If a team of 11 players need to be formed and 6 students must be from A and the others from B, how many ways are there to form the team? How many batting order would be there?