Group 3 R Project 2

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Abstract

The main goal of this project involves creating an MCMC sampler for a logistic regression model and then applying MCMC sampler to NFL data. Credible intervals for parameters will be created using MCMC sampler and through logistic regression model fitting.

Functions for prior, likelihood and posterior

```r
library (dplyr)
library (MASS)
library(TeachingDemos)
```

First, create functions for prior, likelihood and posterior for joint β_0, β_1 distribution including covariate variable.

Based on the assumption that β_0 and β_1 each is normally distributed $\sim N(0, 10^2)$, joint distribution could be found as

$$f(\beta_0, \beta_1) = \frac{1}{2\pi\sigma^2} e^{-\frac{1}{2\sigma^2}(\beta_0^2+\beta_1^2)}$$

.

Function for prior

```r
prior <- function (beta0, beta1) {
  sigma_sq = 100;
  return (  1/(2*sigma_sq*pi)*exp(-(beta0^(2) + beta1^(2))/(2*sigma_sq) ) )
}
```

Model with covariate variable

$$p(x_i) = \frac{1}{1 + e^{-\beta_0 - \beta_1 x_i}}$$

The likelihood for the logistic regression model

$$L(p) \propto \prod_{i=1}^{n} p(x_i)^{y_i}(1 - p(x_i))^{1-y_i}$$

Function for likelihood

```r
likelihood <- function (y, beta0, beta1, x) {
  return (prod((1/(1+exp (-beta0 - beta1*x)))^(y)*(1-1/(1+exp (-beta0 - beta1*x)))^(1-y)))
}
```

Posterior distribution

$$f_{\beta_0,\beta_1|Y_1,...,Y_n}(\beta_0,\beta_1|y_1,...y_n,x_1,...x_n) \propto L(p) * f(\beta_0,\beta_1)$$

Function for posterior

```r
posterior <- function (y, beta0, beta1, x) {
  return (prior(beta0, beta1)*likelihood(y, beta0, beta1, x))
}
```

Sampler function

Create function for

```r
thesampler <- function (y, x, n, niter, beta0start, beta1start, beta0proposalsd, beta1proposalsd) {

  beta0 = rep (0, niter)
  beta1 = rep (0, niter)

  beta0[1] = beta0start
  beta1[1] = beta1start

  for (i in 2:niter) {
    currentbeta0 = beta0[i-1]
    currentbeta1 = beta1[i-1]

    newbeta0 = currentbeta0 + rnorm (1, 0, beta0proposalsd)
    r = posterior(y,newbeta0, currentbeta1, x)/posterior (y, currentbeta0, currentbeta1, x)

    if (runif (1) < r){
      beta0[i] = newbeta0}
    else{
      beta0[i] = currentbeta0}

    newbeta1 = currentbeta1 + rnorm (1, 0, beta1proposalsd)
    r = posterior(y,currentbeta0, newbeta1, x)/posterior (y, currentbeta0, currentbeta1, x)

    if (runif (1) < r){
      beta1[i] = newbeta1}
    else{
      beta1[i] = currentbeta1}
  }

  betas <- data.frame(beta0, beta1)
  names (betas) <- c ("beta0", "beta1")
  return (betas)
}
```

Applying to NFL data

Read NFL data and select only distance and “good” result columns

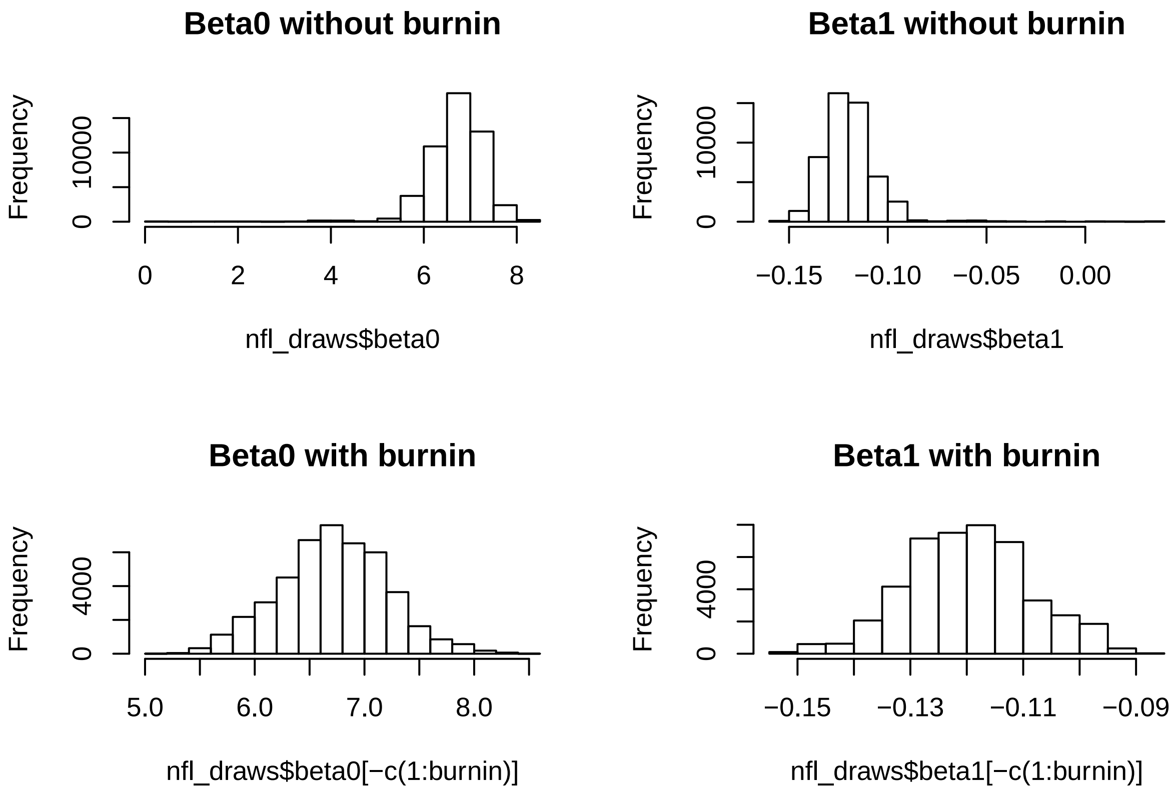
```r
nfl<-read.csv(file = "nfl2008_fga.csv")
nfl <- dplyr::select (nfl, distance, GOOD)
```

Applying

```r
n=nrow(nfl)
nfl_draws <- thesampler (nfl$GOOD, nfl$distance, n, 50000, 0, 0,  0.1, 0.1 )

burnin <- 5000

par(mfrow=c(2,2))
hist (nfl_draws$beta0, main = "Beta0 without burnin")
hist (nfl_draws$beta1, main = "Beta1 without burnin")
hist (nfl_draws$beta0 [-c(1:burnin)], main = "Beta0 with burnin")
hist (nfl_draws$beta1 [-c(1:burnin)], main = "Beta1 with burnin")
```



```r
quantile (c (0.025, 0.975), x = nfl_draws$beta0 [-c(1:burnin)])
```

```
##     2.5%    97.5%
## 5.758546 7.720698
```

```r
quantile (c (0.025, 0.975), x = nfl_draws$beta1 [-c(1:burnin)])
```

```
##        2.5%       97.5%
## -0.14254392 -0.09791457
```

```r
fit <- glm (GOOD ~ distance, family = binomial (link = 'logit'), data = nfl)
summary (fit)
```

```
##
## Call:
## glm(formula = GOOD ~ distance, family = binomial(link = "logit"),
##     data = nfl)
##
## Deviance Residuals:
##     Min       1Q   Median       3Q      Max
## -2.9526   0.2039   0.3478   0.5826   1.2309
##
## Coefficients:
##             Estimate Std. Error z value Pr(>|z|)
## (Intercept)  6.76271    0.54443  12.422   <2e-16 ***
## distance    -0.12084    0.01229  -9.836   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
##     Null deviance: 817.72  on 1038  degrees of freedom
## Residual deviance: 686.93  on 1037  degrees of freedom
## AIC: 690.93
##
## Number of Fisher Scoring iterations: 6
```

```r
confint (fit)
```

```
## Waiting for profiling to be done...
```

```
##                  2.5 %      97.5 %
## (Intercept)  5.7399740  7.87764350
## distance    -0.1457751 -0.09754001
```

```r
#plot (fit)
```

Conclusion