MATHEMATICS FOR PROGRAMMING

RECAP

- Floor vs Ceiling
- Counting number of divisors
- Improving our method
- Prime number check
- Sieve of Eratosthenes
- Digit extraction

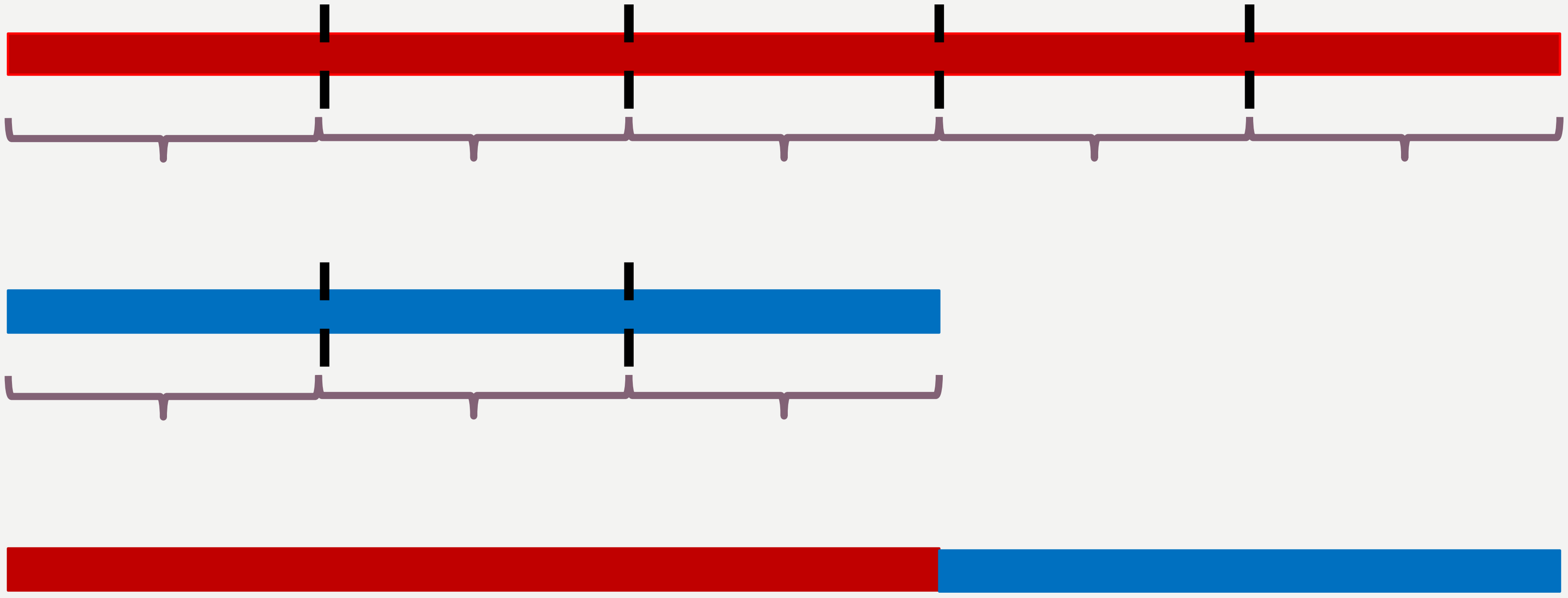
GCD - GREATEST COMMON DIVISOR

- গ.সা..গুঃ গরিষ্ঠ সাধারণ গুণনীয়ক
- Divisor
- Common divisor
- Greatest common divisor
- Example: Find the gcd of 15 and 6
  - Divisors of 15: 1,3,5,15
  - Divisors of 6: 1,2,3,6
  - Common divisors: 1,3
  - GCD: 3
- Co-prime: If $\gcd(a, b) = 1$, a and b are co-prime.
  - Example: 8 and 15

GCD – HOW TO CALCULATE?

- Simple way to find $\gcd(a, b)$
  - Take i to be 1 to $minimum(a, b)$
    - Check if i divides both a and b
    - Largest such i is the gcd
  - Just like the naïve method we learned for finding divisors
    - Too many unnecessary calculations
- Let's learn a clever way- **The Euclidean Algorithm**

GCD – THE EUCLIDEAN ALGORITHM



GCD – THE EUCLIDEAN ALGORITHM

- $\gcd(a, b) = \gcd(b, a\%b)$
- $\gcd(p, 0) = \gcd(0, p) = p$
- Example: $\gcd(15, 6) = \gcd(6, 3) = \gcd(3, 0) = 3$
- Changing order does not matter
  - $\gcd(6, 15) = \gcd(15, 6) = \cdots$
- $\gcd(15, 8) = \gcd(8, 7) = \gcd(7, 1) = \gcd(1, 0) = 1 \rightarrow Co - prime$
- Very fast compared to naïve algorithm

LCM - LEAST COMMON MULTIPLE

- ল.সা.গু.: লঘিষ্ঠ সাধারণ গুণিতক
- Multiple
- Common multiple
- Least common multiple
- Example: Find the lcm of 15 and 6
  - Multiples of 15: 15, 30, 45, 60, ...
  - Multiples of 6: 6, 12, 18, 24, 30, 36, 42, 48, 54, 60, ...
  - Common multiples: 30, 60, ...
  - LCM: 30

LCM – HOW TO CALCULATE?

- Simple way to find lcm(a, b)
  - Take multiples of the larger number and see if the smaller one divides it
  - Computation heavy
- Or you may find it using gcd:
  - $\gcd(a, b) \times lcm(a, b) = a \times b$ [Proof: try yourself. Use examples first to understand how it happens]
  - Example: $lcm(15,6) = \frac{15\times 6}{3} = 30$

FACTORIAL

- A function defined for non-negative integers
  - 0,1,2,3, …
- $n! = n\times(n-1)\times(n-2)\times(n-3)\times\ \dots\ \times3\times2\times1$
- $4! = 4\times3\times2\times1 = 24$
- Exception: $0! = 1$
  - Consider how other factorials are related
  - $5! = 120 \rightarrow divide\ it\ by\ 5 \rightarrow 24$
  - $4! = 24 \rightarrow divide\ it\ by\ 4 \rightarrow 6$
  - $3! = 6 \rightarrow divide\ it\ by\ 3 \rightarrow 2$
  - $2! = 2 \rightarrow divide\ it\ by\ 2 \rightarrow 1$
  - $1! = 1 \rightarrow divide\ it\ by\ 1 \rightarrow stays\ 1$
  - $\mathbf{0!\ =\ 1}$

FACTORIAL – WHY?

- Consider a cricket team with 11 players
- How many batting orders are possible?
- What if we had 2 players?
  - Answer: 2
  - AB or BA
- What about 3 players?
  - Answer: 6
  - ABC, ACB, BAC, BCA, CAB, CBA

FACTORIAL – WHY?

- Now go for 11 players

11	10	9	8	7	6	5	4	3	2	1

- $11 \times 10 \times \cdots \times 1 = 11!$
- These type of problems are part of "Combinatorics”
  - A branch of mathematics
  - Deals with permutation and combination
  - More later

INTRO TO MATRIX/MATRICES

- Collection of numbers
- Arranged in rows and columns

- $A_{i,j} = ?$

$$A = \begin{array}{cc} \text{3 columns} & \\ \begin{bmatrix} -2 & 5 & 6 \\ 5 & 2 & 7 \end{bmatrix} & \text{2 rows} \end{array}$$

- $A_{2,3} = ?$

INTRO TO MATRIX/MATRICES

- How many rows and columns in each of the following matrices?

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ 2 \\ 6 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 9 & -3 & 0 \end{bmatrix}$$

ADDING MATRICES

- Why?
- Let's consider a matrix as an exam result table:
  - Rows indicate individual students (3 students)
  - Columns indicate subject (2 subjects)

$$\begin{bmatrix} 40 & 36 \\ 28 & 32 \\ 30 & 27 \end{bmatrix} + \begin{bmatrix} 19 & 35 \\ 48 & 12 \\ 33 & 48 \end{bmatrix} = \begin{bmatrix} 59 & 71 \\ 76 & 44 \\ 63 & 75 \end{bmatrix}$$

Midterm

Final

POWER AND ROOTS

- **Power**: b^x
- Base and exponent

$$b^x = \underbrace{b \times \cdots \times b}_{x \text{ times}}$$

- $5^3 = 5 \times 5 \times 5 = 125$

POWER AND ROOTS

- **Root**: $\sqrt[n]{a}$ $(n^{th}$ root of $a)$

- Assume: $\sqrt[n]{a} = x$
- Then, $x^n = a$

- $\sqrt[2]{16} = 4$
  - $[4^2 = 16]$
- $\sqrt[3]{27} = 3$
  - $[3^3 = 27]$

INTRO TO SETS

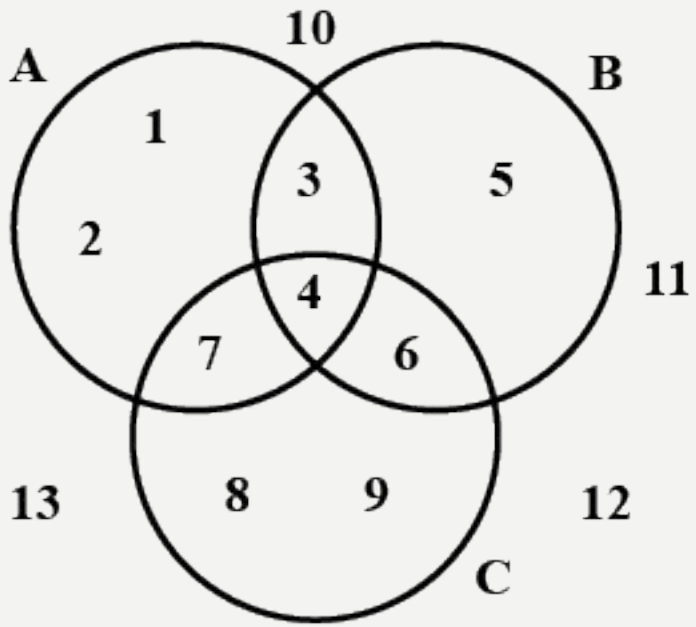
- A collection of objects
  - No specific order or index [Unlike matrix]
- Objects are called elements
- Example
  - Natural number set: $\{1,2,3, \ldots\}$ Infinite set
  - Name of your favorite sports: $\{Cricket, football, hockey, volleyball\}$ Finite set

INTRO TO SETS

- Subset:
  - A is a subset of B if all the elements of A is also in B
  - A = $\{10, 12, 29\}$ and b = $\{101, 12, 10, 29, 32\}$
- Universal Set:
  - Depends on context
  - Example:
    - When talking about numbers, universal set might be the set of Real Numbers
    - In case of set of favorite sports, universal set would be the set of all sports
- Empty/Null set:
  - Set with zero members: { } or Ø

INTRO TO SETS

- $U = \{1,2,3,4,5,6,7,8,9,10,11,12,13\}$
- $A = \{1,2,3,4,7\}$
- $B = \{3,4,5,6\}$
- $C = \{4,6,7,8,9\}$



- Three set operations:
  - Union
  - Intersection
  - Complement

SUMMARY

- GCD and LCM
- Euclidean algorithm for GCD (and LCM too)
- Factorials and a combinatorial problem
- Matrices and an application of them
- Power and roots
- Sets and set operations

- See you after mid!

NEXT

PRACTICE DAY