Statistical Resource Decoupling in Random Access Interference Channels

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***Abstract*—In this work, we introduce a metric to quantify the amount of *channel coupling* present in a random access system. Concentrating on the Z-interference channel, we show that introducing fading channel assumptions into random access analysis exhibits an increase of throughput performance on colliding links; this increase is due to the added channel decoupling caused by the fading channel. We also compare the sum-throughput of random access systems under different channel assumptions and describe how their performance depends on the SNR regime at which the system is operating.**

***Index Terms*—Random access, fading, interference channel.**

I. Introduction

In communication networks, we are interested in understanding how the available resources are used and how to best allocate them. These resources are either shared by several members of the network or they are available to specific members. In the case of wireless networks, one of the prominent resources that is shared by all members is the transmission channel. We refer to this sharing of the same resource by multiple members of a network as *resource coupling*. Concentrating on the channel as the resource of interest, we can think of the wireless users interfering with each other as coupled. Since interference poses negative effects, extensive research has been performed to, essentially, try to *decouple* members of wireless networks. Examples of different approaches are orthogonalizing transmission in either time (TDMA) or frequency (FDMA). [8] These approaches effectively decouple the channel but at the cost of more resources (such as a centralized control unit, larger bandwidth, or perfect channel state knowledge). An alternative approach relies on randomly deciding when to use the channel, thus creating a random access system. In this case we encounter a less effective decoupling (*e.g.*, there exists possibility of collisions) and suboptimal utilization of the channel. The main reason to consider the random access system is the well-known good throughput performance it achieves under situations of little or no channel state information across the network [6].

Typical analysis of random access networks is performed in two major scopes at different layers of the system. At the physical layer, the network is generally studied under the assumptions of fading channel statistics and fully backlogged users trying to communicate. At the network layer, the analysis includes assumptions about the randomness of the arrival traffic and perfect communication over the wireless network (except for collisions). Due to this disagreement in the assumptions at different layers, random access protocols are not optimally designed. In this work we are interested in cross-layer analysis of random access protocols under a more complete set of assumptions for the wireless random access interference network. We introduce the idea of resource decoupling to measure the behavior of random access protocols by comparing them to systems in which resources are completely decoupled based on full knowledge of channel and user status information around the network (such as TDMA). One important step in the explanation of this resource decoupling idea is the introduction of a metric that captures the amount of resource coupling in a network. With channel interference as the measure of coupling, the metric proposed is $\alpha = \frac{INR}{SNR}$. As the interference increases so does the value of α and a value of $\alpha = 0$ signifies the channels will be decoupled. We refer to the user with such metric as being "α-coupled."

The rest of this paper is organized as follows: Section II introduces the system model and defines our resource coupling metric. Section III describes the channel decoupling effect due to fading in our system and shows the plots of throughput performance under different system conditions. Section IV provides an interpretation of the results relating it to the metric, α, that has been introduced. Finally, we conclude our work in Section V.

II. System Model and Metric

A. System Model

The system model analyzed consists of a random access interference Z-channel. Consider the 2-user case shown in Figure 1. Nodes A and B are transmitters trying to communicate with receivers a and b, respectively. In this scenario Node A cannot hear Node B's transmission and vice versa which leads to a hidden node situation. In the random access system, we have the possibility of collisions at receiver a. The system is time-slotted at the symbol level. The receivers are single-user (*i.e.*, the receiver can only decode one user in a time slot. Two users sending simultaneously create a collision). Each one of the transmitters can be in two possible states: *active* or *inactive*. We also consider the channels between the users and the destination and the neighbors to be independent Rayleigh fading, distributed as $\mathcal{CN}(0, \sigma_{xy})$, where xy are the source and destination nodes, respectively. The receiver is assumed to be able to receive the transmitted symbol without error if the

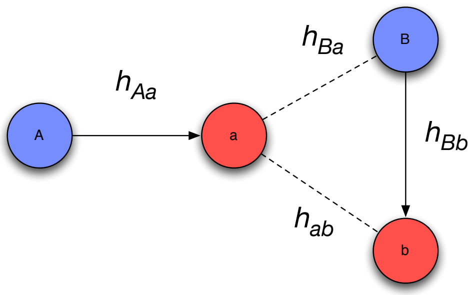

Fig. 1. System Model

channel is not in outage. This joint consideration of collisions and fading is what brings the concept of coupling into our system. Summarizing the possible scenarios in our system at any given time, there are 5 different states for our network:

1) ***Idle*** - Neither user is transmitting
2) ***Successful Transmission*** - Only one user is active *and* the transmission channel is not in outage
3) ***Loss due to channel*** - Only one user is active but the transmitting channel is in outage
4) ***Collision*** - Both users are active *and* both channels interfere with each other
5) ***Capture*** - Both channels are active but their channel qualities are separated enough to not cause interference

These states come from the fact that the receiver tries to decode only one of the signals and treats the other signal (if present) as noise. The transmitters only have channel information about their own channel and do not have information about the state of the other user. The system does not assume channel information at the receiver. One important point to note here is that since we are assuming that the receivers do not have channel knowledge interference cancellation techniques, such as the ones in [2], [3], etc. are not being used.

In this paper we are interested in analyzing the behavior of random access systems so we begin by considering the following random access mechanism. Nodes A and B receive packets with Poisson arrivals of rates η_A and η_B, respectively. The probabilities of each node transmitting at a time slot are G_A and G_B. The access mechanism that we will analyze is the simple slotted Aloha protocol first introduced in [1]. A node transmits if it has a packet to send and it does not consider retransmissions. The only source of randomness in accessing the medium is the Poisson arrivals.

B. Metric

We want to establish the idea that random access protocols achieve resource decoupling (specifically channel decoupling) by randomly selecting when to transmit. In order to quantify the performance of random access protocols we wish to measure the amount of decoupling achieved. The desired metric should give a comparison to the fully decoupled scenario, *i.e.*, when the channel between Nodes B and a does not exist. Since receiver a is trying to decode the transmission from Node A, it treats the transmission from Node B as noise. Node a is able to decode if its channel is not in outage. Considering the interference from Node B as noise, we can say [4] the probability of outage at receiver a when interference is present is

$$P_{out_{Ba}} = Prob\left\{\frac{P_A|h_{Aa}|^2}{N_a + P_B|h_{Ba}|^2} < 2^{R_a} - 1\right\}, \quad (1)$$

where P_A and P_B are the transmission power of users A and B, respectively; N_a is the noise power at receiver a, h_{Aa} and h_{Ba} are the channel magnitudes, and R_a is the desired transmission rate for link A. For the rest of the paper will we use the convention of denoting the probability of outage at receiver a in the presence of interference as $P_{out_{Ba}}$ and P_{out_a} and P_{out_b} as the probabilities of outage at receivers a and b with no interference, respectively. It can be seen from here that if the network was fully decoupled, $h_{Ba} = 0$. Keeping this is mind we define our coupling metric as:

$$\alpha \triangleq \frac{INR}{SNR} = \frac{P_B|h_{Ba}|^2}{\frac{P_A|h_{Aa}|^2}{N_a}} = \frac{P_B|h_{Ba}|^2 N_a}{P_A|h_{Aa}|^2}. \quad (2)$$

This coupling metric is time variant and changes according to channel conditions. We also define an average coupling metric as:

$$\alpha_{avg} \triangleq \frac{\sigma_{Ba} P_B N_a}{\sigma_{Aa} P_A} \quad (3)$$

where σ_{Aa} and σ_{Ba} are the average channel gains of links Aa and Ba, respectively. This average metric will be our metric of interest since it captures the statistics of the network.

Another characteristic of α is that it captures the ratio between the probability of outage at receiver a when interference from Node B is present and when it is not. That is, when no interference is present (the system is decoupled), then $\frac{P_{out_{Ba}}}{P_{out_a}} = 1$ and as the interference increases, $\frac{P_{out_{Ba}}}{P_{out_a}}$ also increases. Hence, the metric has the favorable property of being proportional to this ratio, $\alpha \propto \frac{P_{out_{Ba}}}{P_{out_a}}$.

III. Resource Decoupling Due To Fading

In this section we analyze the performance of the slotted Aloha protocol. We present our results in two major parts. First, we quantify the performance of the wireless network under a constant coupling measure α, *i.e.*, in a Gaussian (non-fading) system. Secondly, we describe how the presence of the fading channel increases the amount of decoupling in the network and we compare this performance to that of the Gaussian channel network. This interesting result encapsulates the major contribution of this work.

A. Gaussian Channel

To create a baseline for comparison in our analysis, we evaluate the performance of the network under non-fading assumptions. In this case our channel in non-varying. Under these conditions, there exist two scenarios: (1) Node B is not within listening range of Node a or (2) Node B is within range of Node a. Given case (1), the system is fully decoupled and transmission is assumed to be collision free if the rate

R_A at which Node A is transmitting is less than the Shannon Capacity [7]

$$R_A \leq \left(1 + \log\left(\frac{P_A}{N_a}\right)\right). \tag{4}$$

If we have an interference system, as in the second scenario, we will have perfect transmission dependent on the power at which Node B is transmitting. Node a is assumed to decode perfectly if

$$R_A \leq \left(1 + \log\left(\frac{P_A}{N_a + P_B}\right)\right). \tag{5}$$

In the cases where either (4) or (5) hold, the channel is completely decoupled and the total throughput of the network is constrained only by the source arrival rates. Since we want to quantify different level of coupling, we wish to concentrate in the case where (5) does not hold (*i.e.*, when there is interference between the two links). In the case that (5) does not hold, we see the typical performance of Aloha with interference on link A and a perfect linear gain on link B. The results are shown in Figure 2. To summarize, if either

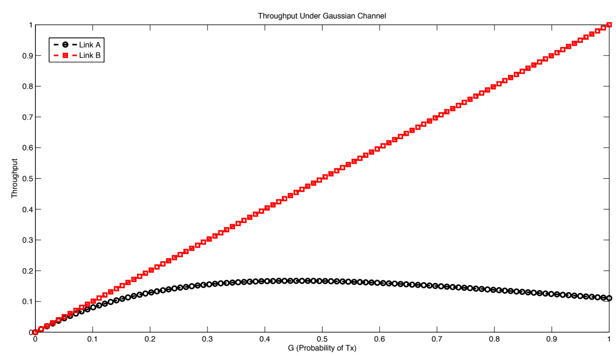

Fig. 2. Throughput Performance Under Gaussian Channel

(4) or (5) hold, then we have a system that a system that is completely decoupled. When we relate this to the metric α, we can see that it gives us a threshold that allows us to determine if the system is fully decoupled. This occurs when

$$\alpha_{ave} \leq \frac{P_B N_a}{(N_a + P_B)2^{R_A - 1}}. \tag{6}$$

It is important to note that under the Gaussian channel assumption, the coupling behavior of the network is bimodal. The system is either coupled and exhibits collisions or it is fully decoupled and shows linear gains with arrival rates. When the system is fully decoupled (*i.e.*, Equation (6) holds) we have linear growth on both links with respect to the probability of transmission.

B. Rayleigh Fading Channel

When we introduce fading to our system, we intuitively expect the link without interference (link B) to show some degradation [9]. The interesting event to analyze is the behavior of the link that is being interfered with. It will be adding new losses due to channel fading, just as the case in link B; however, we also expect the number of collisions to be reduced since we will be entering the *Capture* scenario. The question is: are the capture gains larger than the losses due to fading? From the previously described decoding rules, we derive the throughput of the two links (Θ_A and Θ_B) in the following way:

$$\begin{aligned}\Theta_A &= [P(success|\text{B is not active})P(\text{B is not active}) \\ &\quad + P(success|\text{B is active})P(\text{B is active})]P(\text{A is active}) \\ &\quad = [(1 - P_{out_a})(1 - G_b) + (1 - P_{out_{Ba}})(G_b)]G_a \end{aligned} \tag{7}$$

and

$$\Theta_B = G_b(1 - P_{out_b}), \tag{8}$$

where

$$P_{out_a} = 1 - \exp\left(-\frac{(2^{R_a} - 1)N_a}{P_A}\right), \tag{9}$$

$$P_{out_b} = 1 - \exp\left(-\frac{(2^{R_b} - 1)N_b}{P_B}\right), \tag{10}$$

and

$$P_{out_{Ba}} = 1 - \frac{e^{-\frac{(2^{R_a}-1)N_a}{P_A}}}{(2^{R_a} - 1)(\frac{P_b}{P_a}) + 1}. \tag{11}$$

We plot the results of these throughput equations in Figure 3. When there is no fading, we see that the link not being

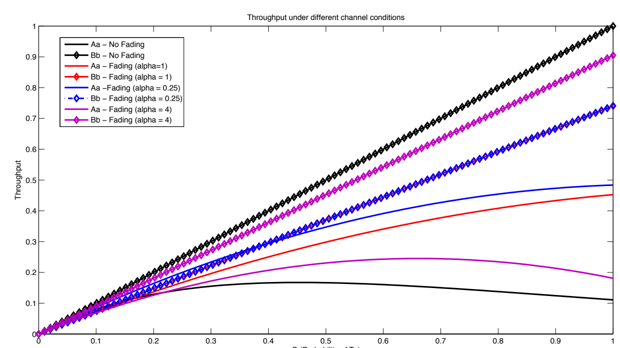

Fig. 3. Throughput Performance under different interference levels

interfered with achieves a constant throughput gain as its own probability of transmitting increases. The link that suffers interference (link A) shows the typical Aloha system behavior. As fading is introduced to the network, we see that the performance of link B starts to suffer since its transmission is no longer perfect, as was expected. The important result to notice is that the throughput of the link with interference is improved. The channel is being decoupled due to the presence of fading and, hence, some collisions are avoided. This results shows that the gains from avoiding collisions is greater than the loss due to fading channel corruption (for an SNR value of at least 1) when our decoding rule assumption is as in Equation 1.

When fading is introduced to the system, channel decoupling becomes a continuous process rather than a discrete bimodal behavior as seen in the Gaussian channel case (see Section III-A). Fading introduces a new source of decoupling by nullifying the effect of the interfering channel between Node B and Node a.

C. Sum Throughput

Having established that the overall throughput behavior for link Ashows gains, we again investigate if the sum-throughput of the system also shows gains. We compare the throughput of the system under the Gaussian channel and the fading channel. We also examine different power distributions for the two users. In each comparison (low SNR and high SNR), we apply an overall power constraint to the system in order to establish a fair comparison.

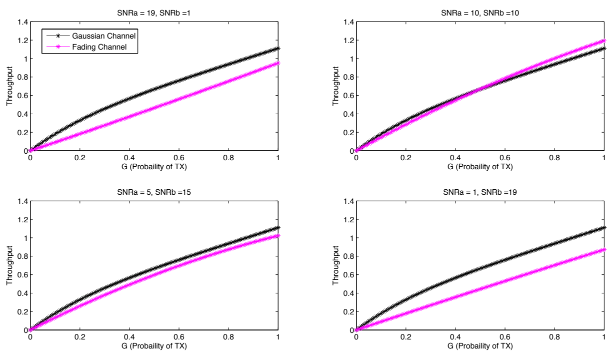

Fig. 4. Sum Throughput Performance - Different power distributions (Low SNR)

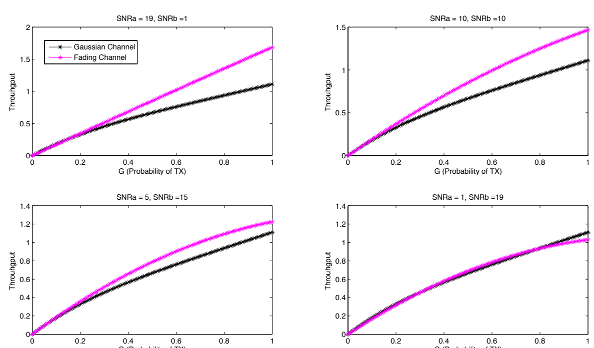

Fig. 5. Sum Throughput Performance - Different power distributions (High SNR)

Figures 4 and 5 show the sum-throughput of the system under conditions of low and high SNR respectively. We point out that the performance of the sum throughput is dependent on the SNR regime at which the system is operating. In the low SNR regime the system with the Gaussian channel assumptions performs better than the one with fading channels assumptions. This trend predominates except for the case in which both transmitters have the same power. In this case, at the higher loads, the system with fading channels shows a higher sum throughput. In the high SNR regime, the pattern reverses and the system with the fading channel assumption displays higher sum throughputs except for the case in which link B has 95% of the power. This is due to fact that, at lower SNRs, the main contributor to performance loss is the channel state. We see the gains of channel decoupling in the high SNR regime, where the main cause of loss is collisions, and the fading channel reduces this effect by decoupling the channel. This results can be explained by referring to our coupling metric, α, defined in equations (2) and (13). We discuss and give an interpretation of the results in the next section.

IV. Discussion on the Results

Given the results shown in the previous section, let us discuss in more detail the characteristics of the α metric proposed as a measure of resource coupling. This channel decoupling metric, α, is an indicator of throughput performance in the random access system. When we compare the throughput of the Gaussian and fading channel systems, we see that link A shows a better performance under the fading channel assumption with the same power constraints. We also see that as the value of α decreases, the throughput of link A increases. This occurs because as the interference at receiver a increases, the probability of a collision increases as well.

A. Throughput of Link A

In the case of the Gaussian channel, we see two behaviors dictated by the threshold shown in Equation (6). If the equation holds, $\Theta_A = G_a$. If it does not hold, we have the well known result for the slotted Aloha system [5], $\Theta_A = G_a(e^{G(1+\zeta)})$, where G is the total system load and ζ is the transmission delay of the system. Our value of α tells us if our throughput performance will consist of linear gains or collision induced Aloha behavior.

In the fading channel case, Θ_A is as defined in Equation (7). Here our value of α is directly related to Θ_A in the form of P_{out_a} and $P_{out_{Ba}}$. As the value of α decreases, one or both of the values of P_{out_a} and $P_{out_{Ba}}$also decrease, hence leading to a higher Θ_A. By keeping track of the coupling metric, we are able to consolidate the effect of both of the outage terms into the one term, α.

There is also an important behavior that we are able to demonstrate using our formulation. In the typical Aloha throughput plots, as the carried load of the system increases, the throughput increases and then starts decrease after some point $(1/(e(1+\eta))$ in slotted Aloha). In Figure 3, we see that for some values of α the throughput is always increasing. By taking the derivative of Equation (7) we have that if

$$P_{out_{Ba}} \leq \frac{1 - P_{out_a}(G+1)}{G} \tag{12}$$

then value of the throughput will be increasing in the interval [0, G).

B. Sum Throughput

The previous discussion is all about the throughput for link A; we now the discuss the sum throughput. The major result is that the overall sum throughput of the system is higher in the Gaussian or the fading channel depending on which SNR regime the system is operating at. The results are discussed in Section III-C. One interpretation of these results can be arrived at using our metric. By looking at Equation 2, we can rewrite it in the form

$$\alpha_{avg} = \frac{\sigma_{Ba} P_B N_a}{\sigma_{Aa} P_A} = \left(\frac{\sigma_{Ba}}{\sigma_{Aa}}\right) \frac{SNR_B N_b}{SNR_A} \quad (13)$$

This formulation shows that at higher SNR values for both SNR_A and SNR_B, the denominator will have a smaller value (since N_b is smaller relative to the SNR). Hence, higher SNR corresponds to a lower α value, which in turn means more decoupling.

V. Conclusion

We have introduced a metric that captures the amount of channel coupling in a random access system. The lower the value for the metric, the more decoupled the system is and the more it performs like a fully decoupled system in terms of throughput. We have shown how introducing fading channel assumptions to a Z-interference channel exhibits a boost in throughput for the interfered link due to this channel decoupling. Finally, we have shown that the sum throughput of the system has a throughput performance under the different channel assumptions that depends on the SNR regime at which the system is performing.

These results lead us to believe that the channel coupling metric α can serve as an important parameter in analyzing and designing random access protocols. By optimizing power allocations, probabilities of transmission and the such, to minimize the value of α, random access protocol throughput performance can be improved.

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