（裝　　訂　　線）

國立成功大學 111 學年度第一 學期考試試卷 National Cheng Kung University Examination Sheet for Academic Year: Semester:				112年 1月 3日 Year Month Day
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院系 College	工 學院 College 航太 系 Department 年 Year 班 Class	評閱成績 Score	100	

1. ① 系統參數誤差，狀態量測誤差，會使回授線性化失效，無法順利線性化

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② 會失去非線性系統的特性，如收斂速度快等，線性化後不一定符合需求

③ 回授線性化設計之控制律可能會超過實際上可實現之範圍

2. 若系統為 n 階，輸出 $y=h(x)$ 在微分 r 次後 $(n\geq r)$ 出現控制輸入訊號 u，則此系統有 r 階的"外部動態"：為系統輸出中"可觀察"之動態，且有 n－r 階之"內部動態"：為系統輸出中"不可觀察"之動態，而"零動態"為使系統輸出 $y(t)=0$ 時之動態

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4.
$$\begin{cases} \dot{x}_1 = x_1 + 2x_1^2 + \sin x_2 \\ \dot{x}_2 = x_2^3 \sin x_1 + u\cos(2x_2) \end{cases}$$

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觀察可知
$$\begin{cases} z_1 = x_1 \\ z_2 = 2x_1^2 + \sin x_2 \end{cases} \Rightarrow \begin{cases} x_1 = z_1 \\ x_2 = \sin^{-1}(z_2 - 2z_1^2) \end{cases}$$

$$\Rightarrow \begin{cases} \dot{z}_1 = \dot{x}_1 = z_1 + z_2 \\ \dot{z}_2 = 4x_1(\dot{x}_1) + \dot{x}_2 \cos x_2 \end{cases}$$

$$\dot{z}_2 = 4x_1(x_1 + 2x_1^2 + \sin x_2) + (x_2^3 \sin x_1 + u\cos(2x_2))\cos x_2$$

$$= 4x_1^2 + 8x_1^3 + 4x_1\sin x_2 + x_2^3 \sin x_1 \cos x_2 + u\cos(2x_2)\cos x_2$$

$\Rightarrow$ 令 $\dot{z}_2 = v$.

$\Rightarrow$ 求出 $a=1$, $b=1$.

設 $\dot{x}=f(x)+g(x)u$，存在 u_0 使 $y(t)=0,\ \forall t\ge 0$，則

$\dot{x}=f(x)+g(x)u_0$ 為零動態

$\dot{x}=f(x)+g(x)u \Rightarrow \begin{cases}\dot{\xi}=A_c\xi+B_c v \to \text{外部動態} \\ \dot{\eta}=q(\xi,\eta) \to \text{內部動態}\end{cases}$ $\quad \dot{\eta}=q(0,\eta) \to$ 零動態

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3. (a) $y=h(x)$, $\dot{y}=\dot{h}(x)=\frac{\partial h}{\partial x}(f+gu)=L_fh+(L_gh)u$.

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$\ddot{y}=\frac{\partial(L_fh)}{\partial x}(f+gu)=L_f^2h+(L_gL_fh)u$.

相對階數2條件：$L_gh=0,\ L_gL_fh\ne 0$ #

(b) $\dddot{y}=\frac{\partial(L_f^2h)}{\partial x}=L_f^3h+(L_gL_f^2h)u$.

相對階數3條件：$L_gh=L_gL_fh=0,\ L_gL_f^2h\ne 0$ #

(c) pole : $-2, -3 \Rightarrow (\lambda+2)(\lambda+3)=\lambda^2+5\lambda+6$

令 $\xi_1=y$，$\xi_2=L_fh$，$\dot{\xi}_2=\ddot{y}=L_f^2h+(L_gL_fh)u$

$$\begin{bmatrix}\dot{\xi}_1\\ \dot{\xi}_2\end{bmatrix}=\begin{bmatrix}\xi_2\\ L_f^2h+L_gL_fhu\end{bmatrix},\ \text{令 } L_f^2h+L_gL_fhu=v.$$

$$\begin{bmatrix}\dot{\xi}_1\\ \dot{\xi}_2\end{bmatrix}=\begin{bmatrix}0&1\\0&0\end{bmatrix}\begin{bmatrix}\xi_1\\ \xi_2\end{bmatrix}+\begin{bmatrix}0\\1\end{bmatrix}v.\ \text{令 } v=-K\xi=-K_1\xi_1-K_2\xi_2$$

$$\begin{bmatrix}\dot{\xi}_1\\ \dot{\xi}_2\end{bmatrix}=\begin{bmatrix}0&1\\-K_1&-K_2\end{bmatrix}\begin{bmatrix}\xi_1\\ \xi_2\end{bmatrix},\ \det\left(\begin{bmatrix}\lambda&-1\\K_1&\lambda+K_2\end{bmatrix}\right)=\lambda^2+\lambda K_2+K_1$$

$\Rightarrow \begin{cases}K_2=5\\K_1=6\end{cases}$ $\Rightarrow u=\frac{1}{L_gL_fh}\left(-6\xi_1-5\xi_2-L_f^2h\right)$ #

$v=-6\xi_1-5\xi_2$

（續寫轉背頁）

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$$\Rightarrow \begin{bmatrix}z_1\\z_2\end{bmatrix}=\begin{bmatrix}x_1\\2x_1^2+\sin x_2\end{bmatrix}=\begin{bmatrix}\phi_1(x_1,x_2)\\ \phi_2(x_1,x_2)\end{bmatrix}\ \#.$$

$v=v(x,u)$

$=4x_1^2+8x_1^3+4x_1\sin x_2+x_2^3\sin x_1\cos x_2+u\cos(2x_2)\cos x_2$ #

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5. $\dot{x}=ax^2+bx^3+u$.

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(a) $\dot{x}=ax^2+bx^3+u=-x$

$\Rightarrow u=-(x+ax^2+bx^3)$ #

(b) $V(x)=\frac{1}{2}x^2$, $\dot{V}(x)=x(ax^2+bx^3+u)$

$\dot{V}(x)=ax^3+bx^4+u=-x^4-x^2$.

$u=-x^2-ax^3-(b+1)x^4$ #

(c). (a)為回授線性化，利用座標轉換和控制轉換產生等義之線性系統，並利用線性控制工具設計控制律.

(b)為逆向步進控制，先求出 Lyapunov 函數 $\dot{V}$ 之表示法，再設計 u 使得 $\dot{V}<0$.

(a) 優：控制方法簡單.

缺：量測誤差使其失效
失去非線性系統特性
可能超過可實現範圍

(b) 優：保留非線性系統特性，設計較直接

缺：不能使用線性控制方法，控制器較複

6. $\begin{cases} \dot{x} = x_1 \sin x_2 \\ \dot{x}_2 = x_2 \cos x_1 + u \end{cases}$ $\quad S(x_1, x_2) = \left(\frac{x_1}{3}\right)^2 + \left(\frac{x_2}{2}\right)^2 - 1$

(a) 切換控制 u_N^+, u_N^- 滿足

$\begin{cases} \nabla S(f + g u_N^+) > 0 \\ \nabla S(f + g u_N^-) < 0 \end{cases}$ 採切換邏輯 $\begin{cases} u_N^+, & S(x_1, x_2) < 0 \\ u_N^-, & S(x_1, x_2) > 0 \end{cases}$

$\Rightarrow S\dot{S} < 0$. 先求 $\dot{S}$ 表示法

$$\dot{S} = \frac{2}{9} x_1 \dot{x}_1 + \frac{1}{2} x_2 \dot{x}_2$$

$$= \frac{2}{9} x_1 (x_1 \sin x_2) + \frac{1}{2} x_2 (x_2 \cos x_1 + u)$$

$$= \frac{2}{9} x_1^2 \sin x_2 + \frac{1}{2} x_2^2 \cos x_1 + \frac{1}{2} x_2 + u.$$

若要保證 $\dot{S} > 0$, 當 $S(x_1, x_2) < 0$ 時.

$$u_N^+ = -\left(\frac{2}{9} x_1^2 \sin x_2 + \frac{1}{2} x_2^2 \cos x_1 + \frac{1}{2} x_2\right) + (x_1^2 + x_2^2).$$

$\rightarrow \dot{S} = x_1^2 + x_2^2 > 0$, 當 $S(x_1, x_2) < 0$ 時.

若要保證 $\dot{S} < 0$, 當 $S(x_1, x_2) > 0$ 時.

$$u_N^- = -\left(\frac{2}{9} x_1^2 \sin x_2 + \frac{1}{2} x_2^2 \cos x_1 + \frac{1}{2} x_2\right) - (x_1^2 + x_2^2)$$

$\Rightarrow \dot{S} = -(x_1^2 + x_2^2) < 0$. 當 $S(x_1, x_2) > 0$ 時.

$\Big\{ u_N^+ = -\left(\frac{2}{9} x_1^2 \sin x_2 + \frac{1}{2} x_2^2 \cos x_1 + \frac{1}{2} x_2\right) + (x_1^2 + x_2^2)$

(b) 令 $u = \hat{K}_y y + K_f y^3 + K_r r$. → 代入 ①

$\bar{K}_y = \hat{K}_y - K_y^*$, $\bar{K}_f = \hat{K}_f - K_f^*$, $\bar{K}_r = \hat{K}_r - K_r^*$

$\hat{K}_y = \bar{K}_y + K_y^*$, $\hat{K}_f = \bar{K}_f + K_f^*$, $\hat{K}_r = \bar{K}_r + K_r^*$

$$\dot{y} + a_p y + c_p y^3 = b_p(\hat{K}_y y + \hat{K}_f y^3 + \hat{K}_r r).$$

$$\dot{y} + (a_p - b_p \hat{K}_y) y + (c_p - b_p \hat{K}_f) y^3 = b_p \hat{K}_r r.$$

$$\dot{y} + (a_p - b_p(\bar{K}_y + K_y^*)) y + (c_p - b_p(\bar{K}_f + K_f^*)) y^3 = b_p(\bar{K}_r + K_r^*$$

$$\dot{y} + \left(\cancel{a_p} - b_p\left(\bar{K}_y + \frac{a_p - a_m}{b_p}\right)\right) y + \left(\cancel{c_p} - b_p\left(\bar{K}_f + \frac{c_p}{b_p}\right)\right) y^3 = b_p\left(\bar{K}_r + \frac{1}{b_p}\right) r$$

$$\dot{y} + (a_m - b_p \bar{K}_y) y - b_p \bar{K}_f y^3 = (1 + b_p \bar{K}_r) r.$$

$$\dot{y} = -(a_m - b_p \bar{K}_y) y + b_p \bar{K}_f y^3 + (1 + b_p \bar{K}_r) r.$$

追蹤誤差 $e = y(t) - y_m(t)$, $\dot{e} = \dot{y}(t) - \dot{y}_m(t)$

$\dot{y}_m = r - a_m y_m$. $\quad -a_m y + a_m y_m = -a_m e$

$$\Rightarrow \dot{e} = -(a_m - b_p \bar{K}_y) y + b_p \bar{K}_f y^3 + (\cancel{1} + b_p \bar{K}_r) r - \cancel{r} + a_m y_m$$

$$= -a_m e + b_p(\bar{K}_y y + \bar{K}_f y^3 + \bar{K}_r r).$$

建立增益調變機制

$$\dot{\hat{K}}_y = \dot{\bar{K}}_y = f_1(e, \bar{K}_y, \bar{K}_f, \bar{K}_r)$$

$$\dot{\hat{K}}_f = \dot{\bar{K}}_f = f_2(e, \bar{K}_y, \bar{K}_f, \bar{K}_r)$$

$$u_N = -\left(\frac{2}{9}x_1^2\sin x_2 + \frac{1}{2}x_2^2\cos x_1 + \frac{1}{2}x_2\right) - (x_1^2 + x_2^2)$$ #.

(b) 平衡控制 u_{eq}. $\dot{S} = 0$ 當 $S(x_1, x_2) = 0$

$$\Rightarrow \dot{S} = \frac{2}{9}x_1^2\sin x_2 + \frac{1}{2}x_2^2\cos x_1 + \frac{1}{2}x_2 u = 0$$

$$\frac{4}{9}x_1^2\sin x_2 + x_2^2\cos x_1 + x_2 u = 0$$

$$\Rightarrow u = \frac{-1}{x_2}\left(\frac{4}{9}x_1^2\sin x_2 + x_2^2\cos x_1\right).$$

$$\Rightarrow u_{eq} = \frac{-1}{x_2}\left(\frac{4}{9}x_1^2\sin x_2 + x_2^2\cos x_1\right)$$

當 $S(x_1, x_2) = 0$ 時 #

7. (a). $\dot{y} + a_p y + c_p y^3 = b_p u$ —① $\dot{y}_m + a_m y_m = r$ —②

$u = K_y^* y + K_f^* y^3 + K_r^* r$. → 代入①

$$\dot{y} + a_p y + c_p y^3 = b_p(K_y^* y + K_f^* y^3 + K_r^* r)$$

$$\begin{cases} \dot{y} + (a_p - b_p K_y^*)y + (c_p - b_p K_f^*)y^3 = b_p K_r^* r \\ \dot{y}_m + a_m y_m = r. \end{cases}$$

$\Rightarrow b_p K_r^* = 1$

$\Rightarrow a_p - b_p K_y^* = a_m$, $K_y^* = \dfrac{a_p - a_m}{b_p}$ # $K_r^* = \dfrac{1}{b_p}$ #

$\Rightarrow c_p - b_p K_f^* = 0$, $K_f^* = \dfrac{c_p}{b_p}$ #

$$\dot{\bar{K}}_r = \dot{\bar{K}}_r = f_3(e, \bar{K}_y, \bar{K}_f, \bar{K}_r)$$

選定 Lyapunov 函數

$$V = \frac{1}{2}e^2 + \frac{1}{2}|b_p|\left(\frac{\bar{K}_y^2}{\gamma_1} + \frac{\bar{K}_f^2}{\gamma_2} + \frac{\bar{K}_r^2}{\gamma_3}\right)$$

$\gamma_1 > 0$, $\gamma_2 > 0$, $\gamma_3 > 0$.

$$\dot{V} = e\dot{e} + |b_p|\left(\frac{\bar{K}_y f_1}{\gamma_1} + \frac{\bar{K}_f f_2}{\gamma_2} + \frac{\bar{K}_r f_3}{\gamma_3}\right)$$

$$= -a_m e^2 + b_p e(\bar{K}_y y + \bar{K}_f y^3 + \bar{K}_r r) + |b_p|\left(\frac{\bar{K}_y f_1}{\gamma_1} + \frac{\bar{K}_f f_2}{\gamma_2} + \frac{\bar{K}_r f_3}{\gamma_3}\right)$$

$\Rightarrow$ 選 $f_1 = -\gamma_1 \mathrm{sgn}(b_p) e y = \dot{\hat{K}}_y$ $\hat{K}_y = \bar{K}_y + K_y^*$

$f_2 = -\gamma_2 \mathrm{sgn}(b_p) e y^3 = \dot{\hat{K}}_f$ $\hat{K}_f = \bar{K}_f + K_f^*$

$f_3 = -\gamma_3 \mathrm{sgn}(b_p) e r = \dot{\hat{K}}_r$ # $\hat{K}_r = \bar{K}_r + K_r^*$.

$\Rightarrow$ 使 $\dot{V} = -a_m e^2 < 0$ $\Rightarrow$ $e = y(t) - y_m(t) \to 0$.