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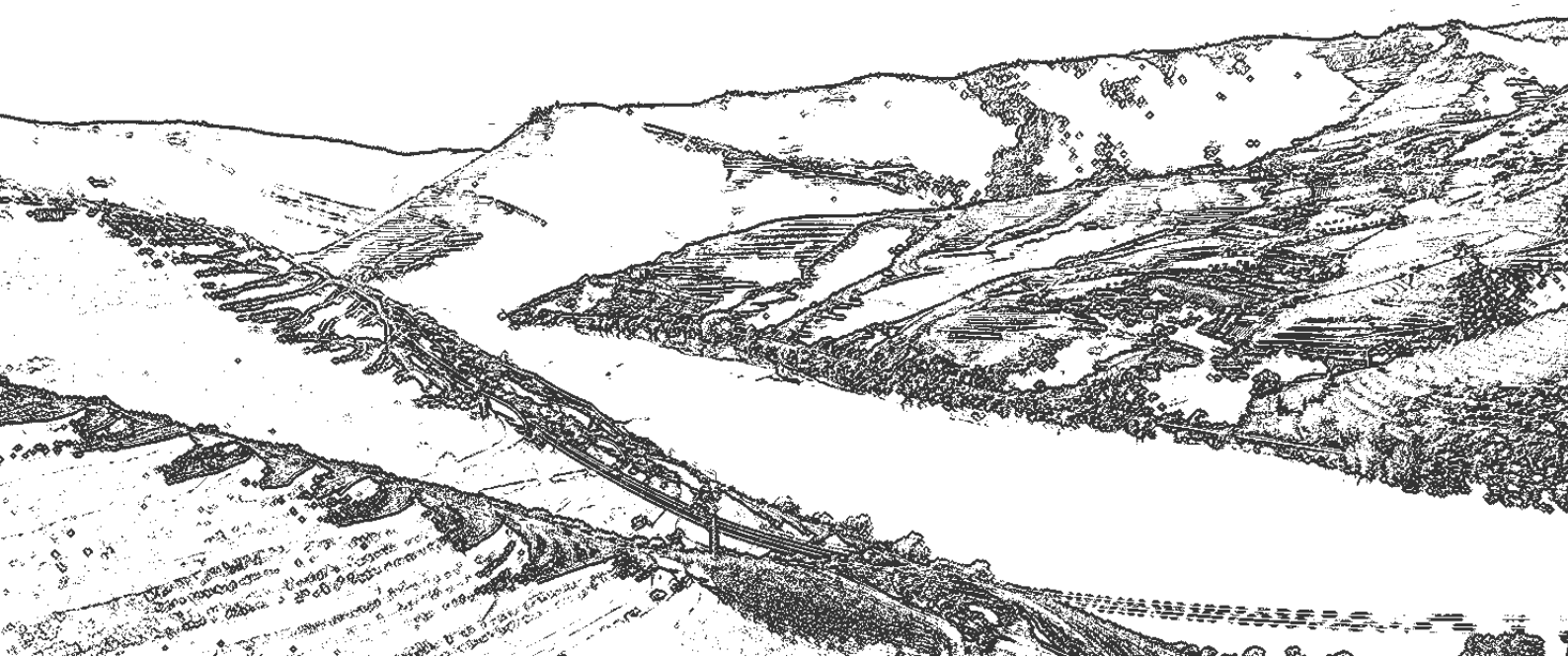
IWANASP 2023

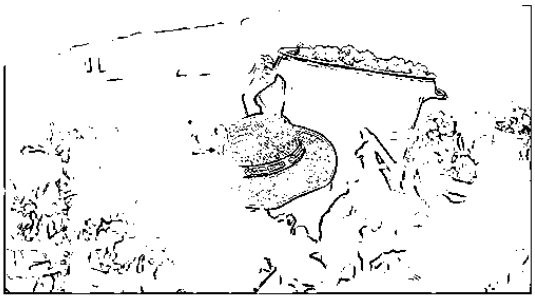
SEVENTH INTERNATIONAL WORKSHOP ON ANALYSIS AND NUMERICAL APPROXIMATION OF SINGULAR PROBLEMS

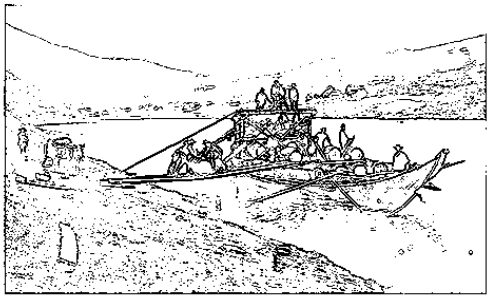
PESO DA RÉGUA, PORTUGAL

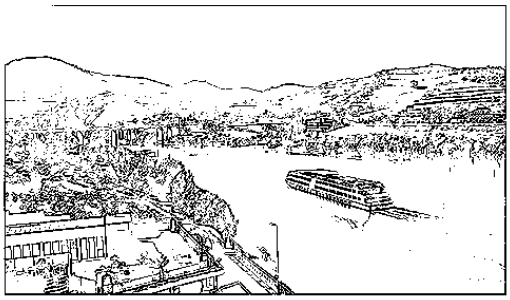
12–14 OCTOBER



BOOK of ABSTRACTS







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More information can be found at:
https://iwanaspconference.wordpress.com/

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About

IWANASP 2023

The IWANASP (International Workshop on Analysis and Numerical Approximation of Singular Problems) workshops are a forum for the presentation and discussion of new and fundamental advances in the numerical mathematics and analysis of differential or integral equations whose coefficients exhibit singularities.

The main goal of this workshop is to bring together mathematicians working on problems of this type in different fields. The workshop will focus on asymptotic properties of the solutions of equations and discretization methods. Topics covered include mathematical modelling of physical phenomena with singularities, numerical solution of singular boundary value problems for ordinary differential equations, numerical integration of functions with singularities, and computational methods for integral equations with singular kernels.

To date, there have been six editions of IWANASP. The first IWANASP was held in 2004 at the Instituto Superior Técnico (Lisbon), the second IWANASP was organized by the Aegean University, Samos (Greece), in 2006, and the third edition was held again in Portugal (Ericeira) in 2008; the fourth edition was organized by the University of Chester (Chester, UK) in 2011. In 2015, the fifth IWANASP was held in the city of Lagos, in the south of Portugal, and the sixth IWANASP was organized in Cagliari, Italy.

This year we honour Professors Teresa Diogo, Neville Ford and Filomena Dias de Almeida, on the occasion of their retirement.

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Paulo Vasconcelos	Faculty of Economics (FEP) of the University of Porto, Portugal
Ewa Weinmueller	Vienna University of Technology – TU Wien, Austria
Yuesheng Xu	Old Dominion University, USA

Timetable

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9:45-10:30	**Arvet Pedas** SMOOTHNESS PROPERTIES OF SOLUTIONS TO INTEGRAL EQUATIONS WITH SINGULARITIES
10:30-11:00	COFFEE BREAK
chair:	Donatella Occorsio
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11:40-12:00	**Ghulam Abbas Khan** THE NUMERICAL ANALYSIS OF SINGULAR FRACTIONAL DIFFERENTIAL EQUATIONS USING CORDIAL INTEGRAL THEORY
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15:30-15:50	**Ângela Ribau** PROPER ORTHOGONAL DECOMPOSITION APPLIED TO THE FLOW AROUND A CYLINDER
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16:10-16:30	**Constantino P. Caetano** PRACTICAL IMPLICATIONS OF USING FRACTIONAL DIFFERENTIAL OPERATORS TO MODEL INFECTIOUS DISEASES
16:30-17:00	COFFEE BREAK
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List of Abstracts – Talks

IS: Invited Speaker, CT: Contributed Talk, CTon: Contributed Talk Online

Invited Speakers

STABILITY RESULTS FOR MULTI-ORDER FRACTIONAL DIFFERENTIAL EQUATION SYSTEMS

Kai Diethelm IS

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Ordinary differential equation systems of fractional order have been a topic of great interest in the scientific community for the last decades. One of the important questions when dealing with such systems is the question for their (asymptotic) stability. In the case when all differential equations of the system have the same order, it is relatively straightforward to generalize the classical methods used for first order equation systems and obtain corresponding results. When the equations have different orders, thus forming a so-called multi-order system, this is not possible in general any more, and hence new approaches need to be developed. In this talk, we will discuss such approaches and present some criteria that can be used to determine whether or not a given multi-order system is asymptotically stable.

ANALYTICAL AND NUMERICAL RESULTS FOR SINGULAR INTEGRAL EQUATIONS

Teresa Diogo IS

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The mathematical modelling of problems in Science and Engineering often leads to differential or integral equations whose coefficients, kernels or solutions have singularities. My research interests have been focused on the numerical solution of integral equations, mainly of the following types: nonlinear Volterra integral (VIEs) with Hammerstein-type singularities, linear VIEs with noncompact operators (cordial VIEs), third kind integral equations. The study of of numerical methods for VIEs whose kernels contain weak singularities or/and whose underlying integral operators are not compact requires special techniques. In this talk we shall give an overview of results and present some ideas for future research.

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[2] Teresa Diogo, Arvet Pedas, Gennadi Vainikko Integral equations of the third kind in L^p spaces Journal of Integral Equations and Applications 32 417-427 2020.
[3] Huiming Song, Zhanwen Yang, Teresa Diogo Collocation methods for cordial Volterra integro-differential equations Journal of Computational and Applied Mathematics 393 113321 2020.

NEAR-INFRARED LIGHT CONTROLLED DRUG DELIVERY SYSTEMS: MATHEMATICAL MODELLING, SIMULATION AND NUMERICAL ANALYSIS

José Augusto Ferreira IS

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Light has been recently classified as magic tool to control drug delivery due to its non-invasive nature, ease of application, and exquisite temporal and spatial control. Near-infrared light responsive targeted drug delivery systems (NIRTDDS) have several advantages over drug delivery systems (DDS) where the drug release is enhanced by other stimuli, such as ultrasound, electric and magnetic fields and heat ([1]). Light controlled DDS are now undergoing extensive research namely in cancer therapy. One prominent aspect of NIRTDD is the possibility of keeping the drug concentration in its optimal therapeutic window using a suitable near-infrared light protocol ([2]). In this talk, we discuss a computational tool based on a non-linear reaction-diffusion system with Dirichlet-Neumann mixed boundary conditions that aims to contribute to the design of light optimal protocols. Numerical methods that can be seen simultaneously as finite difference methods and piecewise linear finite element methods are discussed and their stability and convergence properties are studied. The analysis requires a discrete Green identity as well as a discrete trace inequality. Second-order convergence in space in a discrete H^1-norm and optimal second-order convergence in time in a discrete L^2-norm are established ([3,4]).

Numerical results illustrating the theoretical findings and computational simulations based on a laboratory experiment concerned with light-triggered drug delivery are presented.

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MODELLING AND SIMULATION WITH SINGULAR EQUATIONS

Neville J. Ford

IS

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In this talk we discuss the link between real world systems and the development of appropriate model equations. Our focus is on equations that involve a singularity, and we begin with a discussion of how these singularities reflect the underlying properties and dynamics of the problem being modelled. We discuss approaches to model selection and parameter estimation. We give examples that show how different modelling paradigms can be necessary to develop effective models in different circumstances. We go on to consider examples of well-intentioned approaches to the construction of models which contain fundamental errors, and we see how these errors can show up in approximate solutions and simulations. We give examples that show why the conclusions drawn from the model may be quite unhelpful to the application and we provide some ideas about how to avoid these problems arising.

POINTWISE-IN-TIME A POSTERIORI ERROR CONTROL FOR TIME-FRACTIONAL PARABOLIC EQUATIONS

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I will start with a review of the recent article [1]. For time-fractional parabolic equations with a Caputo time derivative of order $\alpha \in (0,1)$, we give pointwise-in-time a posteriori error bounds in the spatial L_2 and L_∞. Hence, an adaptive time stepping algorithm is applied for the L1 method, which yields optimal convergence rates $2-\alpha$ in the presence of solution singularities. Interestingly, the proposed time stepping algorithm yields the grids similar to a-priori-constructed optimal grids in [2; 3]. In the second part of the talk, we shall discuss recent extensions of the proposed methodology to variable-coefficient multiterm time-fractional subdiffusion equations [4], and to the case of higher-order discretizations [5]. The stable implementation of the proposed algorithm will also be addressed [5].

References

[1] N. Kopteva, Pointwise-in-time a posteriori error control for time-fractional parabolic equations, Appl. Math. Lett., 123 (2022), 107515.
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FRACTIONAL DIFFERENTIAL EQUATIONS AND SINGULARITIES

Juan J. Nieto IS

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We present some basic aspects of fractional calculus and fractional differential equations. For simple linear and nonlinear fractional equations different types of singularities appear. Some real world applications of fractional equations will be discussed.

References

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[3] I. Area, J.J. Nieto, On a Quadratic Nonlinear Fractional Equation. Fractal & Fractional 7 (2023), 469, 8 pages.
[4] J.L. Wei, G.C. Wu, B.Q. Liu, J.J. Nieto, An optimal neural network design for fractional deep learning of logistic growth. Neural Computing and Applications 35 (2023), 10837-10846.

PRODUCT INTEGRATION RULES FOR THE HILBERT TRANSFORM ON $(0, +\infty)$

Donatella Occorsio IS

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The talk deals with the approximation of the Hilbert transform of f of the kind

$$\mathcal{H}(f,t) = \int_0^\infty \frac{f(x)}{x-t}\, e^{-x} x^\alpha dx, \quad t > 0, \quad \alpha > -1,$$

by means of product integration rules obtained by approximating f by some discrete de la Vallée Poussin means based on Laguerre zeros, and recently introduced in [1]. Under suitable assumptions, the new polynomial sequences have uniformly bounded Lebesgue constants in spaces of locally continuous functions presenting possible algebraic singularities in 0 and exponentially growing at infinity, equipped with weighted uniform norms.

The properties of the new sequence of quadrature rules will be investigated both from the theoretical and experimental points of views.

The results presented in this talk are jointly obtained with Woula Themistoclakis.

References

[1] Donatella Occorsio, Woula Themistoclakis, Weighted polynomial approximation on $(0, +\infty)$ by de La Vallée Poussin means 2023 (submitted)

SMOOTHNESS PROPERTIES OF SOLUTIONS TO INTEGRAL EQUATIONS WITH SINGULARITIES

Arvet Pedas

IS

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Estimates for the derivatives of the solutions to weakly singular integral equations of the second kind on a bounded interval is discussed. The kernel of the integral operator may have diagonal and boundary singularities, information about them is given through certain estimates. Showing that the solution belongs to a special weighted space of smooth functions (see [1; 2]), the growth of the derivatives of the solution near the boundary of the domain of integration is described.

This contribution is a joint work with Gennadi Vainikko and Mikk Vikerpuur.

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[1] A. Pedas, G. Vainikko, Integral Equations with Diagonal and Boundary Singularities of the Kernel, J. Anal. Appl., 25 (2006), 487–516.
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LOW RANK APPROXIMATION IN THE COMPUTATION OF FIRST KIND INTEGRAL EQUATIONS WITH TAUTOOLBOX

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`Tau Toolbox` is a mathematical library for solving integro-differential problems. Over the past few years, a class within `Tau Toolbox`, called `polynomial`, has been developed for approximating functions by classical orthogonal polynomials and it is intended to be an easy-to-use yet efficient object-oriented framework. In this work we explain how this class can be useful in the solution of linear ill-posed problems, avoiding the explicit discretization for the use of finite numerical linear algebra techniques. The Tikhonov regularization method and the truncated singular value expansion are implemented. We discuss how the `polynomial` class has been designed to fit the needs of applications and provide a description of the available methods, including low rank approximations for bivariate functions. Numerical experiments illustrate that this approach is capable of efficiently compute good approximations of linear discrete ill-posed problems, even facing perturbed available data function, with no programming effort. Several test problems coming from real applications are used to evaluate the performance and reliability of the solvers.

Joint work with Laurence Grammont from Université de Lyon, Institut Camille Jordan, UMR 5208, France, and Nilson J. Lima from Centro de Matemática da Universidade do Porto and Faculdade de Ciências da Universidade do Porto, Portugal

References

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HOW TO APPROXIMATE SINGULAR ODES AND DAES EFFICIENTLY?

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We deal with boundary value problems for systems of ordinary differential equations with singularities. Typically, such problems have the form

$$z'(t) = F(t, z(t)), \quad t \in (0, 1], \quad B_0 z(0) + B_1 z(1) = \beta,$$

where $\lim_{t\to 0} F(t, z(t)) = \infty$ and $\lim_{t\to 0} \partial F(t, z)/\partial z = \infty$. The analysis is usually done for the model equation

$$z'(t) = \frac{1}{t^\alpha} M z(t) + f(t, z(t)), \quad t \in (0, 1], \quad B_0 z(0) + B_1 z(1) = \beta,$$

where $f(t, z)$ may also be in the form of $g(t, z)/t$ with a smooth function $g(t, z)$. For $\alpha = 1$ the problem has a *singularity of the first kind*, while for $\alpha > 1$ the singularity is commonly referred to as *essential singularity*. We briefly recapitulate the analytical properties of the above problems with a special focus on the most general boundary conditions which guarantee their well-posedness.

To compute the numerical approximation for z we use polynomial collocation, because the method retains its high convergence order even in case of singularities. The usual high-order superconvergence at the mesh points does not hold in general. However, the uniform superconvergence is preserved (up to logarithmic factors). We will discuss how the collocation performs for problems with the inhomogeneity of the form $g(t, z)/t$.

The updated version of the Matlab code `bvpsuite1.1` with the special focus on the above problem class has been implemented. For higher efficiency, estimate of the global error and adaptive mesh selection are provided. The code can be applied to arbitrary order problems in implicit form. Also systems of index 1 differential-algebraic equations (DAEs) are in the scope of the code. We illustrate the performance of the software with a special focus on parameter-dependent problems by means of numerical simulation of models in applications.

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Contributed Talks

Please note that the underlined name indicates the person conducting the presentation, and the abstracts are arranged in alphabetical order based on the presenter's last name.

NUMERICAL SOLUTION OF INTEGRAL ALGEBRAIC EQUATIONS OF EXTRAPOLATION MULTISTEP TWO-STAGE METHODS

Olga S. Budnikova CT

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The report addresses linear systems of Volterra integral equations with an identically degenerate matrix in front of the main part are called integral algebraic equations. Sufficient conditions for the existence of a unique solution are given in terms of matrix pencils in article [1]. The study is devoted of the construction two-stage multistep methods for numerical solution of integral algebraic equations. We present several classes of block-by-block methods. Stable extrapolation two-stage methods identified and the weight coefficients of these algorithms are written.

The study was supported by the Russian Science Foundation, grant No. 22-11-00173.

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ON INTEGRAL-ALGEBRAIC EQUATIONS WITH VARIABLE LIMITS OF INTEGRATION

Mikhail V. Bulatov, Mariia N. Botoroeva

CT

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The report addresses linear systems of integral equations with an identically degenerate matrix in front of the main part. An upper limit of integration equals t, according to the classical Volterra integral equations. A lower limit of integration is at, where $a \in [0; 1)$. Such systems are called integral-algebraic equations with variable limits of integration. Properties of such systems and fundamental differences from standard ones are given. By standard systems we mean cases when the matrix in front of the main part is either identity, or we have one equation of the first kind with a nonzero kernel on the diagonal. Sufficient conditions for the existence of a unique solution are given in terms of matrix pencils. The problems of constructing numerical methods for solving the systems under consideration are discussed. Examples are given.

The study was supported by the Russian Science Foundation, grant No. 22-11-00173.

PRACTICAL IMPLICATIONS OF USING FRACTIONAL DIFFERENTIAL OPERATORS TO MODEL INFECTIOUS DISEASES

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Fractional differential and integral operators arise as a generalisation of classical calculus operators. They have been used to model several phenomena, such as viscoelasticity of materials, pharmacokinetics and epidemiology of infectious diseases. With the emergence of the SARS-CoV-2 virus in Wuhan, China in 2020 and subsequent worldwide pandemic, several groups have developed fractional-order generalisations of the classical first order epidemiological compartmental models to study the dynamics of COVID-19. These showed to provide a better fit to epidemiological data than their classical counterparts [1]. This generalisation consists of a system of fractional differential equations of the Caputo type [2], where the differentiation order is a free parameter, usually taking values between 0 and 1. Nevertheless, some authors claimed that this formulation might incurr in several mathematical problems [3]. In this study we explore the implications of the *ad hoc* substitution of the interger order by a fractional one in the setting of compartmental models for infectious diseases. This includes not only addressing some mathematical issues, but also attempt to provide an accurate epidemiological interpretation of the results obtained from such generalisation.

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ON SOME PROPERTIES OF LINEAR DIFFERENTIAL ALGEBRAIC EQUATIONS WITH RECTANGULAR COEFFICIENT MATRICES AND SINGULAR POINTS IN THE DOMAIN

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In this talk, we consider systems of ordinary differential equations

$$\Lambda_k x := \sum_{i=0}^{k} A_i(t) x^{(i)}(t) = f(t),\ t \in T, \tag{1}$$

where $A_i(t)$ are $\nu \times n$-matrices, $x(t)$ is a desired vector-function and $f(t)$ is a known vector function, $x^{(i)}(t) = (d/dt)^i x(t),\ x^{(0)}(t) = x(t)$, and it is assumed that

$$\operatorname{rank} A_k(t) < \min\{\nu, n\}\ \forall t \in T, \tag{2}$$

which, for $\nu = n$, is equivalent to the equality $\det A_k(t) = 0\ \forall t \in T$. Systems (1) satisfying condition (2) are usually referred to as differential-algebraic equations (DAEs).

We pay a particular attention such systems with singular points on the integration interval. By singular points we understand any point on the segment T, at which the DAE (1) either has no solutions on T, or the dimension of the solution manifold changes, etc. We introduce a formalized notion of a singular point for systems with rectangular matrices of coefficients and give their classification. Numerical issued caused by singular points are discussed followed by a number of examples illustrating the theoretical results.

ENHANCED NUMERICAL SOLUTION OF APPLICATION-ORIENTED REACTION-DIFFUSION EQUATIONS

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In this talk, we focus on the accurate and efficient numerical solution of mathematical models deriving from applications consisting of systems of reaction-diffusion Partial Differential Equations (PDEs). In particular, we will consider models of vegetation phenomena (Eigentler et al. 2019), problems of deterioration and corrosion of metallic materials (Mai et al. 2016, Waschinsky et al. 2021) and of architectural works [1], issues related to production of solar cells (Gagliardi et al. 2017, Maldon et al. 2020). The mentioned problems are characterized by high stiffness and particular properties to be preserved in the discrete setting, forcing the use of specific non-trivial numerical techniques in order to compute the solution accurately and efficiently.

Here, we show techniques for the construction of efficient and strongly problem-oriented numerical methods, which are stable, i.e. able to handle stiffness preserving the main properties of the solution (e.g. long term behavior, any positivity or oscillation frequency) even for large discretization steps [2,3,4]. In particular, we show how the use of Time-Accurate and highly-Stable Explicit (TASE) operators (Calvo et al. 2021) can lead to a class of efficient and parallelizable numerical schemes [5]. Such methods, called TASE-peer, have nice stability properties and involve the solution of a small and fixed number of linear systems per step depending on the Jacobian of the problem (or an approximation thereof) [5,6]. Numerical results testify that TASE methods are competitive in solving the reaction-diffusion models mentioned above [3-6].

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NUMERICAL AND ASYMPTOTIC STUDY OF STRESSES OF PTT, GIESEKUS AND OLDROYD-B NEAR THE STICK-SLIP SINGULARITY

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Flow with singularities poses a challenge in computational fluid dynamics, as the presence of singularities can lead to numerical instabilities, spurious solutions, or even simulation breakdowns. These challenges are even greater for viscoelastic fluid flows, where besides singularity in stress, the emergence of boundary layers is also observed. Singularities in flows can arise due to nonsmooth geometries or abrupt changes in boundary conditions. An example of the former case is flows in reentrant corners, while a typical situation of the latter case is extrusion flows, where the no-slip condition encounters the condition of zero shear stress. Therefore, due to the difficulties associated with high-stress flows, characterizing these singularities is essential, as they can contribute to the development of more stable and accurate numerical schemes.

Thus, this work presents the asymptotic construction of the singular behavior of the polymeric tensor for stick-slip flow [1]. The fluid used in this study is the Oldroyd-B fluid in a Newtonian velocity field [2]. Although this is not the true velocity field for the flow under consideration, this simplification helps us obtain important information about the Oldroyd-B fluid. Furthermore, it is likely that the conclusions drawn from this study can be extended to when we consider the true velocity field. Additionally, the asymptotic results of the PTT and Giesekus models will also be presented. Finally, the asymptotic results will be numerically verified by simulating the constitutive equations along streamlines, considering a Newtonian velocity field, as well as by simulating the set of equations governing a viscoelastic flow [3]. It is worth noting that these numerical verifications will also be performed using the natural stress formulation [4].

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AN EFFICIENT NUMERICAL METHOD FOR SOLVING METASTATIC TUMOR GROWTH MODELS WITH TREATMENT

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The need for better knowledge of the complex biological system which is at the base of the tumor metastatic process is one of the biggest challenges in cancer research. Modeling metastatic tumor growth could help for a better understanding of such process. Particularly this provides fundamental tools in estimating the metastatic state in those cases where the small metastases are invisible to the medical devices. This talk deals with a two-dimensional PDE model introduced in [1] for describing the dynamics of the colony size distribution of metastatic tumors depending on time, which also takes into account a combined cytotoxic/antiangiogenic treatment. The biological observables of interest, such as the total metastatic burden or the cumulative number of metastases, can be represented as a weighted integral of the metastatic density that is the solution of the model. Following an idea in [4] we have first reformulated the PDE model in terms of Volterra integral equations (VIEs) whose unknowns are directly the biological observables of interest and, then, we have proposed a numerical method of Nyström type for approximating their values. To the best of our knowledge the reformulation of the 2D PDE model in terms of VIE has not been considered yet and an efficient numerical method for its resolution is missing. As we will show in the numerical tests, the proposed numerical method for solving VIE on infinite intervals is faster than the one recently proposed in [3] and implemented in the MatLab toolbox [2].

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AN AVERAGED NYSTRÖM-TYPE METHOD FOR 2D FREDHOLM INTEGRAL EQUATIONS ON AN UNBOUNDED DOMAIN

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In this talk, we explore the application of some recent cubature formulae [1] to the numerical solution of second-kind Fredholm integral equations of the type

$$(I - K)f = g, \tag{1}$$

whose solution f defined on the domain $\mathcal{D} = (0, \infty) \times (0, \infty)$ may have algebraic singularities on one or both the positive semi-axes and at most exponential growth at infinity. In (1), I is the identity operator, g is a given right-hand side defined on $\mathcal{D}$, and K is the integral operator defined by

$$(Kf)(\mathbf{y}) = \int_{\mathcal{D}} k(\mathbf{x}, \mathbf{y}) f(\mathbf{x}) w(\mathbf{x}) d\mathbf{x}, \qquad \mathbf{y} \in \mathcal{D},$$

where the kernel function k is known and w is the product of two univariate Laguerre weight functions

$$w(\mathbf{x}) = w_1(x_1) w_2(x_2), \qquad w_i(x_i) = x_i^{\alpha_i} e^{-x_i}, \quad \alpha_i > -1, \quad i = 1, 2.$$

The stability and the convergence of the proposed method will be discussed in suitable weighted spaces, and several numerical tests will be presented.

This is a joint work with D. Lj. Djukić and R. M. Mutavdžić Djukić of the University of Belgrade, Serbia.

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EXPLORING MULTI-SCALE APPROACHES IN MODELLING VISCOELASTIC FLOWS WITH PARTICLES

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DNS offers a potential solution to computational challenges in simulating particle-laden viscoelastic fluids. However, the extensive computational effort required makes DNS impractical for many applications. As an alternative, an unresolved CFD-DEM coupling strategy is commonly used to numerically simulate particle-laden viscoelastic fluids. This study presents the implementation of an unresolved CFD-DEM coupling strategy using a dual-grid multiscale approach. In this approach, the continuum phases (fluids) are solved on a fine grid, while the dispersed phase (particles) is solved on a coarse grid. The effectiveness of the dual-grid approach is evaluated through a benchmark case study known as "Dam Break" which investigates kinetic energy accumulation. A comparison between the single-grid and dual-grid methods is performed. Furthermore, the capability of the dual-grid approach to study non-Newtonian fluids is demonstrated and validated using the "Impacting Drop" benchmark case, where the width of a viscoelastic fluid drop upon impact with a rigid plate is monitored for both the single-grid and dual-grid approaches.

A PRODUCT INTEGRATION METHOD TO SOLVE TWO-DIMENSIONAL LINEAR FREDHOLM INTEGRAL EQUATION IN L1

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We consider the Fredholm integral problem of the second kind

$$(I - K)u \quad = \quad f$$

where I denote the identity operator on X.

In order to obtain an approximation of the solution for this equation, we introduce a finite rank approximation of K, denoted as K_n. This allows us to uniquely solve the approximate equation $(I - K_n)u_n = f$, and we can ensure that the sequence of approximate solutions u_n, converges to the exact solution u, as n approaches infinity.

The paper introduces an extension of the product integration method to the space $X := L^1([a, b] \times [c, d], \mathbb{R})$ [1,2].

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THE NUMERICAL ANALYSIS OF SINGULAR FRACTIONAL DIFFERENTIAL EQUATIONS USING CORDIAL INTEGRAL THEORY

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In our present work, we study the following singular fractional differential equations (SFDEs) with variable coefficients for $f \in C^m[0,T], \alpha, \alpha_k \in \mathbf{R}$,

$$(D_0^\alpha M^\alpha u)(t) = \sum_{k=1}^{l} b_k(t)(D_0^{\alpha_k} M^{\alpha_k} u)(t) + f(t), \quad 0 < t \leq T,$$

where $m < \alpha < m+1, \alpha > \alpha_k \geq 0, b_k \in C^m[0,T], k = 1,2,...,l, m \in \mathbf{N_0}$.
D_0^α is the fractional differential operator of order $\alpha \in [0,\infty)$ which is defined as the inverse operator of Riemann Liouville integral operator J^α and M^α is the multiplication operator which are defined as:

$$(D_0^\alpha v)(t) = \left((J^\alpha)^{-1} v\right)(t), \text{ and } (J^\alpha u)(t) = \frac{1}{\Gamma(\alpha)} \int_0^t (t-s)^{\alpha-1} u(s) ds, \alpha > 0, v \in J^\alpha C([0,T]),$$
$$(M^\alpha u)(t) = t^\alpha u(t), \quad t \in (0,T], \alpha \in \mathbb{R}, \quad u \in C([0,T]),$$

where Γ is the Euler gamma function.
Thus, by employing cordial theory, recently introduced by Prof. Gennadi Vainikko [1,2], we can convert the SFDE into a cordial integral equation in the following format

$$v(t) = \sum_{k=1}^{l} b_k(t)(V_{\phi_{\alpha,\alpha_k}} v)(t) + f(t), \quad 0 < t \leq T,$$

where $V_{\phi_{\alpha,\alpha_k}}$ is the cordial integral operator with core function $\phi_{\alpha,\alpha_k} \in L^1(0,1)$ defined as

$$(V_{\phi_{\alpha,\alpha_k}} u)(t) = \int_0^1 \phi_{\alpha,\alpha_k}(x) u(tx) dx, \qquad \phi_{\alpha,\alpha_k}(x) = \frac{1}{\Gamma(\alpha-\alpha_k)}(1-x)^{\alpha-\alpha_k-1} x^{\alpha_k},$$

We explored the results for the unique solvability condition for cordial integral equation [3] and developed a polynomial collocation method ([2]) for the numerical investigation of proposed class of SFDE . We investigate the convergence of the proposed method and we present several examples to illustrate the performance of the numerical method.

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ANALYSIS OF LOCAL BIFURCATIONS IN FRACTIONAL ORDER DYNAMICAL SYSTEMS USING THE CENTER MANIFOLD METHOD

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Fractional calculus is an important field of applied mathematics. The main advantage of studying fractional-order differential equation models is that they are naturally related to systems with memory, which is particularly prevalent in most biological systems [1-3].

In this work, we propose a predator-prey model of fractional order with Allee effect associated with prey species production and Evelev functional response. We present results on the positivity and boundedness of the solutions of the fractional-order dynamical system. We also investigate the local stability of the equilibrium points. Additionally, we discuss local bifurcations (Hopf, saddle-node, transcritical) using bifurcation theory and derive conditions for their existence using the center manifold theorem [1,3].

To validate the theoretical results and illustrate the complex dynamical behaviors of the system, we perform numerical simulations. These simulations demonstrate that the present model exhibits chaotic and complex behavior. Finally, we compare the system to discrete systems with Holling-type functional response in terms of their dynamical behaviors.

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A COLLOCATION BASED APPROACH FOR THE NUMERICAL SOLUTION OF A CLASS OF SINGULAR FRACTIONAL INTEGRO-DIFFERENTIAL EQUATIONS

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Previously, the unique solvability of singular fractional differential equations was studied [1]. We now consider singular fractional integro-differential equations of the form

$$(D_0^{\alpha} M^{\alpha} u)(t) = b_1(t)(D_0^{\alpha_1} M^{\alpha_1} u)(t) + b(t)(Vu)(t) + f(t), \quad 0 < t \le T, \tag{1}$$

where the multiplication operator M^{ν} is defined by

$$(M^{\nu} u)(t) = t^{\nu} u(t), \quad 0 < t \le T, \quad \nu \in \mathbb{R}, \quad u \in C[0,T],$$

V is a certain type of Volterra integral operator, $\alpha, \alpha_1, \in \mathbb{R}$, and

$$m < \alpha \le m+1, \quad \alpha > \alpha_1 \ge 0, \quad b, b_1, f \in C^m[0,T], \quad m \in \mathbb{N}_0 = \{0,1,2,\dots\}.$$

By $C^m[0,T]$ ($m \in \mathbb{N}_0$) we denote the space of m times continuously differentiable functions u on $[0,T]$; $C^0[0,T] = C[0,T]$. In equation (1) the fractional differential operator D_0^{μ}, of order $\mu \in [0,\infty)$, is defined as the inverse of the Riemann-Liouville integral operator

$$(J^{\mu} u)(t) = \frac{1}{\Gamma(\mu)} \int_0^t (t-s)^{\mu-1} u(s) ds, \quad u \in C[0,T], \quad t > 0, \ \mu > 0; \quad J^0 = I,$$

where I is the identity mapping and Γ the Euler gamma function.

In the talk we present some results about the unique solvability of equations of the form (1) and introduce a scheme based on piecewise polynomial collocation for finding the numerical solution of such equations. We also give results of numerical experiments.

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NEURAL FIELDS WITH DIFFUSION: QUALITATIVE ANALYSIS AND NUMERICAL APPROXIMATION

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We are concerned with the analytical and numerical solution of the following integro-differential equation

$$\partial_t v(x,\xi,t) = \nu \partial^2_{\xi\xi} v(x,\xi,t) - \gamma v(x,\xi,t) + \int_\Omega W(x,x',\xi,\xi') S(v(x',\xi',t)) dx' d\xi' + G(x,\xi,t), \tag{1}$$

for $(x,\xi,t) \in \Omega_T = \Omega \times [0,T]$, where $\Omega = \mathbb{T}^n \times U,\ U = (0,L)$ and $\mathbb{T}^n$ is the $n-$dimensional torus, being equivalent to the quotient space $(\mathbb{R}/2\pi\mathbb{Z})^n$. We search for a solution v of (1), being a periodic function on x, defined on whole $\mathbb{R}^n$,

$$v(x+2\pi k,\xi,t) = v(x,\xi,t) \qquad \text{for } x \in \mathbb{R}^n \quad \text{and} \quad (\xi,t) \in U_T = U \times [0,T], \tag{2}$$

and identifying v with its restriction to the fundamental domain $[0,2\pi)^n$. We also assume that no current leaks away at the boundaries $\xi = 0$ and $\xi = L$, which corresponds to the Neumann boundary conditions

$$\partial_\xi v(x,0,t) = \partial_\xi v(x,L,t) = 0, \qquad \forall t \in [0,T]. \tag{3}$$

Moreover, we have the initial condition

$$v(x,\xi,0) = v_0(x,\xi), \qquad (x,\xi) \in \Omega. \tag{4}$$

Here γ, ν are known physical positive constants and W, S, G are given functions.

In the case $\nu = 0$ equation (1) is known as the neural field equation and is used since the years 70 of the last century to model the activity of interacting neurons in certain regions of the brain. From the physical point of view, the introduction of the diffusion term is justified as taking into account the dendritic processing, as discussed in [1]. In the cited paper, the authors have presented an efficient numerical algorithm for the approximation of the solution of (1)-(4). Here we develop an analytical study of the problem (1)-(4), focusing on how the solution of this problem depends on the parameter ν. Then we carry out some numerical experiments that illustrate the theoretical findings.

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PROPER ORTHOGONAL DECOMPOSITION APPLIED TO THE FLOW AROUND A CYLINDER

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Numerical simulations of fluid flows generate a significant amount of data, but some relevant flow structures can go undetected if a comprehensive analysis is not carried out. In transient situations it is very challenging to attempt to identify these flow structures which evolve over time while being advected along the domain. By adequately decomposing the entire dataset into smaller sets, important structures present in the main flow are identified. The Proper Orthogonal Decomposition (POD) method is one such technique that is able to reduce the amount of significant data, to obtain a better and global understanding of the embedded flow structures by analysing specific modes, and subsequently it is able to reconstruct the flow main characteristics.

In this work, simulations of a planar flow past a cylinder are carried out, for a generalized Phan-Thien–Tanner fluid at a very low Reynolds number ($Re = 0.01$) and at two different Weissenberg numbers ($Wi = 1.2,\ 1.25$). At these two different Weissenberg numbers, a flow phenomenon similar to a Von KÃ¡rman vortex street is observed downstream of the cylinder and through the POD method a decomposition into a generator base is performed, followed by a reconstruction of the flow for the two fluids and a frequency analysis of specific modes. Then, changing the frequencies of the modes time coefficients of the POD applied to the flow at $Wi = 1.2$, a new reconstruction is made for $Wi = 1.25$, and its results compare well with the results of the simulation for $Wi = 1.25$.

GLOBAL APPROXIMATION METHODS FOR NONLINEAR INTEGRAL EQUATIONS OF HAMMERSTEIN TYPE

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We consider an Hammerstein integral equation of the form

$$f(x) + \int_{-1}^{1} k(x,y)h(y,f(y))v^{\alpha,\beta}(y)\,dy = g(x), \qquad |x| \leq 1$$

where k and h are known functions defined on $[-1,1]^2$ and $[-1,1] \times \mathbb{R}$ respectively, $v^{\alpha,\beta}(y) = (1-y)^{\alpha}(1+y)^{\beta}$, $\alpha, \beta > -1$, is a Jacobi weight, g, defined in $[-1,1]$, is also known, while f is the unknown function.

The Hammerstein integral equations appear in nonlinear physical phenomena such as electromagnetic fluid dynamics, reformulation of boundary value problems with a nonlinear boundary condition.

The most popular numerical methods for solving these equations are collocation, Galerkin and Nyström methods, based on piecewise polynomial approximation (see for instance [1] and the references therein).

Here we propose methods based on the polynomial approximation and in particular a Nyström method based on the Gaussian rule. We prove convergence and stability of the proposed methods in uniform norm and show that the approximating functions behave like the best polynomial approximation of the unknown f.

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ON DIFFERENTIAL EQUATIONS WITH EXPONENTIAL NONLINEARITIES

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We consider the second order ordinary differential equations arising in the heat-conductivity theory. These equations often contain nonlinearities of exponential type [1], like this one,

$$x''(t) + ax'(t) + Fe^x = 0,$$

or this one

$$x''(t) + Fe^{-\frac{\theta}{x}} = 0.$$

These equations are considered along with the boundary conditions of the type

$$x(-1) = 0 = x(1).$$

The main problems are the existence of positive solutions, the number of solutions and the dependence of the number of solutions on parameters.

Our goal is to present the results on the number of positive solutions, their dependence on parameters and peculiar properties of the phase plane.

We will present examples to illustrate our analytical and numerical approach.

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ON A QUALITATIVE STUDY OF INTEGRAL-ALGEBRAIC EQUATIONS

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The report addresses a qualitative study of systems of the form

$$A(t)x(t) + \int_0^t K\left(t, s, x(s)\right) ds = f(t) \tag{1}$$

and

$$B(t)y(t) + \int_0^t (t-s)^{-\alpha} L(t,s)y(s)ds = g(t),\ 0 < \alpha < 1, \tag{2}$$

where $0 \leq s \leq t \leq 1$, $A(t)$, $B(t)$, $L(t,s)$ are given $(n \times n)$-matrices, $K\left(t, s, x(s)\right)$, $f(t)$, $g(t)$ are given n-dimensional vector-functions, $x(t)$ and $y(t)$ are unknown n-dimensional vector-functions.

Consider the case when

$$\det A(t) \equiv 0 \ \text{ and } \ \det B(t) \equiv 0. \tag{3}$$

Such systems are usually called integral-algebraic equations. The fundamental differences of systems (1), (2) with condition (3) from the classical systems of equations of the second kind are emphasized, namely, the solutions of these systems may not exist or may be nonunique.

Conditions for the existence and uniqueness of a solution to systems (1), (2) with condition (3) are given in terms of matrix pencils.

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HIGH ORDER NUMERICAL METHOD FOR A SUBDIFFUSION PROBLEM

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We consider an equation that captures a subdiffusive behaviour which appears in many physical contexts [1]. This equation involves a time Riemann-Liouville fractional derivative of order $1-\alpha$, for $\alpha \in (0,1)$, on the right hand side of the differential equation and the diffusive coefficient can depend on space and time. An high order numerical method for the subdiffusion problem is derived based on the fractional splines, [2,3], of degree $\beta \in (1,2]$. The main purpose of this work is to apply the fractional splines to approximate the fractional integral, incorporated in the definition of the Riemann-Liouville fractional derivative, and hence explain how to solve the subdiffusion problem using this approach. It is discussed how the rate of convergence of the numerical method depends on the solution, the degree of the spline and the order of the fractional derivative.

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EXACT SOLUTION FOR A NUMERICAL APPROXIMATION OF THE SIR MODEL

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We derive a nonstandard finite difference scheme for the Susceptible–Infected–Removed continuous model proposed by Bailey [1]. We prove that our discretized system is dynamically consistent with its continuous counterpart and we derive its exact solution. We end with the analysis of the long-term behavior of susceptible, infected and removed individuals, illustrating our results with numerical simulations.

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A COLLOCATION METHOD BASED ON CENTRAL PART INTERPOLATION FOR FRACTIONAL INTEGRO-DIFFERENTIAL EQUATIONS

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The central part interpolation approach was introduced in [1] for solving Fredholm integral equations of the second kind and it has shown accuracy and numerical stability advantages compared to standard piecewise polynomial collocation methods, including collocation at Chebyshev knots [2]. This approach has been modified in [3] to solve fractional differential equations. In the present talk we continue this analysis for solving a class of fractional integro-differential equations involving Caputo fractional differential operators. To this end, we first reformulate the problem as a weakly singular Volterra integral equation of the second kind. Then, we use a smoothing change of variables to improve the boundary behaviour of the exact solution of the underlying problem. After that, we construct a collocation method based on central part interpolation by continuous piecewise polynomials on the uniform grid. Finally, we derive the optimal convergence estimates of the presented method and test the theoretical results with some numerical experiments.

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List of Posters

THE ROLE OF THE MITTAG-LEFFLER FUNCTION IN VISCOELASTIC DIFFERENTIAL EQUATIONS

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Fractional viscoelastic models are a class of mathematical models used to describe the complex mechanical behaviour of materials and systems that exhibit viscoelasticity. Unlike traditional viscoelastic models, which assume integer-order derivatives to represent the stress-strain relationship, fractional viscoelastic models employ fractional calculus operators to account for the non-local and memory-dependent nature of the material's response. These models have gained significant interest in recent years due to their ability to capture more accurate behaviours in various physical systems. By incorporating fractional derivatives, these models can better characterise the long-term memory effects and hereditary properties of viscoelastic materials, making them particularly useful in areas like biomechanics, material engineering, and geophysics, where complex behaviours and time-dependent responses are prevalent. Through their flexibility and enhanced descriptive power, fractional viscoelastic models have become valuable tools for understanding and predicting the mechanical properties of a wide range of materials and systems.

The main limitation of these models lies in the challenge of achieving frame-invariance and easily interpretable physical constants. Recently, researchers have made decisive steps in developing *intermediate* models that attempt to blend the favourable attributes of both classical and fractional viscoelastic models [1-5].

This study aims to present a comprehensive overview of various viscoelastic models developed to date while introducing innovative approaches to improve the modelling of complex viscoelastic fluids.

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A GENERALISATION OF THE INTEGRAL MAXWELL MODEL: FRAME INVARIANCE AND ANALYTICAL SOLUTIONS

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In this work we have developed a Maxwell-type linear viscoelastic model using the Mittag-Leffler function in the relaxation modulus and the new constitutive equation is given by

$$\boldsymbol{\sigma}(t) = \int_{-\infty}^{t} G(t-t')\frac{d\boldsymbol{\gamma}(t')}{dt'}dt' \tag{1}$$

where $G(t-t') = S_0\Gamma(\beta)E_{\alpha,\beta}\left(-\frac{t-t'}{\lambda}\right)$ is the relaxation modulus, γ is the deformation tensor and $\boldsymbol{\sigma}$ is the stress tensor.

To ensure the frame invariance of the model, we employed an invariant strain measure, namely the Finger tensor $\boldsymbol{B}(t,t')$, instead of the infinitesimal strain $\boldsymbol{\gamma}(t,t')$, resulting in the Lodge-ML model [1].

However, despite the advancements made in the Lodge-ML model, certain limitations were encountered. To overcome these limitations, we further developed a nonlinear integral viscoelastic model known as the generalised Kaye-Bernstein-Kearsley-Zapas model (gK-BKZ). In this model, the stress tensor is defined by

$$\boldsymbol{\sigma}(\mathbf{x},t) = \int_{-\infty}^{t} m\left(t-t'\right)h\left(I_1,I_2\right)\mathbf{B}_{t'}(\mathbf{x},t)dt' \tag{2}$$

where m is the memory function given by $m(t-t') = -\frac{\partial G(t-t')}{\partial t'}$, $h\left(I_1,I_2\right)$ is a decay function and $\mathbf{B}_{t'}(\mathbf{x},t)$ is the Finger tensor. In this decay function, I_1 and I_2 are respectively the first and the second invariants of $\mathbf{B}_{t'}(\mathbf{x},t)$ [2]. The gK-BKZ model retains the desired properties of frame invariance and provides a steady shear viscosity along with a non-zero first normal stress coefficient dependent on the strain rate $\dot{\gamma}$.

This is a joint work with R. T. Leiva, L. L. Ferás, A. Castelo, M. L. Morgado, J. Bertoco and A. M. Afonso

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