Transport Equation.

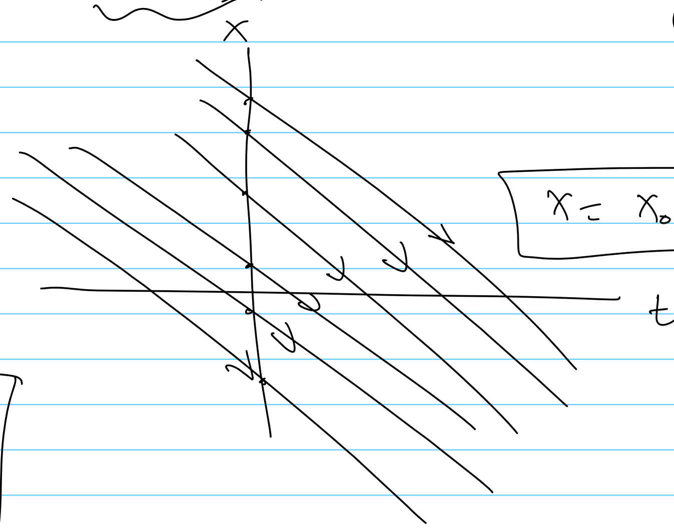
$$u_t = c\,u_x \quad , \quad u(0,x) = f(x).$$

$$u_t - c\,u_x = 0$$

$$\begin{pmatrix} u_t \\ u_x \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -c \end{pmatrix} = 0$$

$$(u_t , u_x) \cdot (1 , -c) = 0$$

$u(t,x)$ = constant in direction $(1,-c)$



$$\boxed{x = x_0 - ct}$$

$$\boxed{\begin{aligned} u(0,x) &= f(x) \\ u(0,x_0) &= f(x_0) \end{aligned}}$$

$$u(t, x_0 - ct) = C$$

$$u(0, x_0) = \boxed{f(x_0)}$$

$$u(t, x_0 - ct) = f(x_0)$$

$$x = x_0 - ct$$

$$x_0 = x + ct$$

$$\boxed{u(t, x) = f(x+ct)}$$

Fourier Transfor

$$u_t - c u_x = 0 \quad , \quad u(0, x) = f(x)$$

$u, f \to 0$
$|x| \to \infty$
$(x \to \pm\infty)$

$$\hat{u} = \frac{1}{2\pi} \int_{\mathbb{R}} u(x) e^{-ikx} dx$$

$$\hat{u}_x = ik \hat{u}$$

$$\hat{u}_t - c \cdot (ik) \hat{u} = 0$$

$$\hat{u}_t - ikc \hat{u} = 0$$

$$\hat{u}(0, k) = \hat{f}(k)$$

$y' - ay = 0$
$y = y_0 e^{at}$

$$\hat{u}(t,k) = \hat{f}(k) e^{ikct}$$

$$u(t,x) = \int_{\mathbb{R}} \hat{f}(k) e^{ikct} \cdot e^{ikx} dk$$

$$u(t,x) = \int_{\mathbb{R}} \hat{f}(k) e^{ik(x+ct)} dk.$$

$$\boxed{\int_{\mathbb{R}} \hat{f}(k) e^{ik(h(x))} dk = f(h(x))}$$

$$\boxed{u(t,x) = f(x+ct)}$$

$$u_t = f' \cdot c$$

$$u_x = f' \cdot 1$$

$$u_t - c u_x = 0$$

$$cf' - cf' = 0$$