TEACHING STATEMENT

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I believe that mathematics is an essential component of any education, not so much because of the technical content of a calculus or linear algebra course, but because of the rigorous reasoning required by these and other mathematical subjects. Ultimately, the effective student, be the subject history, economics, chemistry, literature, politics, or any other, is the student who can present a well-reasoned argument. The rigor of mathematics provides an environment with unambiguous statements and established rules of deduction. Practice in formal reasoning helps a student develop a more sophisticated understanding of what it means for an argument to be convincing and thereby helps the student make a convincing argument in any setting.

Though teaching styles liked the flipped classroom are gaining popularity, the typical picture of a mathematics course, or of a university course in general, is that of a class of students learning quietly together. Students attend the same lecture and are assigned the same homework, but they largely operate independently of each other. In this statement, I focus on the role that collaborative learning, what I sometimes like to call "learning loudly" plays in my teaching. Collaborative effort is a fundamental part of professional research mathematics, but it is too often under-emphasized in the actual teaching of mathematics courses. I have performed a wide variety of teaching roles as a graduate student at Cornell University, as a visiting lecturer at Appalachian State University, and as a post-doctoral fellow at Ghent University. I have instructed basic courses from pre-calculus to multivariable and vector calculus, and I have acted as a teaching assistant and an instructor for courses in mathematical logic at both undergraduate and graduate levels. I have also collaborated on research projects with students at both the REU and PhD levels. At all these levels of teaching, I have found that learning loudly greatly enriches learning mathematics.

Discussing mathematics provides students with rapid feedback that bare lecture and homework do not. This is especially critical for beginning students and for students whose main focus is outside of mathematics because these students tend to be either too tentative or too careless when first learning to reason formally. When learning loudly, students quickly identify and correct their misconceptions and thereby refine their mathematical thinking. Learning loudly also provides students with a friendly environment that bolsters their mathematical confidence. Students can have trouble starting homework problems because they are too afraid to try anything. When working with others, students see that no one ever gets hurt from a couple of false starts and that doing mathematics generates a lot of crumpled paper and eraser dust. Collaboration gives students time to ask questions, to get answers, and, most importantly, to think.

Cornell's calculus courses stress student collaboration by design, and teaching these courses inspired much of my enthusiasm for learning loudly. Cornell's Calculus I & II organize informal homework sessions. As an instructor and co-organizer, I lead many of these sessions myself to ensure that they ran smoothly and that the students were learning as loudly as possible. Student collaboration is a formal component of Cornell's Calculus for Engineers. In this course, students spend one of their two weekly recitation meetings working together on workshops, which are multi-step problems designed to both simulate the engineering environment and illustrate recent course material more deeply than the usual homework problems do. Students in my Pre-calculus and Calculus I courses at Appalachian State collaborated in much the same way that my Cornell calculus students did. Every week, I reserved one class meeting for collaborative problems solving.

The students also did small group projects similar in spirit to the Cornell workshop problems. In my Mathematical Logic II course at Ghent University, I have the students do their homework exercises in teams.

Learning loudly pervades my office hours. When students come to me with questions about their homework, I always prefer to err by giving away too much of the answers than too little. I believe that students learn more from working through problems with a little help than from a lot of red ink. I work to organize and focus the students' ideas and help them solve their problems. The following situation is typical. I first set a student to work on his or her problem at the board. Working at the board helps the student articulate his or her difficulty upon encountering it and encourages him or her to ask questions more helpful than "how do you do this one?" The student's eventual sticking point develops into a meaningful conversation among everyone at office hours. I ask questions about what we have learned, what what we might try, and what might work. I encourage the other students at office hours to offer suggestions and work them out. We handle each difficulty in this way until we solve the problem.

Learning clear and concise writing is a major component of learning mathematics at any level, and learning loudly helps students with mathematical presentation as well as with mathematical content. Talking about mathematics exercises the students' mathematical communication skills. This practice improves the students' explanations of mathematics when speaking with others and when writing their homework solutions. It also improves the students' understanding of mathematics that is explained to them, both in a lecture and in a text. When I work with undergraduate researchers or with PhD students, I consider one of my responsibilities to be helping these students to write their discoveries as clearly as possible.

My enthusiasm for collaborative learning is not meant to be at the expense of clear lecturing. In order to make my lectures as effective as possible, I develop them to make new material as familiar as possible. This goal is inspired by intuitive psychological principles connecting understanding and memory first explicitly brought to my attention by cognitive psychologist Daniel T. Willingham's book *Why Don't Students Like School?* People best understand what is familiar to them, and the more someone knows about a subject, the easier it is for him or her to learn new things in that subject. Thus, when teaching multivariable calculus, I remind my students over and over that they already know all the calculus that I will teach them. I emphasize that the key calculus concepts do not change from the single variable case to the multivariable case. "Limit" and "continuity" mean the same thing in $\mathbb{R}^3$ as they did in $\mathbb{R}$, we just have to adapt our notion of distance to $\mathbb{R}^3$. "Riemann sum" and "integral" mean the same thing in $\mathbb{R}^3$ as they did in $\mathbb{R}$, we just have to adapt our notion of partition to regions in $\mathbb{R}^3$.

More philosophically, I believe that introductory mathematics courses paint too shallow a picture of mathematics. Low level courses are often based on calculation, sometimes to the point that I fear students with no further experience in mathematics believe the sole purpose of mathematics is to justify the calculations we make. Instructors are quick to inspire advanced students with the unity, elegance, and creativity of mathematics. There is no reason that calculus students cannot appreciate the aesthetics of mathematics, and these are the students who most need the aesthetics brought to their explicit attention as they are the furthest from seeing it for themselves. I believe that introducing students to the aesthetics of mathematics has practical benefits in the classroom. If a student sees calculus as a chest full of specialized tools, then he or she will be stuck when not knowing which tool to use. If instead the student sees calculus as a unified theory of change, then he or she will use his or her knowledge of this theory to accommodate unfamiliar problems. I make this point to my students by explaining that they should not think "what *do* I do to solve this problem?" but should rather think "what *can* I do to solve this problem?"

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