MATHEMATICS FOR PROGRAMMING

RECAP

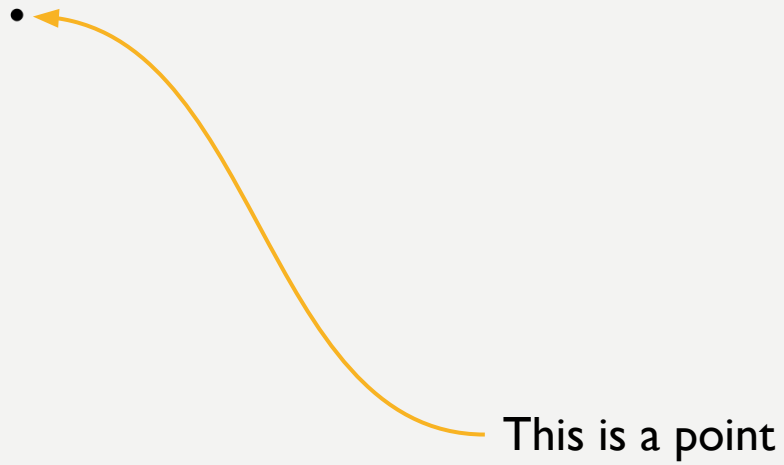
- GCD and LCM
- Euclidean algorithm for GCD (and LCM too)
- Factorials and a combinatorial problem
- Matrices and an application of them
- Power and roots
- Sets and set operations

BASIC GEOMETRY

- Points and lines
- Coordinate System
- Angles
- Some geometric shapes

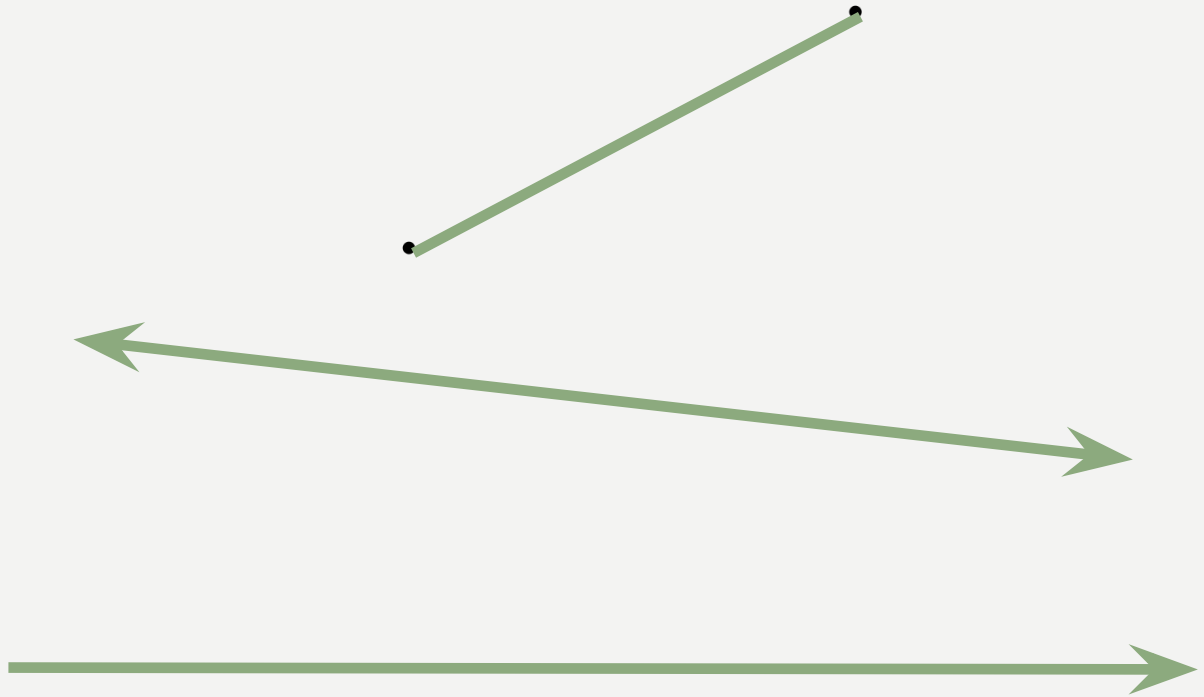
POINT

- Just a dot
- Location only
- No length, width, shape, size



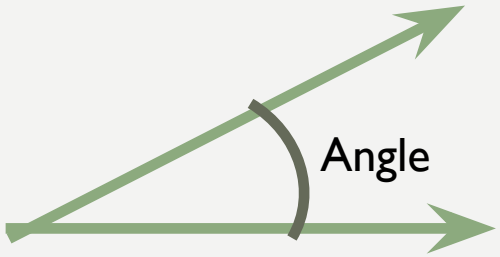
LINE

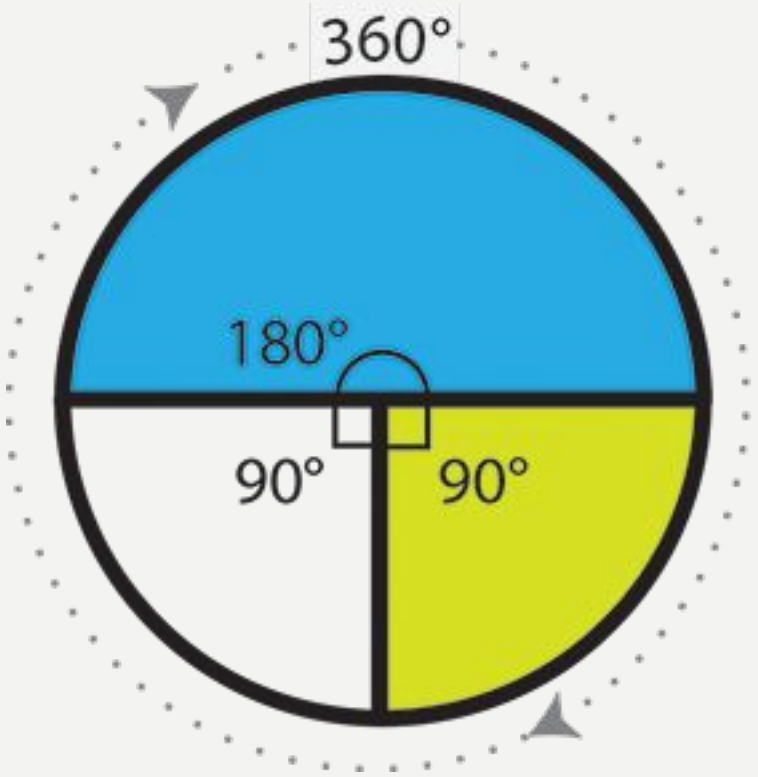
- Two different points
- Connect them
- Take the straight path
  - Also the shortest
- This is called line ***segment***
  - Finite length
- ***Line***: extended in both ends
  - Infinite length
- ***Ray***: extended in one end
  - Infinite length



ANGLE

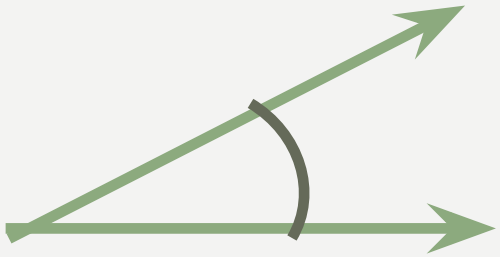
- Between two rays extending from the same point
- Circular arc to annotate
- Full rotation = 360 degrees



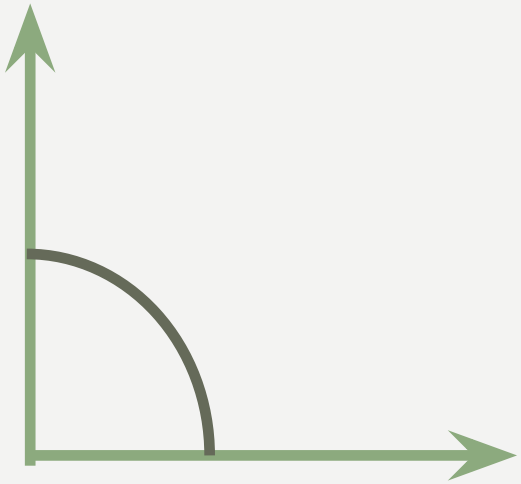


ANGLE

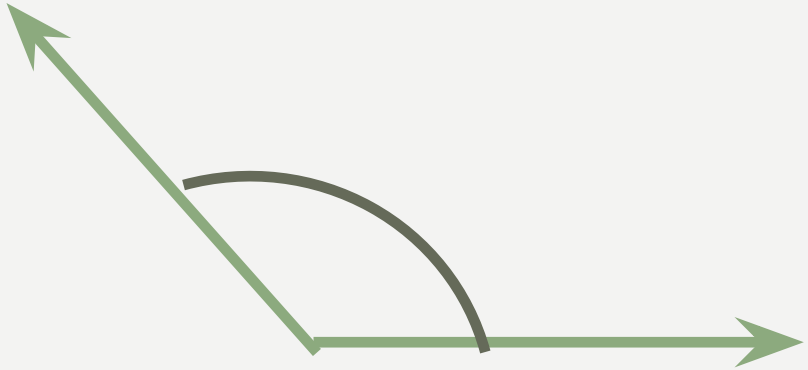
Acute Angle (<90°)



Right Angle (=90°)

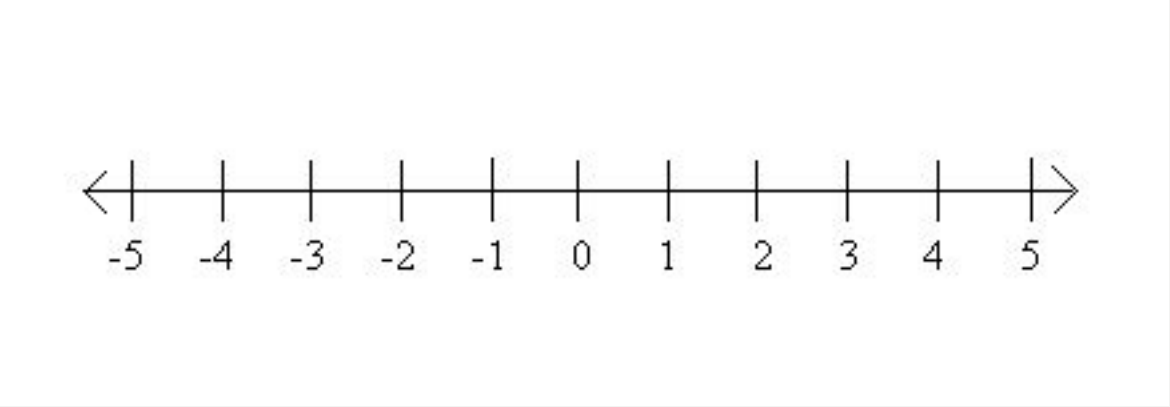


Obtuse Angle (>90°)



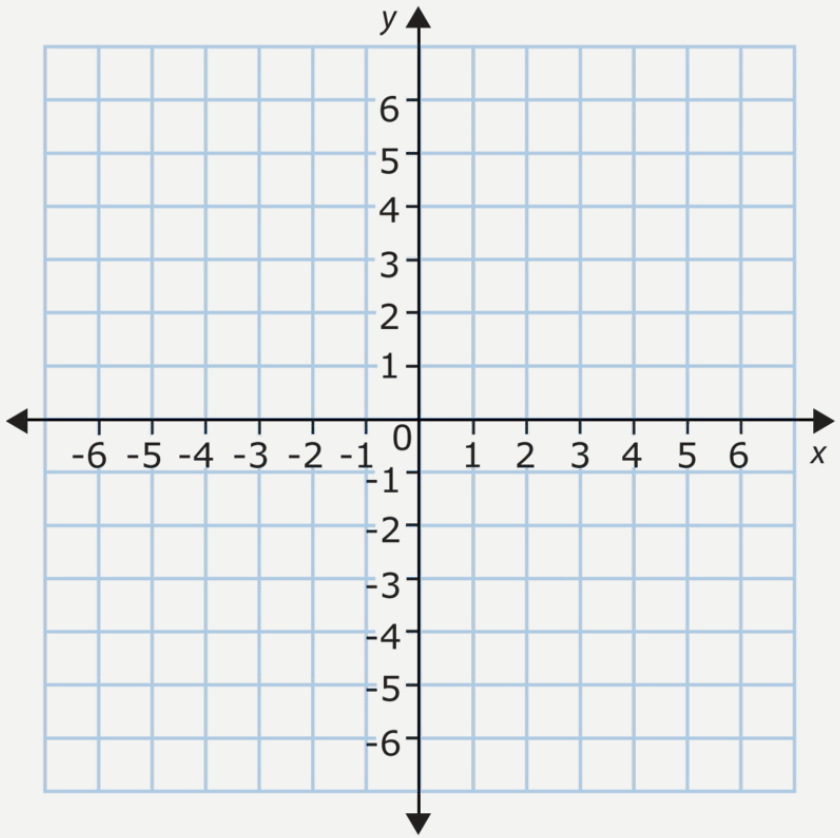
COORDINATE SYSTEM

- How do you navigate in real life?
  - Reference point, directions, distance etc
- How to identify a point in space?
  - Coordinate System!
  - Remember number line? One-dimensional coordinate system.



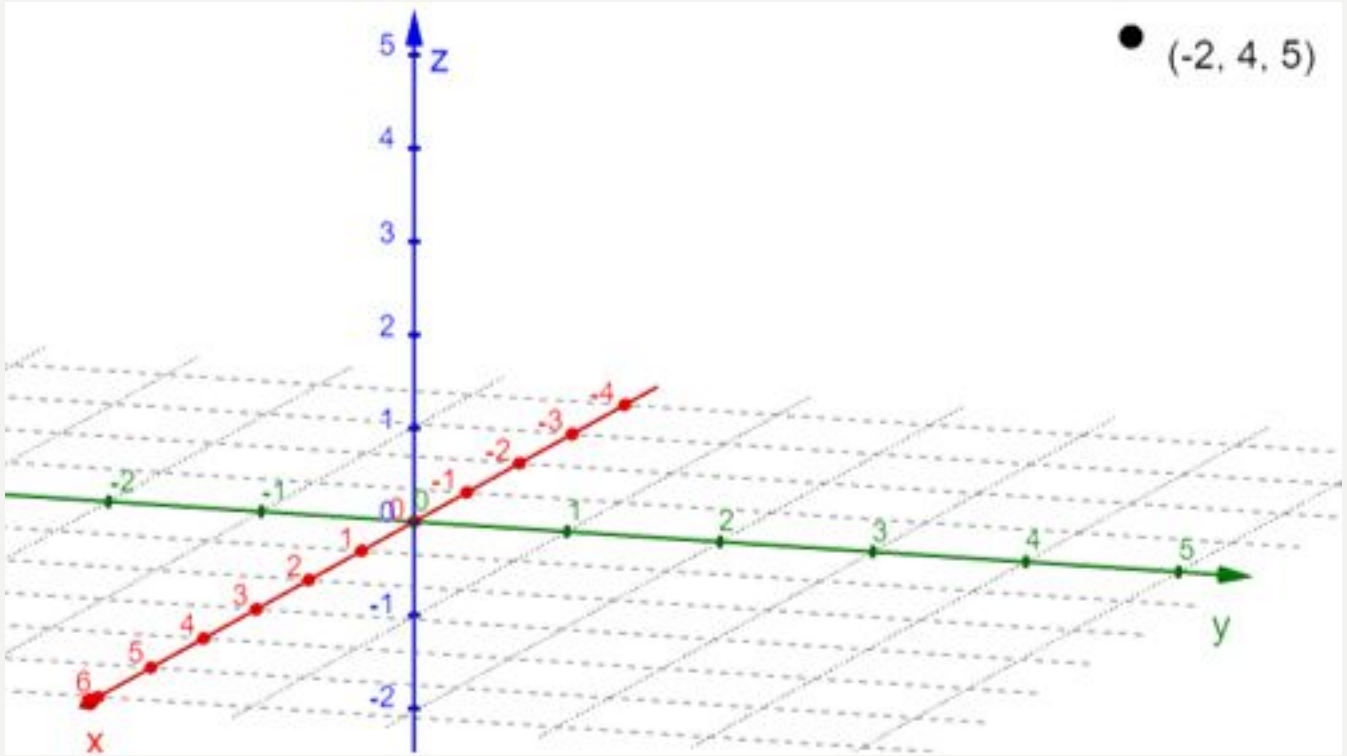
COORDINATE SYSTEM

- 2D coordinate system
- Take two perpendicular lines
  - Right angle / 90 degrees
  - Horizontal: X axis
  - Vertical: Y axis
- Intersecting point: Origin (0,0)
- Any point can be identified as a pair of numbers
  - (x,y)
  - Where is (-2, 5)?



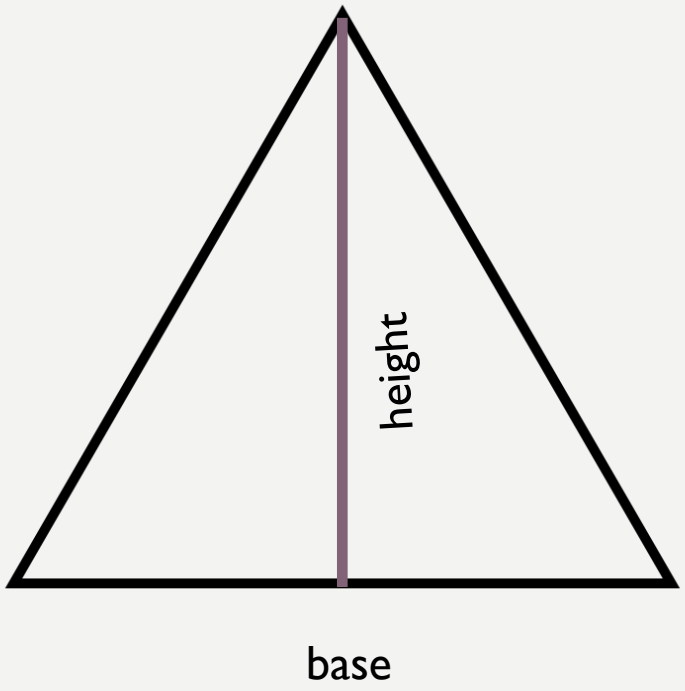
COORDINATE SYSTEM

- How about 3D coordinate system?
  - (x,y,z)
  - Hard to visualize on paper
  - But easier in real life



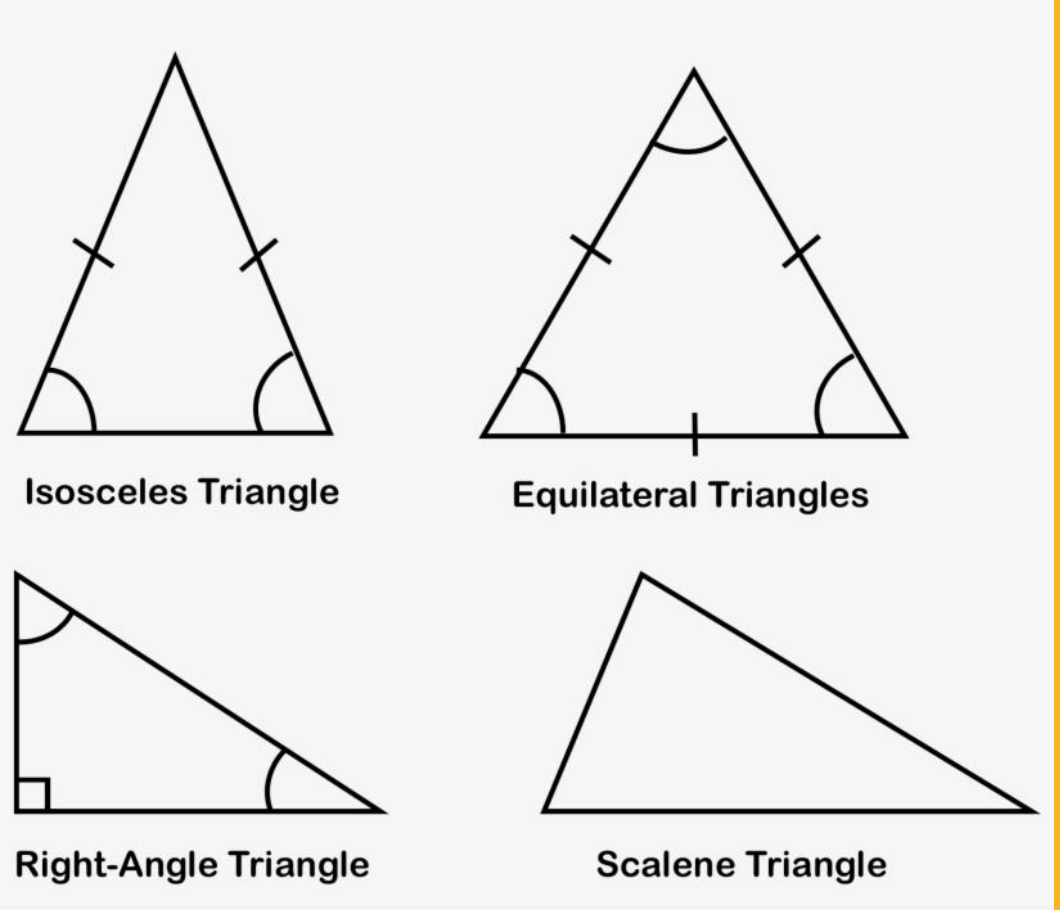
TRIANGLE

- 3 points and 3 line segments
- Sum of three angles = $180°$
- Area = $\frac{1}{2} * base * height$



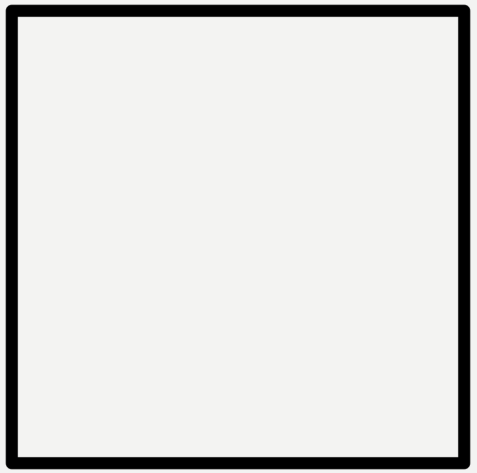
TRIANGLE

- Based on equality between sides
  - Equilateral
  - Isosceles
  - Scalene



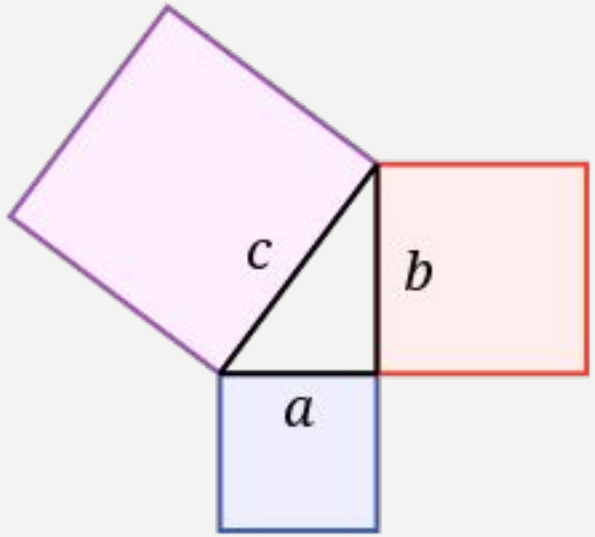
SQUARE

- 4 points and 4 line segments
- All sides are equal
- Each angle is $90°$
- Sum of four angles is $360°$
- **Area**: $(side\_length)^2$
- Length of **diagonal**: $\sqrt{2} \times (side\_length)$



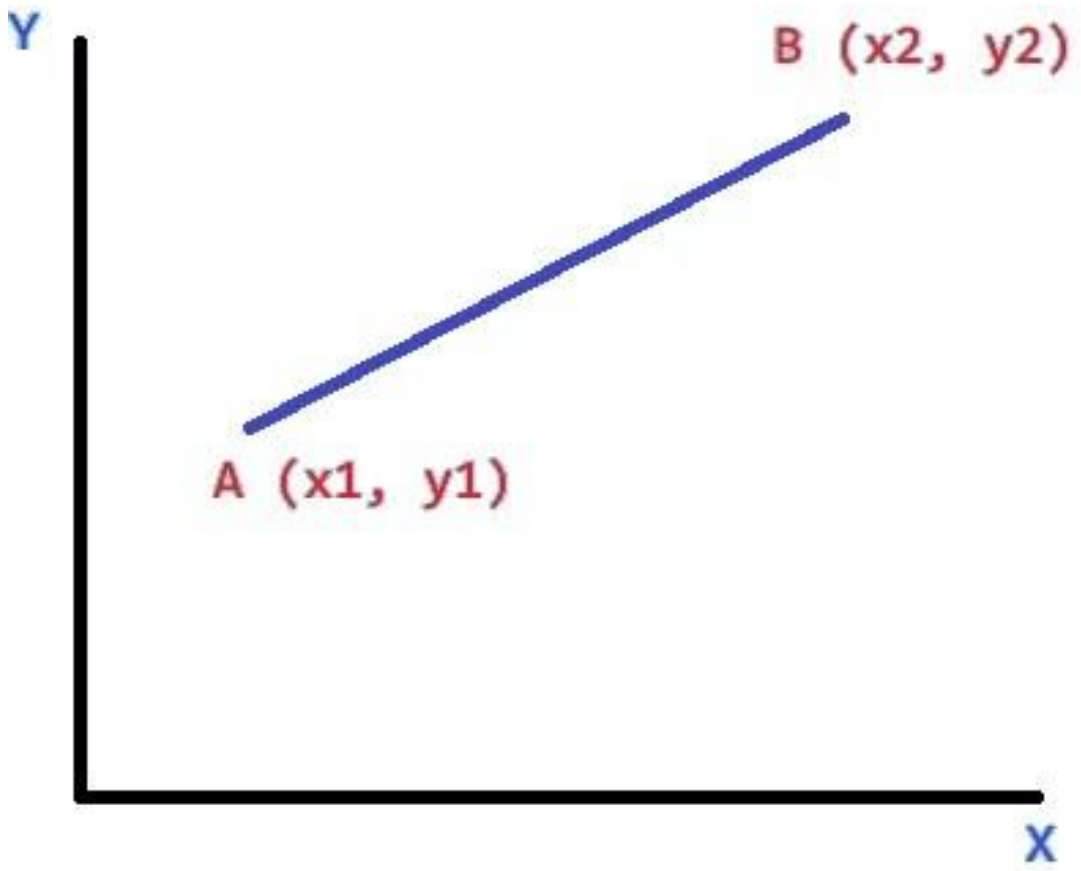
PYTHAGOREAN THEOREM

- Pythagorean theorem
  - $c^2 = a^2 + b^2$
- Right-angle triangle and square
- Example:
  - $5^2 = 3^2 + 4^2$
  - $13^2 = 5^2 + 12^2$



DISTANCE BETWEEN TWO POINTS

- Can you apply Pythagorean Theorem to get distance between A and B?
- $d_{AB} = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

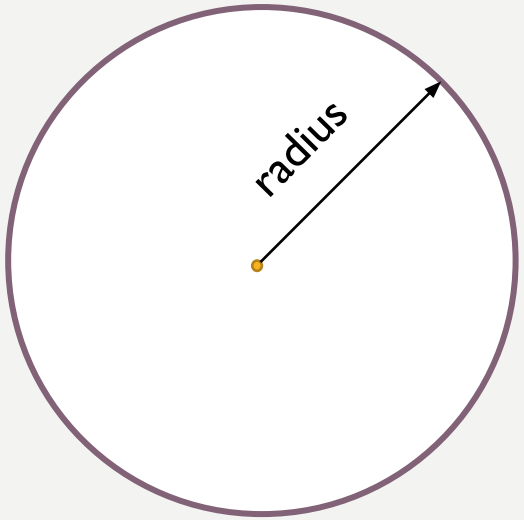


RECTANGLE

- 4 points and 4 line segments
- Opposite sides are equal
- Each angle is 90°
- Sum of four angles is 360°
- Area: $length \times height$

CIRCLE

- Center
- Radius
- Diameter
- Area = $\pi \times (radius)^2$



SERIES

- Sequence: List of numbers with some order or pattern
  - 1,3,5,7,9
  - 1,1,2,3,5,8,13, …

- Series: Sum of elements in a sequence

$$S_1 = 1 + 3 + 5 + 7 + 9$$

$$S_2 = 1 + 1 + 2 + 3 + 5 + 8 + 13 + \cdots$$

- Finite vs Infinite series
- Arithmetic vs Geometric Series

$$1 + 6 + 11 + 16 + 21 + \cdots$$

$$2 + 6 + 18 + 54 + \cdots$$

1+2+3+…+N = ?

- $1 + 2 + 3 + 4 + 5 = ?$
- Let,

$$S = 1 + 2 + 3 + 4 + 5$$

$$S = 5 + 4 + 3 + 2 + 1$$

$$(+), 2S = 6 + 6 + 6 + 6 + 6$$

$$2S = 5 \times 6$$

$$S = 15$$

1+2+3+…+N = ?

- $1 + 2 + 3 + \cdots + N = ?$
- Let,

$$S = 1 + 2 + 3 + \cdots + N$$

$$S = N + (N-1) + (N-2) + \cdots + 1$$

$$(+), 2S = (N+1) + (N+1) + (N+1) + \cdots + (N+1)$$

$$2S = N \times (N+1)$$

$$S = \frac{N \times (N+1)}{2}$$

SERIES (FINITE SUM)

- Arithmetic series

  $S = 2 + 5 + 8 + \cdots$

  Find the 100^{th} number in this series.

  Find the sum of first 100 numbers.

SERIES (FINITE SUM)

-
- n^{th} term in arithmetic series: $a + (n-1)\times d$

$\rightarrow$ 100^{th} number = $2 + 99\times 3 = 299$

- Sum of first n terms: $\frac{n}{2}\{2a + (n-1)\times d\}$

$\rightarrow$ sum of first 100 numbers = $\frac{100}{2}\{2\times 2 + (100-1)\times 3\} = 15050$

SUMMARY

- Basic Geometry
  - Points – Lines
  - Rectangle – Square
  - Circle
- Coordinate system
  - 2D and 3D – Distance between points
- Sequence and Series
- Finding sum of series

PRACTICE DAY

- Find the straight line (shortest) distance between two points in a 3-dimensional space.
  - Points: $A(-10,23,5)$ & $B(31,-11,76)$
- Find the area of the green region if the side length of the square is 10
- Find the sum of first n odd numbers. Calculate for $n = 10,100,1000$