非線性控制作業

第二章

系級：系統所碩一

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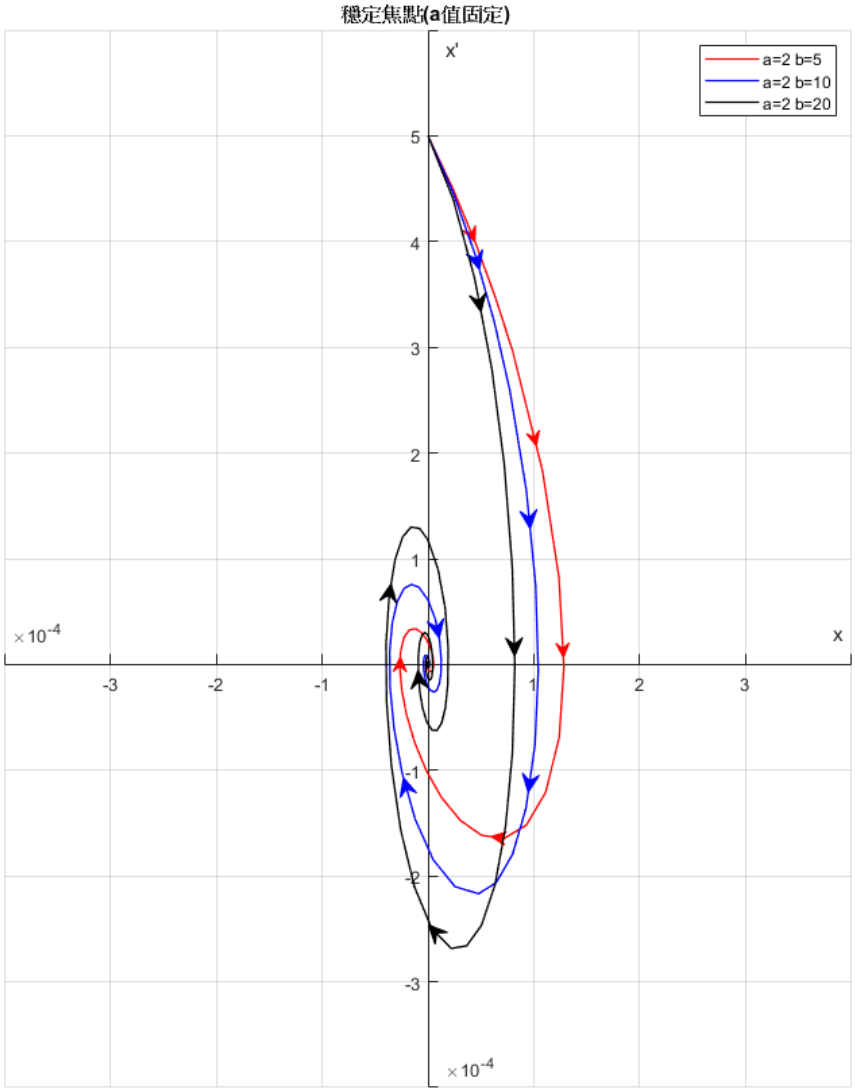
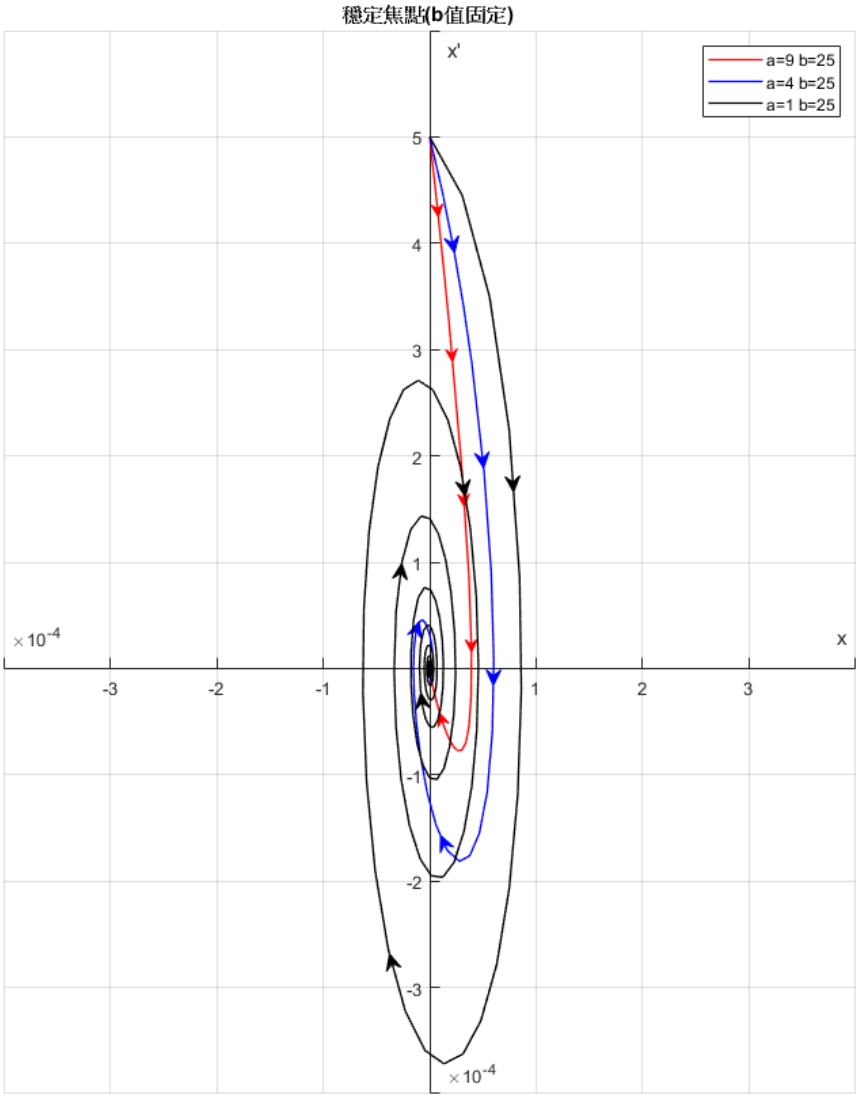
1. 考慮(2.4.3)式，選取 6 種不同的$(a,\ b)$值，使得特徵方程式$\lambda^2 + a\lambda + b = 0$ 所求得到的 2 個特徵值的位置剛好對應到圖 2.4.1 的 6 種情形。針對這 6 種不同的$(a,\ b)$值，畫出(2.4.3) 式的相平面軌跡，並比較圖 2.4.1 的軌跡，驗證所得結果的正確性。

1.1. 穩定焦點(stable focus)：共軛複數根，實部為負。若要滿足穩定焦點條件，則 a 與 b 為 $a > 0$ 且 $a^2 < 4b$。



圖(1.1.1)

在圖(1.1.1)中，在滿足 a 與 b 條件下，a 值保持固定，調動 b 值，可觀察到圖形變化，當 b 值越大，系統振盪變大 。

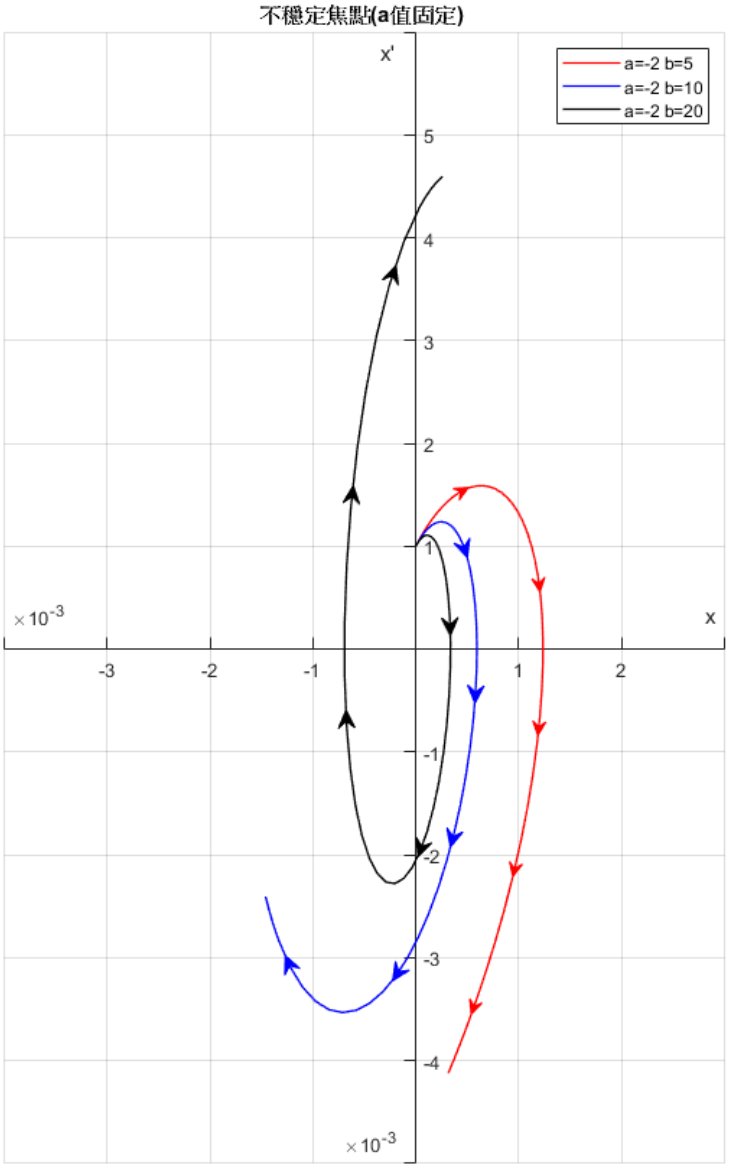


圖(1.1.2)

在圖(1.1.2)中，在滿足 a 與 b 條件下，b 值保持固定，調動 a 值，可觀察到圖形變化，當 a 值越小，系統振盪變大 。

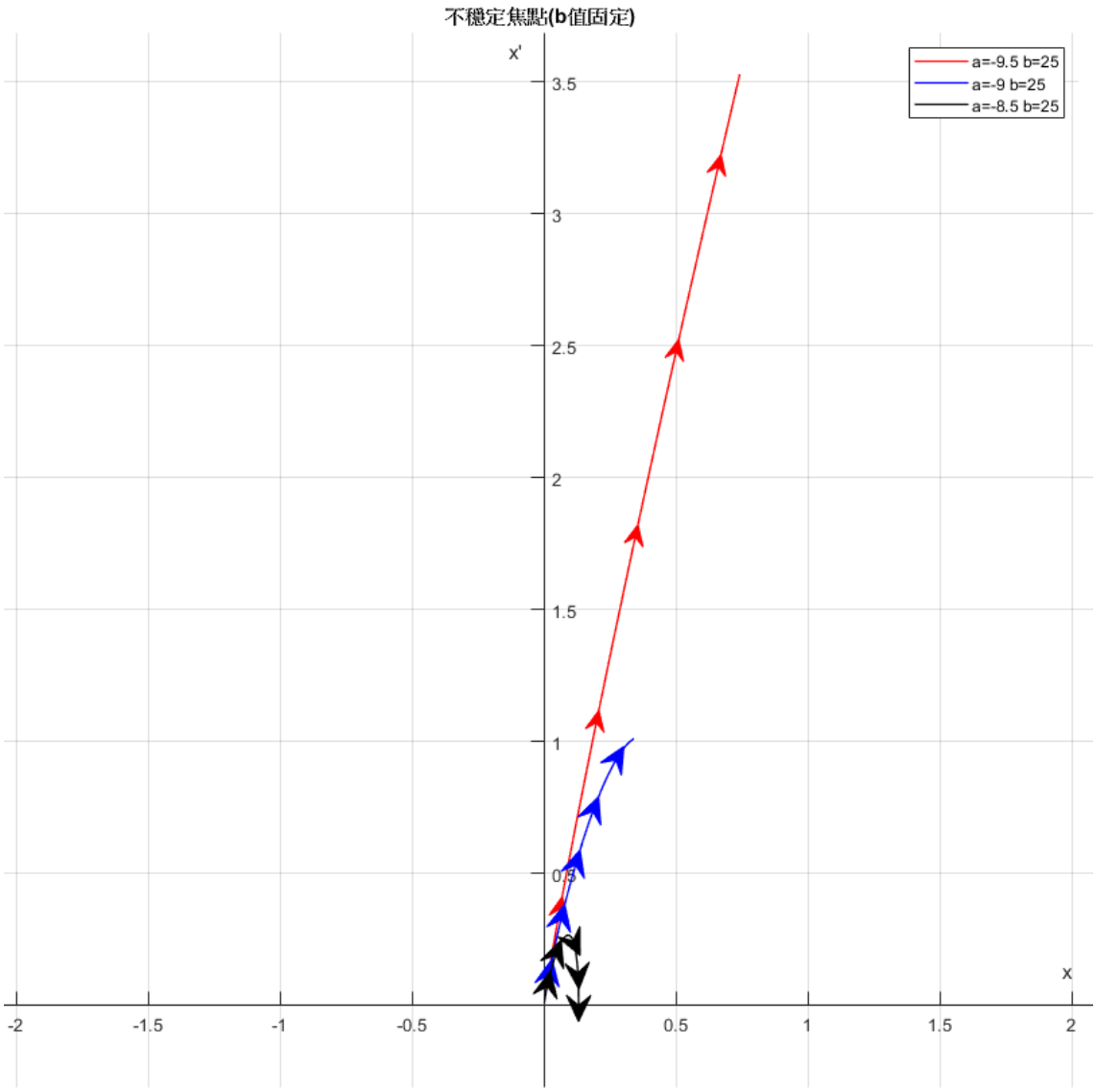
經過不同的係數組合以及不同的初始條件反覆測試，發現不同的係數以及初始條件會造成不同的圖形結果，但最終軌跡都會收斂於原點。

1.2. 不穩定焦點(unstable focus)：共軛複數根，實部為正。若要滿足不穩定焦點，a 與 b 條件為 $a < 0$ 且 $a^2 < 4b$。



圖(1.2.1)

在圖(1.2.1)中，在滿足 a 與 b 條件下，a 值保持固定，調動 b 值，可觀察到圖形變化，當 b 值越大，軌跡遠離初始點速度愈快。

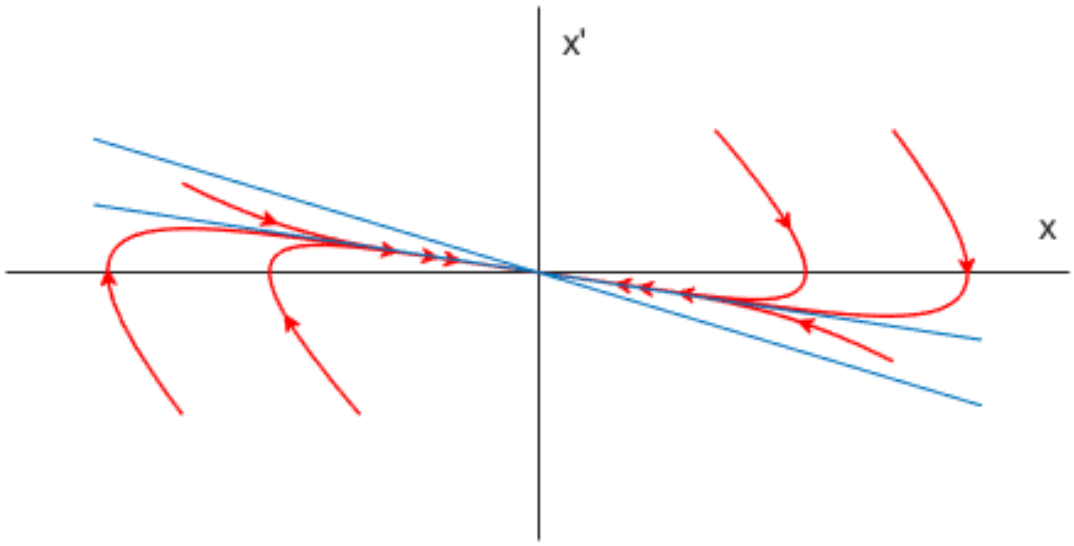


圖(1.2.2)

在圖(1.2.2)中，在滿足 a 與 b 條件下，b 值保持固定，調動 a 值，可觀察到圖形變化，當 a 值越小，軌跡遠離初始點速度愈快。

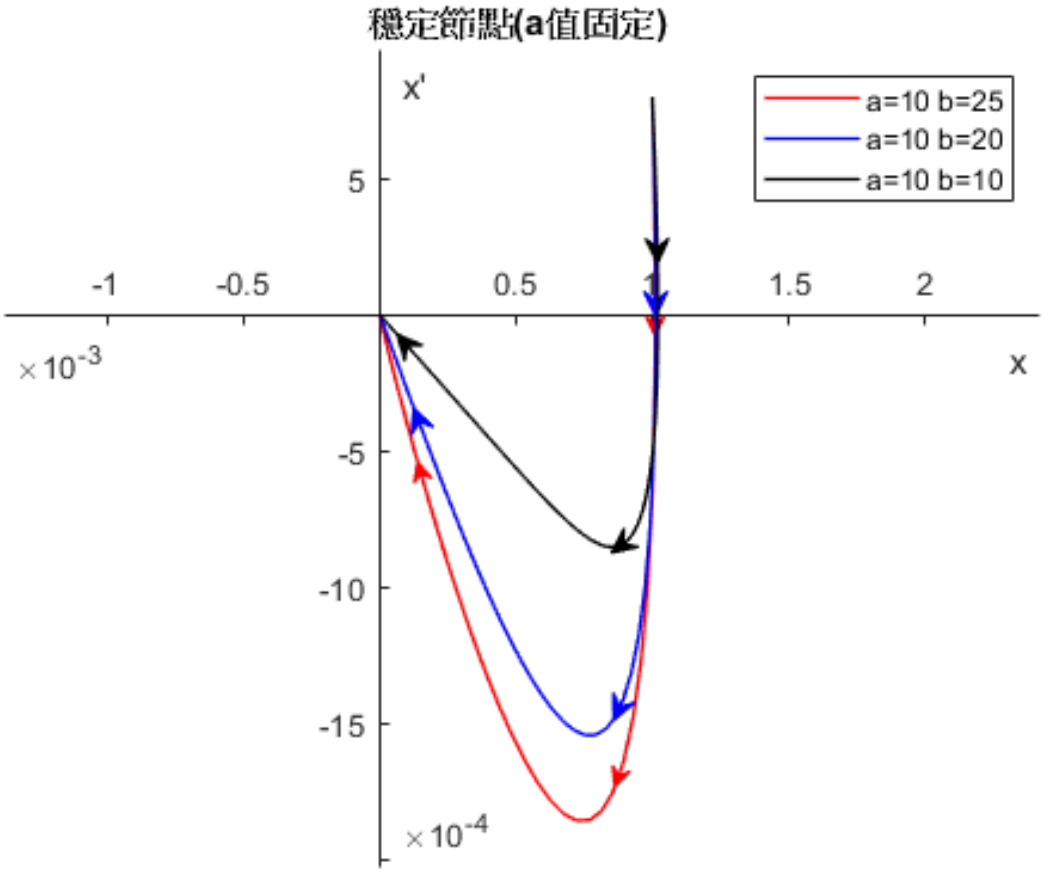
經過不同的係數組合以及不同的初始條件反覆測試，發現不同的係數以及初始條件會造成不同的圖形結果，但最終軌跡都會遠離於原點。

1.3. 穩定節點(stablenode)：二根皆為負實數。若要滿足穩定節點條件，則 a 與 b 為 $a > 0$ 且 $a^2 \geqq 4b$。



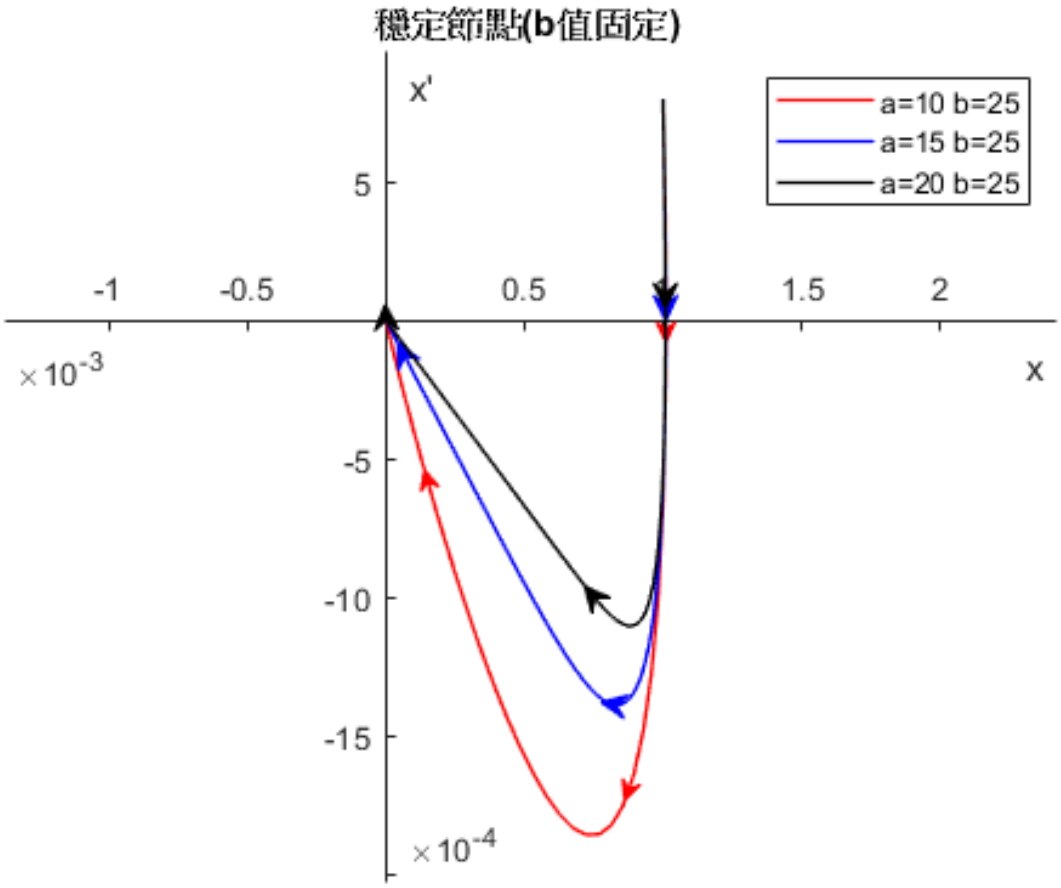
圖(1.3.1)

圖(1.3.1)為 a=1 與 b=1/8 代入方程式中，並且設一組不相同的初始條件，由圖形可知，即使初始條件不相同，但系統最終都會收斂原點。



圖(1.3.2)

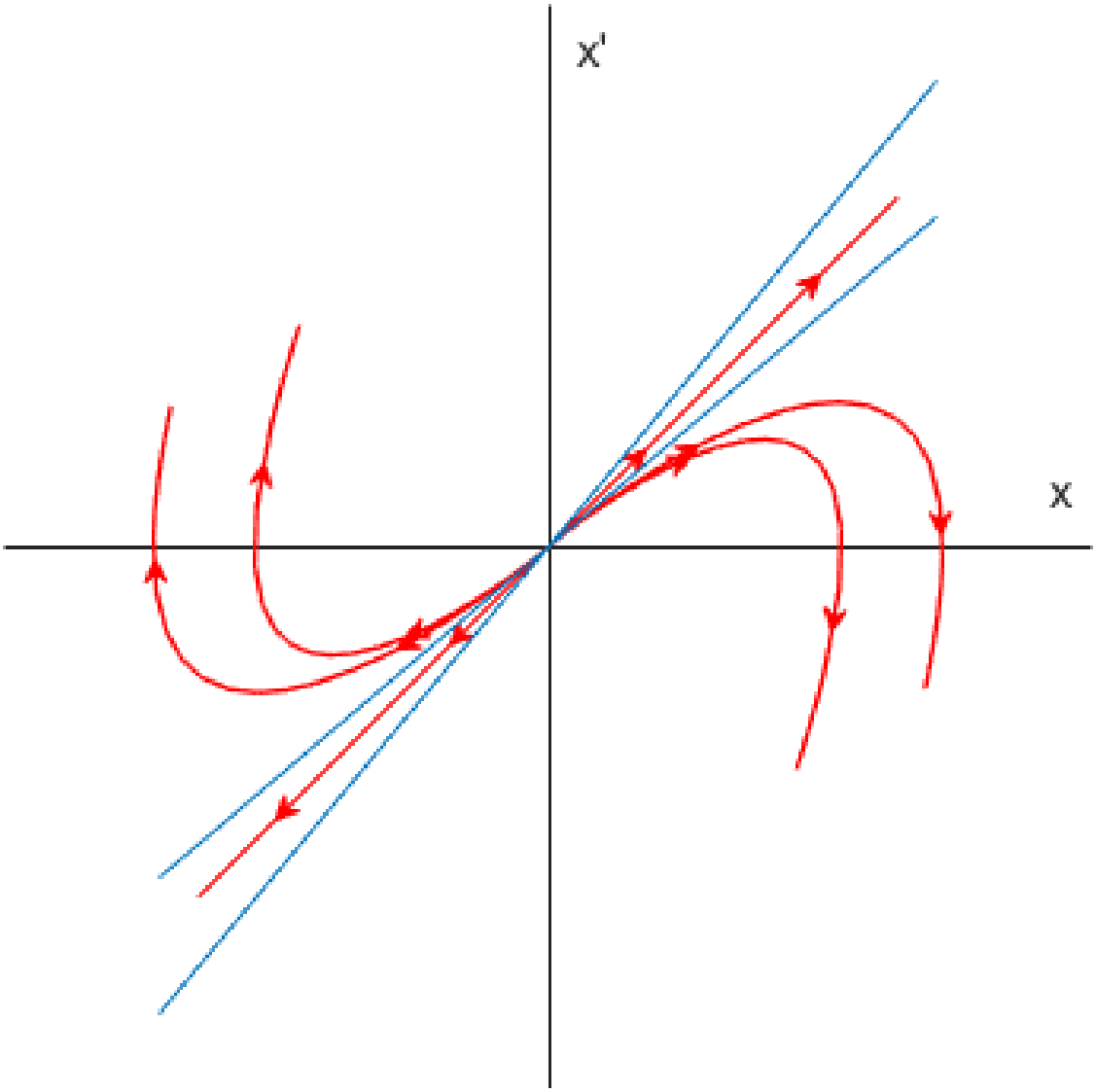
在圖(1.3.2)中，在滿足 a 與 b 條件下，a 值保持固定，調動 b 值，可觀察到圖形變化，當 b 值越小，系統振盪變小。



圖(1.3.3)

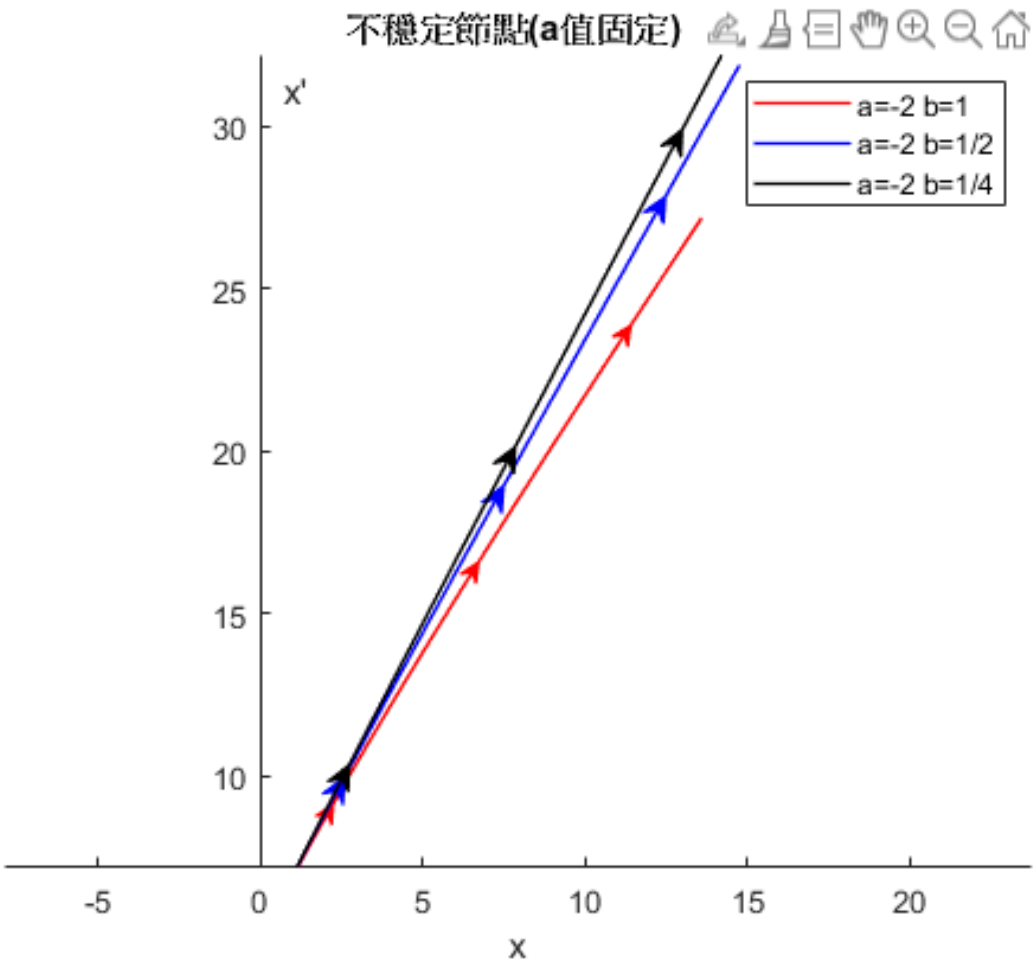
在圖(1.3.3)中，在滿足 a 與 b 條件下，b 值保持固定，調動 a 值，可觀察到圖形變化，當 a 值越大，系統振盪變小。

1.4. 不穩定節點(unstable node)：二根皆為正實數。若要滿足不穩定節點條件，則 a 與 b 為 $a < 0$ 且 $a^2 \geqq 4b$。



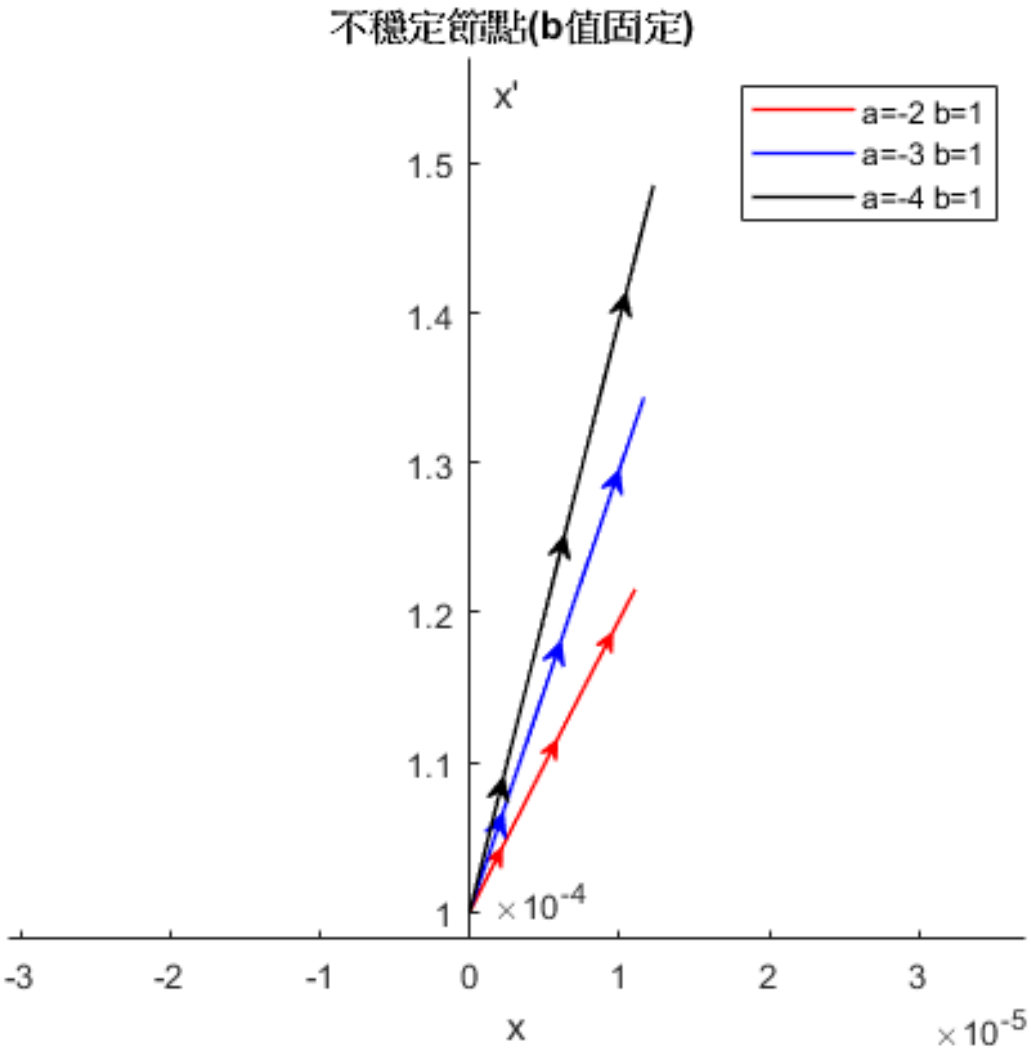
圖(1.4.1)

圖(1.4.1)為 a=-2 與 b=1 代入方程式中，並且設一組不相同的初始條件，由圖形可知，即使初始條件不相同，但系統最終都會遠離原點。



圖(1.4.2)

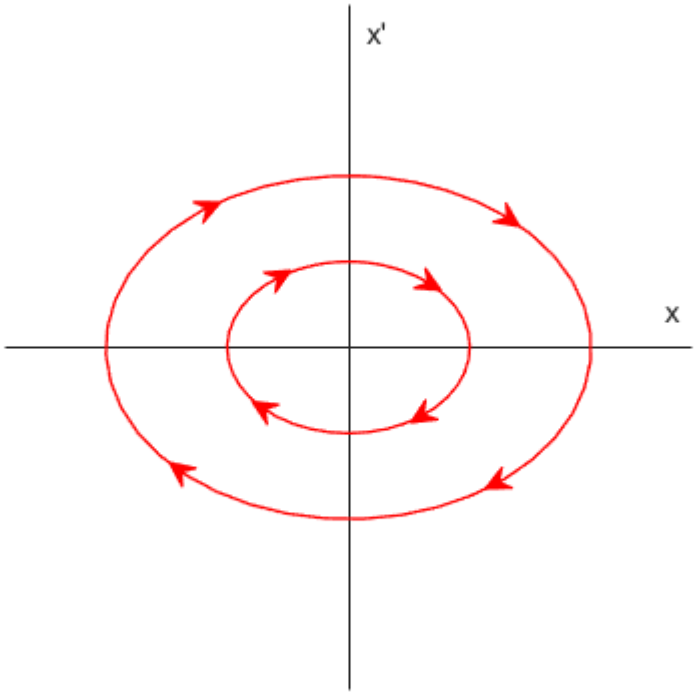
在圖(1.4.2)中，在滿足 a 與 b 條件下，a 值保持固定，調動 b 值，可觀察到圖形變化，當 b 值越小，軌跡遠離初始點速度愈快。



圖(1.4.3)

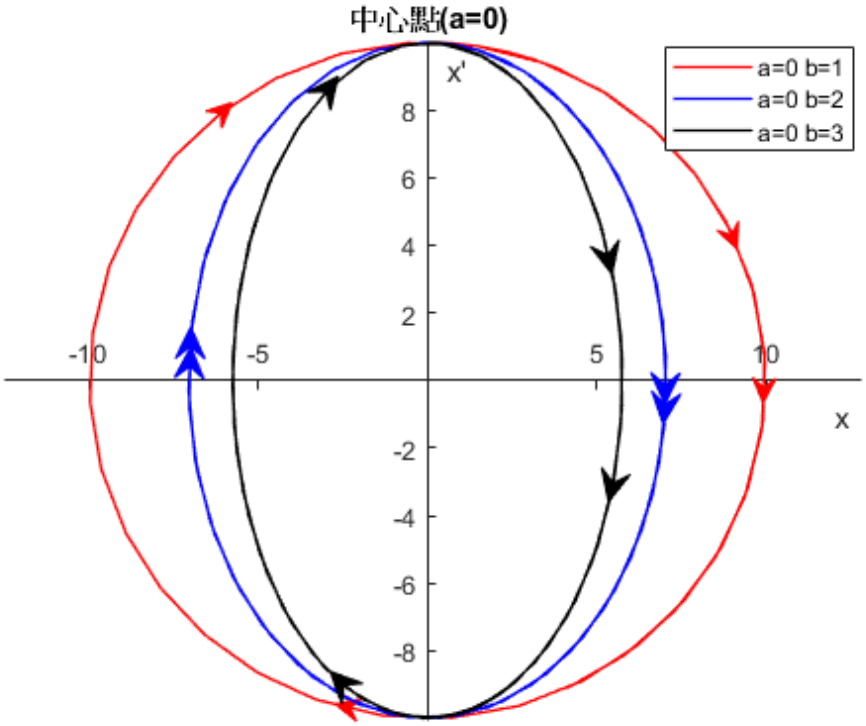
在圖(1.4.3)中，在滿足 a 與 b 條件下，b 值保持固定，調動 a 值，可觀察到圖形變化，當 a 值越小，軌跡遠離初始點速度愈快。

1.5. 中心點(center)：虛軸上之共軛複數根。若要滿足中心點條件，則a與b為$a = 0$ 且 $b > 0$。



圖(1.5.1)

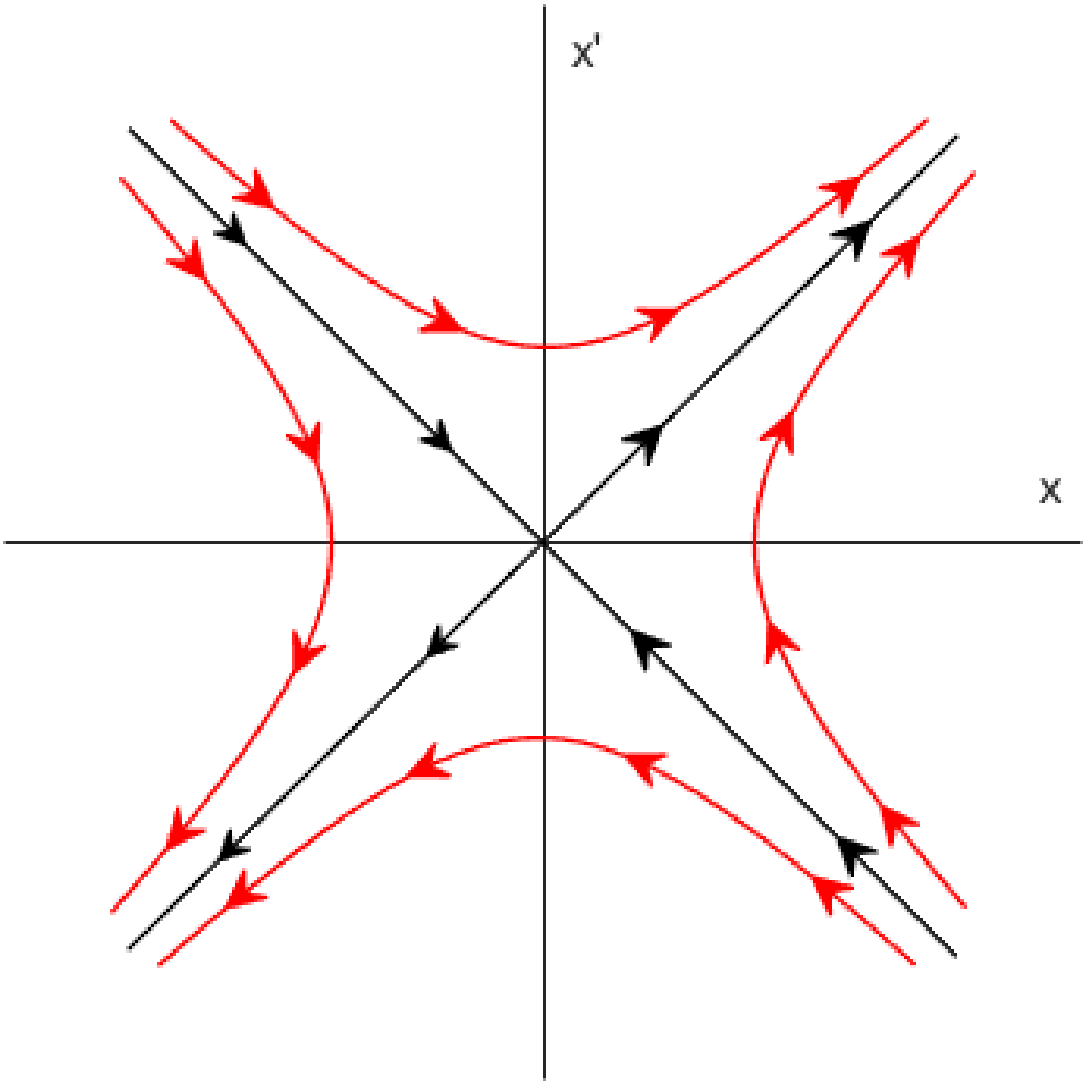
圖(1.5.1)為 $a=0$ 與 $b=1/2$ 代入方程式中，並且設一組不相同的初始條件，由圖形可知，即使初始條件不相同，但系統依然會循著固定軌跡。



圖(1.5.2)

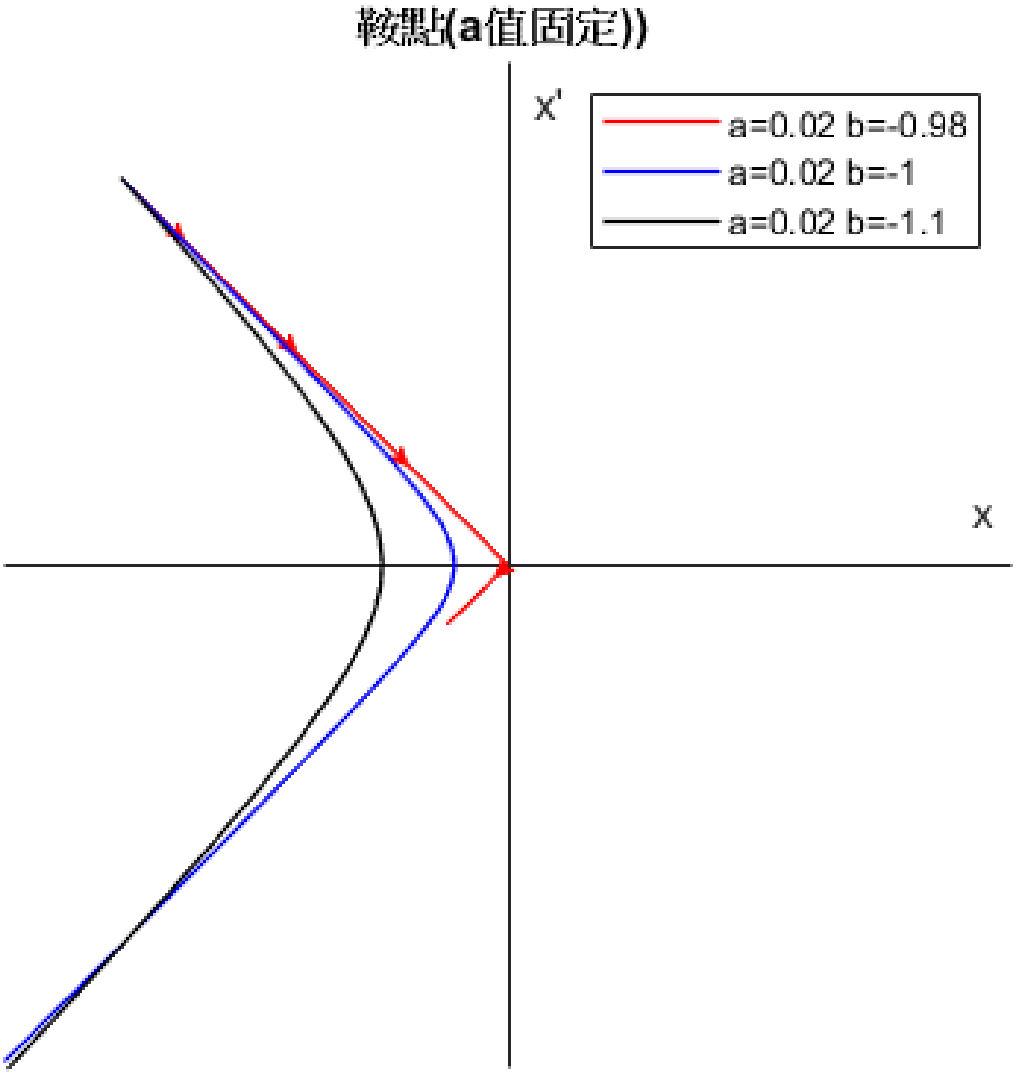
在圖(1.5.2)中，在滿足 a 與 b 條件下，a=0，調動 a 值，可觀察到圖形變化，當 b 值越大，收斂速度愈快。

1.6. 鞍點(saddle point)：二實根，一正一負。若要滿足中心點條件，則a與b為 $a^2 \geqq 4b$ 且 $b \neq 0$。



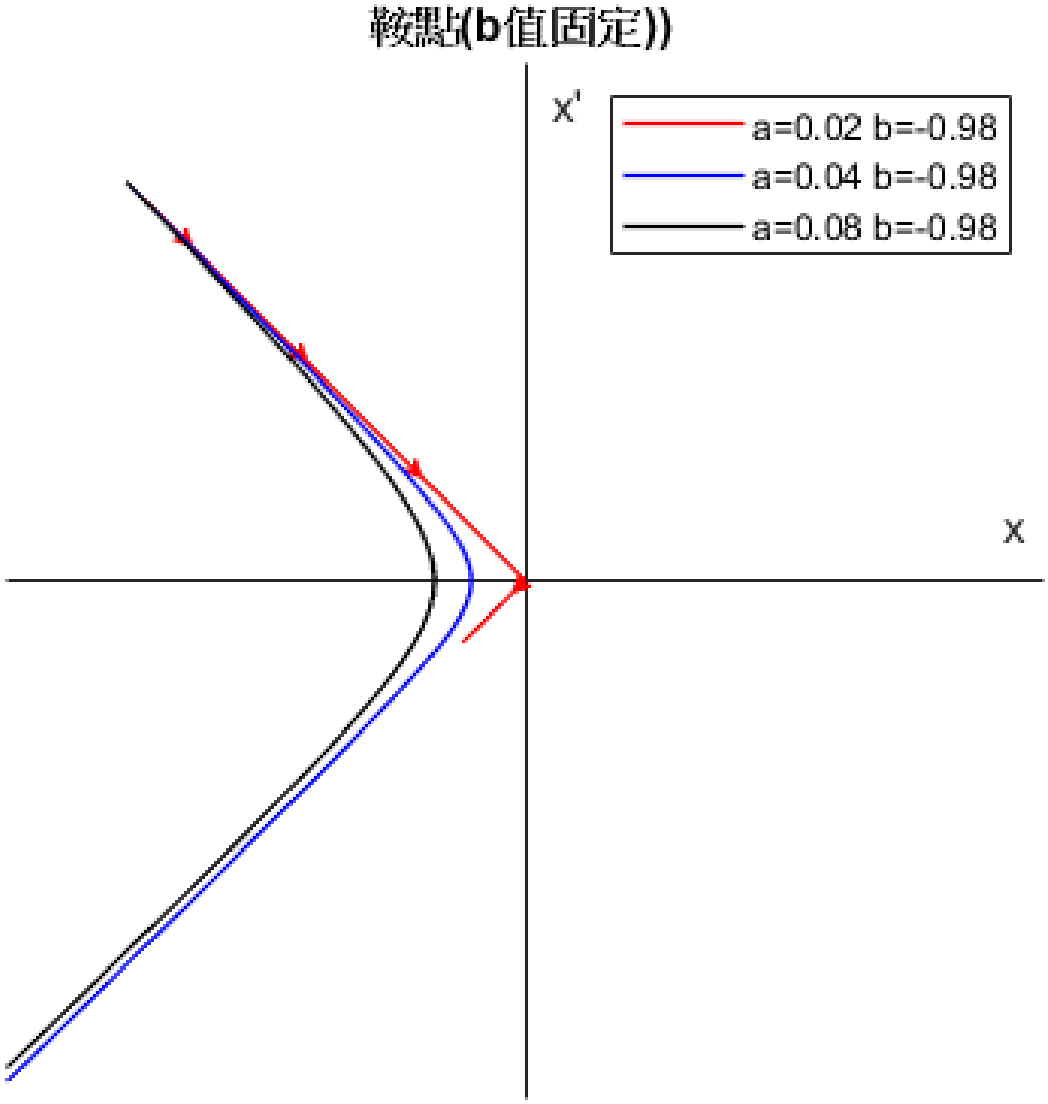
圖(1.6.1)

圖(1.6.1)為 a=0.02 與 b=-0.98 代入方程式中，並且設一組不相同的初始條件，由圖形可知，即使初始條件不相同，但軌跡都會接近原點後又遠離。



圖(1.6.2)

在圖(1.6.2)中，在滿足 a 與 b 條件下，a 值固定，調動 b 值，可觀察到圖形變化，當 b 值越小，軌跡速度愈快，且轉折處離原點越遠。



圖(1.6.3)

在圖(1.6.3)中，在滿足 a 與 b 條件下，b 值固定，調動 a 值，可觀察到圖形變化，當 a 值越大，軌跡速度愈快，且轉折處離原點越遠。

2. 試以座標變換

$$r = (x_1^2 + x_2^2)^{1/2}\text{，}\ \theta = \tan^{-1}(x_2/x_1)$$

求下列三組非線性系統的解析解

(a) $\dot{x}_1 = x_2 + x_1(x_1^2 + x_2^2 - 1)$，

$\dot{x}_2 = -x_1 + x_2(x_1^2 + x_2^2 - 1)$

(b) $\dot{x}_1 = x_2 + x_1(x_1^2 + x_2^2 - 1)$，

$\dot{x}_2 = -x_1 + x_2(x_1^2 + x_2^2 - 1)$

(c) $\dot{x}_1 = x_2 - x_1(x_1^2 + x_2^2 - 1)$，

$\dot{x}_2 = -x_1 - x_2(x_1^2 + x_2^2 - 1)$

由所得到的極座標方程式預測各個系統是否存在極限圓，其穩定性如何(穩定？半穩定? 不穩定?)

其次再以 Matlab 分別畫出以上三組方程式的相平面軌跡圖，驗證解析解 的預測是否正確性。

(a) $\dot{x}_1 = x_2 + x_1(x_1^2 + x_2^2 - 1)$，

$\dot{x}_2 = -x_1 + x_2(x_1^2 + x_2^2 - 1)$

引入極座標變數，則(a)式可化成

$$\dot{r} = r(r^2 - 1) \quad , \quad \dot{\theta} = -1 \qquad (2.1)$$

- 若一開始 $r = 1$，則 $dr/dt = 0$，此說明若一旦進入極限圓 $r = 1$，則將一直維持在極限圓之上。
- 若一開始 $r > 1$ 時，則有$\dot{r} > 0$，代表 r 將一直增加到 ∞，而遠離極限圓。
- 若一開始 $r < 1$ 時，則有$\dot{r} < 0$，表示 r 將一直遞減到 0，而遠離極限圓。

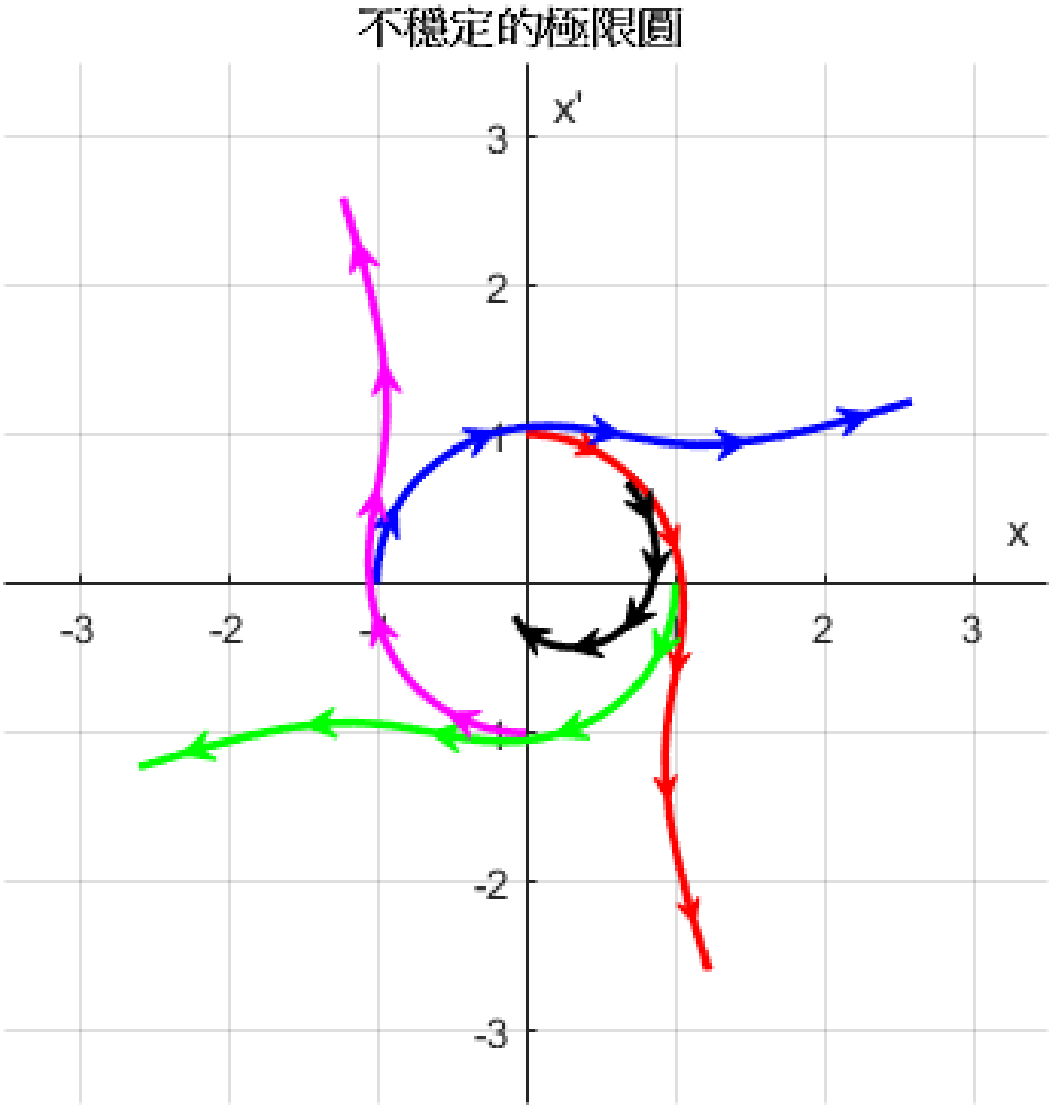
由以上三個特徵知道非線性系統所具有的極限圓是不穩定的，因為其圓內、外的軌跡最後都遠離極限圓之上。實際上，(2.1)式有解析解的存在，其解可表之如下：

$$r(t) = \frac{1}{\sqrt{1+Ce^{2t}}} \quad , \quad C_0 = r_0^{-2} - 1 \ , \ \theta(t) = \theta_0 - t$$

故當時間 $t \to \infty$時，$r(t) \to 0$，即

$$\lim_{n\to\infty} r(t) = 0$$

此式說明了所謂的極限圓是指時間趨近於無窮大的『極限』情形下，相平面軌跡最終所遠離的『圓』。



圖(2.1)

由圖(2.1)中，可觀察到圓內、外的軌跡皆會遠離極限圓，代表此極限圓為不穩定的，與解析解分析之結果一至。

(b) $\dot{x}_1 = x_2 - x_1(x_1^2 + x_2^2 - 1)^2$

$\dot{x}_2 = -x_1 - x_2(x_1^2 + x_2^2 - 1)^2$

答:

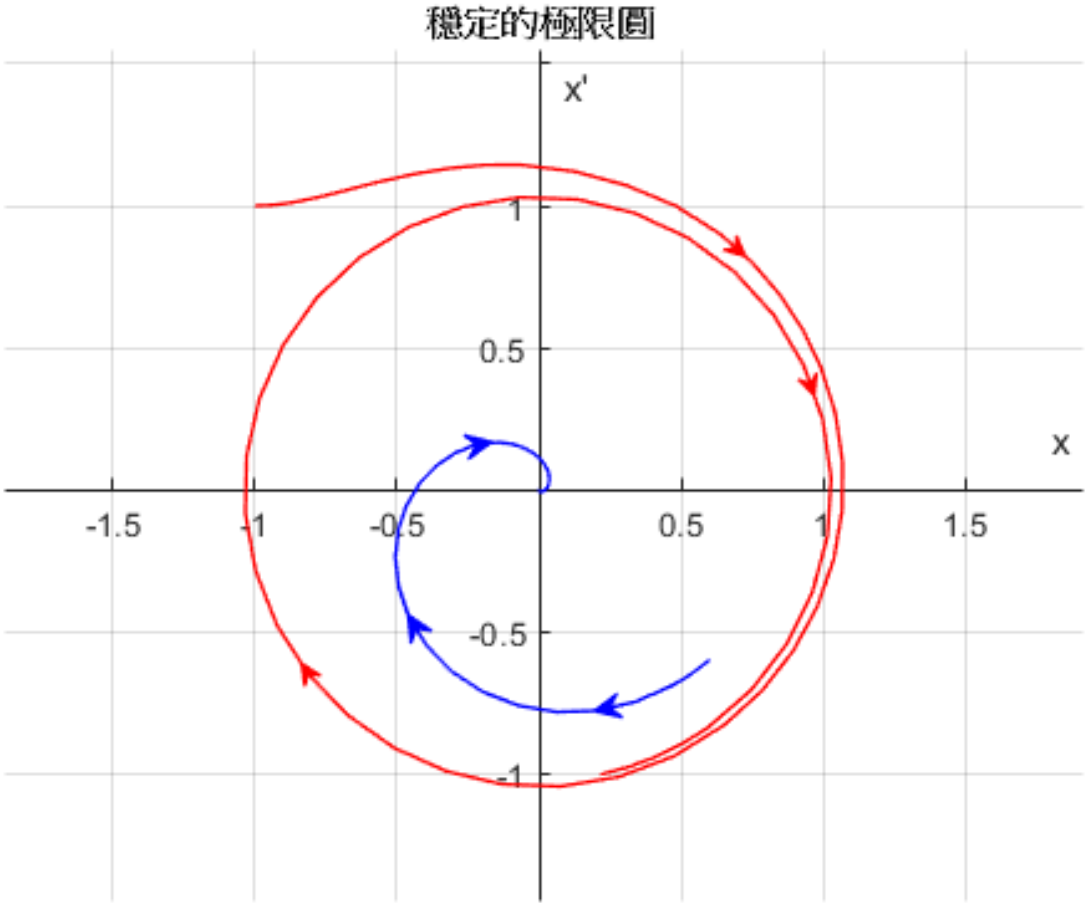
引入極座標變數，則(b)式可化成

$$\dot{r} = -r(r^2 - 1)^2 \quad , \quad \dot{\theta} = -1 \qquad (2.1)$$

- 若一開始 $r = 1$，則 $dr / dt = 0$，此說明若一旦進入極限圓 $r = 1$，則將一直維持在極限圓之上。
- 若一開始 $r > 1$ 時，則有$\dot{r} < 0$，代表 r 將一直遞減到 1，而後進入極限圓。
- 若一開始 $r < 1$ 時，則有$\dot{r} < 0$，表示 r 將一直遞減到 0，而後遠離極限圓。

由以上三個特徵知道非線性系統所具有的極限圓是半穩定的，因為其圓內、外的軌跡最後都遠離極限圓之上。實際上，(2.2)式有解析解的存在，其解可表之如下：

$$ln(r) - \frac{1}{2} ln|r^2 - 1| - \frac{1}{2(r^2-1)} = -t + C \quad , \quad , \theta(t) = \theta_0 - t$$



圖(2.2)

由圖(2.2)中，可觀察到圓內的軌跡會遠離極限圓，圓外的軌跡會進入極限圓，皆代表此極限圓為半穩定的，與解析解之結果一至。

(c) $\dot{x}_1 = x_2 - x_1(x_1^2 + x_2^2 - 1)$，

$\dot{x}_2 = -x_1 - x_2(x_1^2 + x_2^2 - 1)$

引入極座標變數，則(c)式可化成

$$\dot{r} = -r(r^2 - 1) \quad , \quad \dot{\theta} = -1 \qquad (2.1)$$

- 若一開始 $r = 1$，則 $dr/dt = 0$，此說明若一旦進入極限圓 $r = 1$，則將一直維持在極限圓之上。
- 若一開始 $r > 1$ 時，則有 $\dot{r} < 0$，代表 r 將一直遞減到 1，而進入極限圓。
- 若一開始 $r < 1$ 時，則有 $\dot{r} > 0$，表示 r 將一直增加到 1，而進入極限圓。

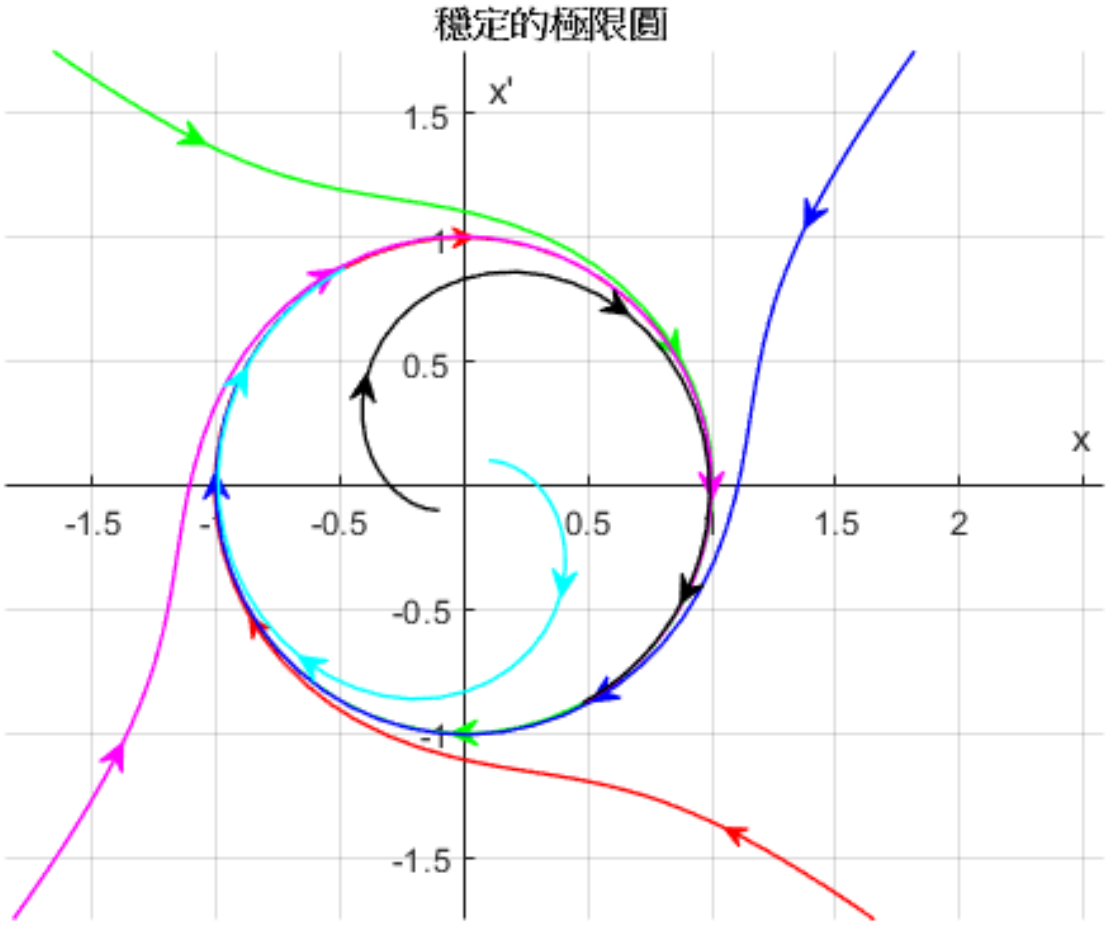
由以上三個特徵知道非線性系統所具有的極限圓是穩定的，因為其圓內、外的軌跡最後都遠離極限圓之上。實際上，(2.1)式有解析解的存在，其解可表之如下：

$$r(t) = \frac{1}{\sqrt{1+Ce^{-2t}}} \quad , \quad C_0 = r_0^{-2} - 1 \ , \ \theta(t) = \theta_0 - t$$

故當時間 $t \to \infty$時，$r(t) \to 1$，即

$$\lim_{n\to\infty} r(t) = 1$$

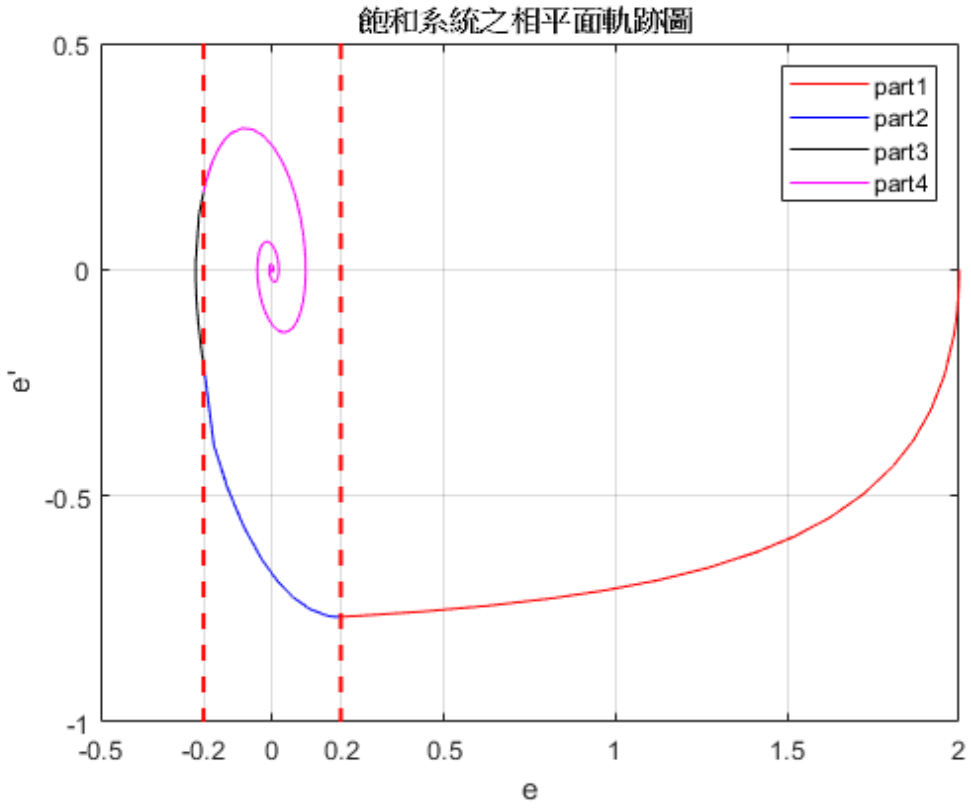
此式說明了所謂的極限圓是指時間趨近於無窮大的『極限』情形下，相平面軌跡最終所進入的『圓』。



圖(2.3)

由圖(2.3)中，可觀察到圓內、外的軌跡皆會進入極限圓，代表此極限圓為穩定的，與解析解之結果一至。

3. 利用 Matlab 畫出圖 2.7.7 所示飽和系統的相平面軌跡圖，其中採用下列的參數設定： $T = 1$，$K = 4$，$M0 = 0.2$，$c_0 = 0.2$。比較圖 2.7.8 的手繪圖以及圖 2.7.9 的電腦繪製圖， 你所得到的軌跡圖是否與之相符？是否能得到比手繪圖更精確的結果？



圖(3.1)

將系統代入參數$T = 1$，$K = 4$，$M0 = 0.2$，$c_0 = 0.2$。由 Matlab 畫出之相平面軌跡圖，與課本之圖 2.7.8 的手繪圖及圖 2.7.9 電腦繪製圖比較而知，圖軌跡皆相符。而電腦繪圖相較於手繪圖在趨近於平衡點時軌跡圖形更加準確。

圖(1.1.1)

```
clc
close all
clear all
[t, y] = ode45(@vdp1, [0 10], [0.00 0.0005]);
a1=arrowPlot(y(:,1), y(:,2), 'number', 5,'color', 'r', 'LineWidth', 1, 'scale', 0.8,
'ratio', 'equal');
hold on;
[t, y] = ode45(@vdp2, [0 10], [0.00 0.0005]);
a2=arrowPlot(y(:,1), y(:,2), 'number', 5,'color', 'b', 'LineWidth', 1, 'scale', 0.8,
'ratio', 'equal');
hold on;
[t, y] = ode45(@vdp3, [0 10], [0.00 0.0005]);
a3=arrowPlot(y(:,1), y(:,2), 'number', 5,'color', 'black', 'LineWidth', 1, 'scale', 0.8,
'ratio', 'equal');

ax = gca;                    % gets the current axes
ax.XAxisLocation = 'origin';    % sets them to zero
ax.YAxisLocation = 'origin';    % sets them to zero
ax.Box ='off';               % switches off the surrounding box
% ax.XTick = [];   % sets the tick marks
% ax.YTick = [];    % sets the tick marks

axis([-0.0004, 0.0004, -0.0004, 0.0006]);
xlabel('x'); ylabel('x'');
grid on;

legend([a1 a2 a3 ],'a=2 b=5','a=2 b=10 ','a=2 b=20');
title('Ã-©wµJÂI(a-È©T©w) ');
%-------------
function dy = vdp1(t, y)
a=2
b=5
dy = [y(2); -a*y(2)-b*y(1)];
end
function dy = vdp2(t, y)
a=2
b=10
dy = [y(2); -a*y(2)-b*y(1)];
end
function dy = vdp3(t, y)
a=2
b=20
dy = [y(2); -a*y(2)-b*y(1)];
end
```

圖(1.1.2)

```
clc
close all
clear all
[t, y] = ode45(@vdp1, [0 10], [0.00 0.0005]);
a1=arrowPlot(y(:,1), y(:,2), 'number', 5,'color', 'r', 'LineWidth', 1, 'scale', 0.8,
'ratio', 'equal');
hold on;
[t, y] = ode45(@vdp2, [0 10], [0.00 0.0005]);
a2=arrowPlot(y(:,1), y(:,2), 'number', 5,'color', 'b', 'LineWidth', 1, 'scale', 0.8,
'ratio', 'equal');
hold on;
[t, y] = ode45(@vdp3, [0 10], [0.00 0.0005]);
a3=arrowPlot(y(:,1), y(:,2), 'number', 5,'color', 'black', 'LineWidth', 1, 'scale', 0.8,
'ratio', 'equal');

ax = gca;                    % gets the current axes
ax.XAxisLocation = 'origin';    % sets them to zero
ax.YAxisLocation = 'origin';    % sets them to zero
ax.Box ='off';               % switches off the surrounding box
% ax.XTick = [];   % sets the tick marks
% ax.YTick = [];    % sets the tick marks

axis([-0.0004, 0.0004, -0.0004, 0.0006]);
xlabel('x'); ylabel('x'''');
grid on;

legend([a1 a2 a3 ],'a=9 b=25','a=4 b=25 ','a=1 b=25');
title('Ã-©wµJÂI(b-È©T©w) ');
%-------------
function dy = vdp1(t, y)
a=9                          %½Õ¾ãa¡Bb-È
b=25
dy = [y(2); -a*y(2)-b*y(1)];
end
function dy = vdp2(t, y)
a=4                          %½Õ¾ãa¡Bb-È
b=25
dy = [y(2); -a*y(2)-b*y(1)];
end
function dy = vdp3(t, y)
a=1                          %½Õ¾ãa¡Bb-È
b=25
dy = [y(2); -a*y(2)-b*y(1)];
end
```

圖(1.2.1)

```
clc
close all
clear all
[t, y] = ode45(@vdp1, [0 1.5], [0.0 0.001]);
a1=arrowPlot(y(:,1), y(:,2), 'number', 5,'color', 'r', 'LineWidth', 1, 'scale', 0.8,
'ratio', 'equal');
hold on;
[t, y] = ode45(@vdp2, [0 1.5], [0.0 0.001]);
a2=arrowPlot(y(:,1), y(:,2), 'number', 5,'color', 'b', 'LineWidth', 1, 'scale', 0.8,
'ratio', 'equal');
hold on;
[t, y] = ode45(@vdp3, [0 1.5], [0.0 0.001]);
a3=arrowPlot(y(:,1), y(:,2), 'number', 5,'color', 'black', 'LineWidth', 1, 'scale', 0.8,
'ratio', 'equal');

ax = gca;                    % gets the current axes
ax.XAxisLocation = 'origin';    % sets them to zero
ax.YAxisLocation = 'origin';    % sets them to zero
ax.Box ='off';               % switches off the surrounding box
% ax.XTick = [];   % sets the tick marks
% ax.YTick = [];    % sets the tick marks
%
 axis([-0.004, 0.003, -0.005, 0.006]);
xlabel('x'); ylabel('x'''');
grid on;

legend([a1 a2 a3 ],'a=-2 b=5','a=-2 b=10 ','a=-2 b=20');
title('¤£Ã-©wµJÂI(a-È©T©w) ');
%-------------
function dy = vdp1(t, y)
a=-2                         %½Õ¾ãa¡Bb-È
b=5
dy = [y(2); -a*y(2)-b*y(1)];
end
function dy = vdp2(t, y)
a=-2                         %½Õ¾ãa¡Bb-È
b=10
dy = [y(2); -a*y(2)-b*y(1)];
end
function dy = vdp3(t, y)
a=-2                         %½Õ¾ãa¡Bb-È
b=20
dy = [y(2); -a*y(2)-b*y(1)];
end
```

圖(1.2.2)

```
clc
close all
clear all
[t, y] = ode45(@vdp1, [0 1], [0.0 0.01]);
a1=arrowPlot(y(:,1), y(:,2), 'number', 5,'color', 'r', 'LineWidth', 1, 'scale', 0.8,
'ratio', 'equal');
hold on;
[t, y] = ode45(@vdp2, [0 1], [0.0 0.01]);
a2=arrowPlot(y(:,1), y(:,2), 'number', 5,'color', 'b', 'LineWidth', 1, 'scale', 0.8,
'ratio', 'equal');
hold on;
[t, y] = ode45(@vdp3, [0 1], [0.0 0.01]);
a3=arrowPlot(y(:,1), y(:,2), 'number', 5,'color', 'black', 'LineWidth', 1, 'scale', 0.8,
'ratio', 'equal');

ax = gca;                    % gets the current axes
ax.XAxisLocation = 'origin';    % sets them to zero
ax.YAxisLocation = 'origin';    % sets them to zero
ax.Box ='off';               % switches off the surrounding box
% ax.XTick = [];   % sets the tick marks
% ax.YTick = [];    % sets the tick marks

% axis([-0.004, 0.003, -0.005, 0.006]);
xlabel('x'); ylabel('x'''');
grid on;
legend([a1 a2 a3 ],'a=-9.5 b=25','a=-9 b=25 ','a=-8.5 b=25');
title('¤£Ã-©wµJÂI(b-È©T©w) ');
%-------------
function dy = vdp1(t, y)
a=-9.5                       %½Õ¾ãa¡Bb-È
b=25
dy = [y(2); -a*y(2)-b*y(1)];
end
function dy = vdp2(t, y)
a=-9                         %½Õ¾ãa¡Bb-È
b=25
dy = [y(2); -a*y(2)-b*y(1)];
end
function dy = vdp3(t, y)
a=-8.5                       %½Õ¾ãa¡Bb-È
b=25
dy = [y(2); -a*y(2)-b*y(1)];
end
```

圖(1.3.1)

```
clc
close all
clear all

[t, y] = ode45(@vdp1, [0 100], [0.001 0.0008]);
arrowPlot(y(:,1), y(:,2), 'number', 2,'color', 'r', 'LineWidth', 1, 'scale', 2, 'ratio',
'equal');
hold on;
[t, y] = ode45(@vdp1, [0 100], [0.002 0.0008]);
arrowPlot(y(:,1), y(:,2), 'number', 2,'color', 'r', 'LineWidth', 1, 'scale', 2, 'ratio',
'equal');
hold on;
[t, y] = ode45(@vdp1, [0 100], [-0.001 -0.0008]);
arrowPlot(y(:,1), y(:,2), 'number', 2,'color', 'r', 'LineWidth', 1, 'scale', 2, 'ratio',
'equal');
hold on;
[t, y] = ode45(@vdp1, [0 100], [-0.002 -0.0008]);
arrowPlot(y(:,1), y(:,2), 'number', 2,'color', 'r', 'LineWidth', 1, 'scale', 2, 'ratio',
'equal');
hold on;
[t, y] = ode45(@vdp1, [0 100], [-0.002 0.0005]);
arrowPlot(y(:,1), y(:,2), 'number', 2,'color', 'r', 'LineWidth', 1, 'scale', 2, 'ratio',
'equal');
hold on;
[t, y] = ode45(@vdp1, [0 100], [0.002 -0.0005]);
arrowPlot(y(:,1), y(:,2), 'number', 2,'color', 'r', 'LineWidth', 1, 'scale', 2, 'ratio',
'equal');
hold on;
line([-0.0025,0.0025],[0.00075,-0.00075])
hold on;
line([-0.0025,0.0025],[0.00038,-0.00038])

ax = gca;                    % gets the current axes
ax.XAxisLocation = 'origin';    % sets them to zero
ax.YAxisLocation = 'origin';    % sets them to zero
ax.Box = 'off';              % switches off the surrounding box
ax.XTick = [];   % sets the tick marks
ax.YTick = [];    % sets the tick marks
axis([-0.003, 0.003, -0.0015, 0.0015]);
xlabel('x'); ylabel('x'');

%-------------
function dy = vdp1(t, y)
a=1                          %½Õ¾ãa¡Bb-È
b=1/8
dy = [y(2); -a*y(2)-b*y(1)];
end
```

圖(1.3.2)

```
clc
close all
clear all

[t, y] = ode45(@vdp1, [0 100], [0.001 0.0008]);
a1=arrowPlot(y(:,1), y(:,2), 'number', 3,'color', 'r', 'LineWidth', 1, 'scale', 1,
'ratio', 'equal');
hold on;
[t, y] = ode45(@vdp2, [0 100], [0.001 0.0008]);
a2=arrowPlot(y(:,1), y(:,2), 'number', 3,'color', 'b', 'LineWidth', 1, 'scale', 1,
'ratio', 'equal');
hold on;
[t, y] = ode45(@vdp3, [0 100], [0.001 0.0008]);
a3=arrowPlot(y(:,1), y(:,2), 'number', 3,'color', 'black', 'LineWidth', 1, 'scale', 1,
'ratio', 'equal');


ax = gca;                    % gets the current axes
ax.XAxisLocation = 'origin';    % sets them to zero
ax.YAxisLocation = 'origin';    % sets them to zero
ax.Box = 'off';               % switches off the surrounding box
% ax.XTick = [];   % sets the tick marks
% ax.YTick = [];    % sets the tick marks

% axis([-0.003, 0.003, -0.0015, 0.0015]);
xlabel('x'); ylabel('x'''');
legend([a1 a2 a3 ],'a=10 b=25','a=10 b=20 ','a=10 b=10');
title('Ã-©w¸`ÂI(a-È©T©w) ');


%-------------
function dy = vdp1(t, y)
a=10
b=25
dy = [y(2); -a*y(2)-b*y(1)];
end
function dy = vdp2(t, y)
a=10
b=20
dy = [y(2); -a*y(2)-b*y(1)];
end
function dy = vdp3(t, y)
a=10
b=10
dy = [y(2); -a*y(2)-b*y(1)];
end
```

圖(1.3.3)

```
clc
close all
clear all

[t, y] = ode45(@vdp1, [0 100], [0.001 0.0008]);
a1=arrowPlot(y(:,1), y(:,2), 'number', 3,'color', 'r', 'LineWidth', 1, 'scale', 1,
'ratio', 'equal');
hold on;
[t, y] = ode45(@vdp2, [0 100], [0.001 0.0008]);
a2=arrowPlot(y(:,1), y(:,2), 'number', 3,'color', 'b', 'LineWidth', 1, 'scale', 1,
'ratio', 'equal');
hold on;
[t, y] = ode45(@vdp3, [0 100], [0.001 0.0008]);
a3=arrowPlot(y(:,1), y(:,2), 'number', 3,'color', 'black', 'LineWidth', 1, 'scale', 1,
'ratio', 'equal');

ax = gca;                    % gets the current axes
ax.XAxisLocation = 'origin';    % sets them to zero
ax.YAxisLocation = 'origin';    % sets them to zero
ax.Box = 'off';               % switches off the surrounding box
% ax.XTick = [];   % sets the tick marks
% ax.YTick = [];    % sets the tick marks

% axis([-0.003, 0.003, -0.0015, 0.0015]);
xlabel('x'); ylabel('x''');
legend([a1 a2 a3 ],'a=10 b=25','a=15 b=25 ','a=20 b=25');
title('Ã-©w¸`ÂI(b-È©T©w) ');


%-------------
function dy = vdp1(t, y)
a=10
b=25
dy = [y(2); -a*y(2)-b*y(1)];
end
function dy = vdp2(t, y)
a=15
b=25
dy = [y(2); -a*y(2)-b*y(1)];
end
function dy = vdp3(t, y)
a=20
b=25
dy = [y(2); -a*y(2)-b*y(1)];
end
```

圖(1.4.1)

```
clc
close all
clear all
[t, y] = ode45(@vdp1, [0 99.3], [10 9.9]);
arrowPlot(y(:,1), y(:,2), 'number', 2,'color', 'r', 'LineWidth', 1, 'scale', 2, 'ratio',
'equal');
hold on;
[t, y] = ode45(@vdp1, [0 99.3], [-10 -9.9]);
arrowPlot(y(:,1), y(:,2), 'number', 2,'color', 'r', 'LineWidth', 1, 'scale', 2, 'ratio',
'equal');
hold on;
[t, y] = ode45(@vdp1, [0 99.2], [10 9.8997]);
arrowPlot(y(:,1), y(:,2), 'number', 2,'color', 'r', 'LineWidth', 1, 'scale', 2, 'ratio',
'equal');
hold on;
[t, y] = ode45(@vdp1, [0 99.2], [-10 -9.8997]);
arrowPlot(y(:,1), y(:,2), 'number', 2,'color', 'r', 'LineWidth', 1, 'scale', 2, 'ratio',
'equal');
hold on;
[t, y] = ode45(@vdp1, [0 94.3], [10 10]);
arrowPlot(y(:,1), y(:,2), 'number', 2,'color', 'r', 'LineWidth', 1, 'scale', 2, 'ratio',
'equal');
hold on;
[t, y] = ode45(@vdp1, [0 94.3], [-10 -10]);
arrowPlot(y(:,1), y(:,2), 'number', 2,'color', 'r', 'LineWidth', 1, 'scale', 2, 'ratio',
'equal');
hold on;
line([-1e+42,1e+42],[-0.85e+42,0.85e+42])
hold on;
line([-1e+42,1e+42],[-1.2e+42,1.2e+42])

ax = gca;                    % gets the current axes
ax.XAxisLocation = 'origin';    % sets them to zero
ax.YAxisLocation = 'origin';    % sets them to zero
ax.Box = 'off';              % switches off the surrounding box
ax.XTick = [];   % sets the tick marks
ax.YTick = [];    % sets the tick marks

axis([-1.4e+42, 1.4e+42, -1.4e+42, 1.4e+42]);
xlabel('x'); ylabel('x'');


%-------------
function dy = vdp1(t, y)
```

```
a=-2                    %½Õ¾ãa¡Bb-È
b=1
dy = [y(2); -a*y(2)-b*y(1)];
end
```

圖(1.4.2)

```
clc
close all
clear all
[t, y] = ode45(@vdp1, [0 1], [0 5]);
a1=arrowPlot(y(:,1), y(:,2), 'number', 3,'color', 'r', 'LineWidth', 1, 'scale', 1,
'ratio', 'equal');
hold on;
[t, y] = ode45(@vdp2, [0 1], [0 5]);
a2=arrowPlot(y(:,1), y(:,2), 'number', 3,'color', 'b', 'LineWidth', 1, 'scale', 1,
'ratio', 'equal');
hold on;
[t, y] = ode45(@vdp3, [0 1], [0 5]);
a3=arrowPlot(y(:,1), y(:,2), 'number', 3,'color', 'black', 'LineWidth', 1, 'scale', 1,
'ratio', 'equal');

ax = gca;                    % gets the current axes
ax.XAxisLocation = 'origin';    % sets them to zero
ax.YAxisLocation = 'origin';    % sets them to zero
ax.Box = 'off';              % switches off the surrounding box
% ax.XTick = [];  % sets the tick marks
% ax.YTick = [];   % sets the tick marks
% axis([-1.4e+42, 1.4e+42, -1.4e+42, 1.4e+42]);
xlabel('x'); ylabel('x'''');
xlabel('x'); ylabel('x'''');
legend([a1 a2 a3 ],'a=-2 b=1','a=-2 b=1/2 ','a=-2 b=1/4');
title('¤£Ã-©w¸`ÂI(a-È©T©w) ');

%-------------
function dy = vdp1(t, y)
a=-2
b=1
dy = [y(2); -a*y(2)-b*y(1)];
end
function dy = vdp2(t, y)
a=-2
b=1/2
dy = [y(2); -a*y(2)-b*y(1)];
end
function dy = vdp3(t, y)
a=-2
b=1/4
dy = [y(2); -a*y(2)-b*y(1)];
end
```

圖(1.4.3)

```
clc
close all
clear all
[t, y] = ode45(@vdp1, [0 0.1], [0 0.0001]);
a1=arrowPlot(y(:,1), y(:,2), 'number', 3,'color', 'r', 'LineWidth', 1, 'scale', 2,
'ratio', 'equal');
hold on;
[t, y] = ode45(@vdp2, [0 0.1], [0 0.0001]);
a2=arrowPlot(y(:,1), y(:,2), 'number', 3,'color', 'b', 'LineWidth', 1, 'scale', 2,
'ratio', 'equal');
hold on;
[t, y] = ode45(@vdp3, [0 0.1], [0 0.0001]);
a3=arrowPlot(y(:,1), y(:,2), 'number', 3,'color', 'black', 'LineWidth', 1, 'scale', 2,
'ratio', 'equal');

ax = gca;                    % gets the current axes
ax.XAxisLocation = 'origin';    % sets them to zero
ax.YAxisLocation = 'origin';    % sets them to zero
ax.Box = 'off';               % switches off the surrounding box
% ax.XTick = [];   % sets the tick marks
% ax.YTick = [];    % sets the tick marks
% axis([-1.4e+42, 1.4e+42, -1.4e+42, 1.4e+42]);
xlabel('x'); ylabel('x'');
xlabel('x'); ylabel('x'');
legend([a1 a2 a3 ],'a=-2 b=1','a=-3 b=1 ','a=-4 b=1');
title('¤£Ã-©w¸`ÂI(b-È©T©w) ');

%-------------
function dy = vdp1(t, y)
a=-2
b=1
dy = [y(2); -a*y(2)-b*y(1)];
end
function dy = vdp2(t, y)
a=-3
b=1
dy = [y(2); -a*y(2)-b*y(1)];
end
function dy = vdp3(t, y)
a=-4
b=1
dy = [y(2); -a*y(2)-b*y(1)];
end
```

圖(1.5.1)

```
clc
close all
clear all
[t, y] = ode45(@vdp1, [0 9], [0 20]);
arrowPlot(y(:,1), y(:,2), 'number', 4,'color', 'r', 'LineWidth', 1, 'scale', 2, 'ratio',
'equal');
hold on;
[t, y] = ode45(@vdp1, [0 9], [0 10]);
arrowPlot(y(:,1), y(:,2), 'number', 4,'color', 'r', 'LineWidth', 1, 'scale', 2, 'ratio',
'equal');

ax = gca;                    % gets the current axes
ax.XAxisLocation = 'origin';    % sets them to zero
ax.YAxisLocation = 'origin';    % sets them to zero
ax.Box = 'off';              % switches off the surrounding box
ax.XTick = [];   % sets the tick marks
ax.YTick = [];    % sets the tick marks
axis([-40, 40, -40, 40]);
xlabel('x'); ylabel('x'''');
%-------------
function dy = vdp1(t, y)
a=0                  %½Õ¾ãa¡Bb-È
b=0.5
dy = [y(2); -a*y(2)-b*y(1)];
end
```

圖(1.5.2)

```
clc
close all
clear all
[t, y] = ode45(@vdp1, [0 1], [0 10]);
a1=arrowPlot(y(:,1), y(:,2), 'number', 4,'color', 'r', 'LineWidth', 1, 'scale', 1.5,
'ratio', 'equal');
hold on;
[t, y] = ode45(@vdp2, [0 1], [0 10]);
a2=arrowPlot(y(:,1), y(:,2), 'number', 4,'color', 'b', 'LineWidth', 1, 'scale', 1.5,
'ratio', 'equal');
hold on;
[t, y] = ode45(@vdp3, [0 1], [0 10]);
a3=arrowPlot(y(:,1), y(:,2), 'number', 4,'color', 'black', 'LineWidth', 1, 'scale', 1.5,
'ratio', 'equal');

ax = gca;                    % gets the current axes
ax.XAxisLocation = 'origin';    % sets them to zero
ax.YAxisLocation = 'origin';    % sets them to zero
ax.Box = 'off';              % switches off the surrounding box
% ax.XTick = [];   % sets the tick marks
% ax.YTick = [];    % sets the tick marks
% axis([-40, 40, -40, 40]);
xlabel('x'); ylabel('x''');
legend([a1 a2 a3 ],'a=0 b=1','a=0 b=2 ','a=0 b=3');
title('¤¤¤ßÂI(a=0) ');
%-------------
function dy = vdp1(t, y)
a=0                  %½Õ¾ãa¡Bb-È
b=1
dy = [y(2); -a*y(2)-b*y(1)];
end
function dy = vdp2(t, y)
a=0                  %½Õ¾ãa¡Bb-È
b=2
dy = [y(2); -a*y(2)-b*y(1)];
end
function dy = vdp3(t, y)
a=0                  %½Õ¾ãa¡Bb-È
b=3
dy = [y(2); -a*y(2)-b*y(1)];
end
```

圖(1.6.1)

```
clc
close all
clear all


[t, y] = ode45(@vdp1, [0 41.9], [-10 10]);
arrowPlot(y(:,1), y(:,2), 'number', 4,'color', 'black', 'LineWidth', 1, 'scale', 1.5,
'ratio', 'equal');
hold on;

grid on
ax = gca;                    % gets the current axes
ax.XAxisLocation = 'origin';    % sets them to zero
ax.YAxisLocation = 'origin';    % sets them to zero
ax.Box = 'off';               % switches off the surrounding box
ax.XTick = [];   % sets the tick marks
ax.YTick = [];    % sets the tick marks

axis([-13, 13, -13, 13]);
xlabel('x'); ylabel('x'');


%-------------
function dy = vdp1(t, y)
a=0.02
b=-0.98
dy = [y(2); -a*y(2)-b*y(1)];
end
```

圖(1.6.2)

```
clc
close all
clear all

[t, y] = ode45(@vdp1, [0 40], [-10 10]);
a1=arrowPlot(y(:,1), y(:,2), 'number', 4,'color', 'r', 'LineWidth', 1, 'scale', 1.5,
'ratio', 'equal');
hold on;
[t, y] = ode45(@vdp2, [0 40], [-10 10]);
a2=arrowPlot(y(:,1), y(:,2), 'number', 4,'color', 'b', 'LineWidth', 1, 'scale', 1.5,
'ratio', 'equal');
hold on;
[t, y] = ode45(@vdp3, [0 40], [-10 10]);
a3=arrowPlot(y(:,1), y(:,2), 'number', 4,'color', 'black', 'LineWidth', 1, 'scale', 1.5,
'ratio', 'equal');
grid on
ax = gca;                    % gets the current axes
ax.XAxisLocation = 'origin';    % sets them to zero
ax.YAxisLocation = 'origin';    % sets them to zero
ax.Box = 'off';               % switches off the surrounding box
ax.XTick = [];   % sets the tick marks
ax.YTick = [];    % sets the tick marks

axis([-13, 13, -13, 13]);
xlabel('x'); ylabel('x''');
legend([a1 a2 a3 ],'a=0.02 b=-0.98','a=0.02 b=-1 ','a=0.02 b=-1.1');
title('¾bÂI(a-È©T©w)) ');
%-------------
function dy = vdp1(t, y)
a=0.02
b=-0.98
dy = [y(2); -a*y(2)-b*y(1)];
end
function dy = vdp2(t, y)
a=0.02
b=-1
dy = [y(2); -a*y(2)-b*y(1)];
end
function dy = vdp3(t, y)
a=0.02
b=-1.1
dy = [y(2); -a*y(2)-b*y(1)];
end
```

圖(1.6.3)

```
clc
close all
clear all

[t, y] = ode45(@vdp1, [0 40], [-10 10]);
a1=arrowPlot(y(:,1), y(:,2), 'number', 4,'color', 'r', 'LineWidth', 1, 'scale', 1.5,
'ratio', 'equal');
hold on;
[t, y] = ode45(@vdp2, [0 40], [-10 10]);
a2=arrowPlot(y(:,1), y(:,2), 'number', 4,'color', 'b', 'LineWidth', 1, 'scale', 1.5,
'ratio', 'equal');
hold on;
[t, y] = ode45(@vdp3, [0 40], [-10 10]);
a3=arrowPlot(y(:,1), y(:,2), 'number', 4,'color', 'black', 'LineWidth', 1, 'scale', 1.5,
'ratio', 'equal');
grid on
ax = gca;                    % gets the current axes
ax.XAxisLocation = 'origin';    % sets them to zero
ax.YAxisLocation = 'origin';    % sets them to zero
ax.Box = 'off';               % switches off the surrounding box
ax.XTick = [];   % sets the tick marks
ax.YTick = [];    % sets the tick marks

axis([-13, 13, -13, 13]);
xlabel('x'); ylabel('x''');
legend([a1 a2 a3 ],'a=0.02 b=-0.98','a=0.04 b=-0.98 ','a=0.08 b=-0.98');
title('¾bÂI(b-È©T©w)) ');

%-------------
function dy = vdp1(t, y)
a=0.02
b=-0.98
dy = [y(2); -a*y(2)-b*y(1)];
end
function dy = vdp2(t, y)
a=0.04
b=-0.98
dy = [y(2); -a*y(2)-b*y(1)];
end
function dy = vdp3(t, y)
a=0.08
b=-0.98
dy = [y(2); -a*y(2)-b*y(1)];
end
```

圖(2.1)

```
clc
close all
clear all
[t,y] = ode45(@vdp1, [0 2.7], [0 1.002]);
arrowPlot(y(:,1), y(:,2), 'number', 5,'color', 'r',
'LineWidth', 2, 'scale', 1.5, 'ratio', 'equal');
hold on;
[t,y] = ode45(@vdp1, [0 2.7], [1.002 0]);
arrowPlot(y(:,1), y(:,2), 'number', 5,'color', 'g',
'LineWidth', 2, 'scale', 1.5, 'ratio', 'equal');
hold on;
[t,y] = ode45(@vdp1, [0 2.7], [-1.002 0]);
arrowPlot(y(:,1), y(:,2), 'number', 5,'color', 'b',
'LineWidth', 2, 'scale', 1.5, 'ratio', 'equal');
hold on;
[t,y] = ode45(@vdp1, [0 2.7], [0 -1.002]);
arrowPlot(y(:,1), y(:,2), 'number', 5,'color', 'm',
'LineWidth', 2, 'scale', 1.5, 'ratio', 'equal');
hold on;
[t,y] = ode45(@vdp1, [0 2.7],  [0.68 0.68]);
arrowPlot(y(:,1), y(:,2), 'number', 5,'color', 'black',
'LineWidth', 2, 'scale', 1.5, 'ratio', 'equal');

ax = gca;                          % gets the current axes
ax.XAxisLocation = 'origin';     % sets them to zero
ax.YAxisLocation = 'origin';     % sets them to zero
ax.Box ='off';                    % switches off the
surrounding box
% ax.XTick = [];   % sets the tick marks
% ax.YTick = [];    % sets the tick marks

axis([-3.5, 3.5, -3.5, 3.5]);
xlabel('x'); ylabel('x''');
grid on;
title('?Ã-©wªº·¥--¶ê ');

%-------------
function dydt = vdp1(t, y)
dydt=zeros(2,1)
dydt(1)=y(2)+y(1)*(y(1)^2+y(2)^2-1);
dydt(2)=-y(1)+y(2)*(y(1)^2+y(2)^2-1);
end
```

圖(2.2)

```
clc
close all
clear all
[t,y] = ode45(@vdp1,  [0 10], [-1 1]);
arrowPlot(y(:,1), y(:,2), 'number', 3,'color', 'r',
'LineWidth', 1, 'scale', 1, 'ratio', 'equal');
hold on;
[t,y] = ode45(@vdp1,  [0 100], [0.6 -0.6]);
arrowPlot(y(:,1), y(:,2), 'number', 3,'color', 'b',
'LineWidth', 1, 'scale', 1, 'ratio', 'equal');

ax = gca;                          % gets the current axes
ax.XAxisLocation = 'origin';     % sets them to zero
ax.YAxisLocation = 'origin';     % sets them to zero
ax.Box ='off';                    % switches off the
surrounding box
% ax.XTick = [];   % sets the tick marks
% ax.YTick = [];    % sets the tick marks

% axis([-2, 2, -2, 2]);
xlabel('x'); ylabel('x''');
grid on;
title('Ã-©wªº·¥--¶ê ');

%-------------
function dydt = vdp1(t, y)
dydt=zeros(2,1)
dydt(1)=y(2)-y(1)*(y(1)^2+y(2)^2-1)^2;
dydt(2)=-y(1)-y(2)*(y(1)^2+y(2)^2-1)^2;
end
```

圖(2.3)

```
clc
close all
clear all
[t,y] = ode45(@vdp1,  [0 5], [2 -2]);
arrowPlot(y(:,1), y(:,2), 'number', 3,'color', 'r',
'LineWidth', 1, 'scale', 1, 'ratio', 'equal');
hold on;
[t,y] = ode45(@vdp1,  [0 5], [-2 2]);
arrowPlot(y(:,1), y(:,2), 'number', 3,'color', 'g',
'LineWidth', 1, 'scale', 1, 'ratio', 'equal');
hold on;
[t,y] = ode45(@vdp1,  [0 5], [2 2]);
arrowPlot(y(:,1), y(:,2), 'number', 3,'color', 'b',
'LineWidth', 1, 'scale', 1, 'ratio', 'equal');
hold on;
[t,y] = ode45(@vdp1,  [0 5], [-2 -2]);
arrowPlot(y(:,1), y(:,2), 'number', 3,'color', 'm',
'LineWidth', 1, 'scale', 1, 'ratio', 'equal');
hold on;
[t,y] = ode45(@vdp1,  [0 5], [0.1 0.1]);
arrowPlot(y(:,1), y(:,2), 'number', 3,'color', 'c',
'LineWidth', 1, 'scale', 1, 'ratio', 'equal');
hold on;

[t,y] = ode45(@vdp1, [0 5], [-0.1 -0.1]);
arrowPlot(y(:,1), y(:,2), 'number', 3,'color', 'k',
'LineWidth', 1, 'scale', 1, 'ratio', 'equal');


ax = gca;                          % gets the current axes
ax.XAxisLocation = 'origin';     % sets them to zero
ax.YAxisLocation = 'origin';     % sets them to zero
ax.Box ='off';                    % switches off the
surrounding box
% ax.XTick = [];   % sets the tick marks
% ax.YTick = [];    % sets the tick marks

% axis([-2, 2, -2, 2]);
xlabel('x'); ylabel('x''');
grid on;
title('Ã-©wªº·¥--¶ê ');

%-------------
function dydt = vdp1(t, y)
dydt=zeros(2,1)
dydt(1)=y(2)-y(1)*(y(1)^2+y(2)^2-1);
dydt(2)=-y(1)-y(2)*(y(1)^2+y(2)^2-1);
end
```

圖(3.1)

```
clear all
close all
clc
% part 1
[t,x] = ode45(@vdp2,[0 10],[2 0])
for i = 1:length(x(:,1))
    if x(i,1) >= 0.2
       a(i,1) = x(i,1)
       a(i,2) = x(i,2)
    else
        break
    end
end
x2_init =
interp1(x(:,1),x(:,2),0.2)
n = length(a(:,1))
a(n+1,1) = 0.2
a(n+1,2) = x2_init
a1=plot(a(:,1),a(:,2),'r')
hold on

% part 2
[t,x] = ode45(@vdp1,[0 10],[0.2
x2_init])
for i = 1:length(x(:,1))
    if x(i,1) <= 0.2 && x(i,1) >= -
0.2
        b(i,1) = x(i,1)
        b(i,2) = x(i,2)
    else
        break
    end
end
x2_init = interp1(x(:,1),x(:,2),-
0.2)
n = length(b(:,1))
b(n+1,1) = -0.2
b(n+1,2) = x2_init
a2=plot(b(:,1),b(:,2),'b')

% part3
[t,x] = ode45(@vdp3,[0 10],[-0.2
x2_init])
for i = 1:length(x(:,1))
    if x(i,1) <= -0.2
        c(i,1) = x(i,1)
        c(i,2) = x(i,2)
    else
        break
    end
end
m = length(x(:,1))
x2_init = interp1([-
0.213129004914689;-
0.189069522520330],[0.1197916618204
87;0.211315897170702],-0.2)
n = length(c(:,1))
c(n+1,1) = -0.2
c(n+1,2) = x2_init
a3=plot(c(:,1),c(:,2),'black')

% part4
[t,x] = ode45(@vdp1,[0 10],[-0.2
x2_init])
for i = 1:length(x(:,1))
    if x(i,1) <= 0.2 & x(i,1) >= -
0.2
        d(i,1) = x(i,1)
        d(i,2) = x(i,2)
    else
        break
    end
end
a4=plot(d(:,1),d(:,2),'m')

hold on;
plot([0.2,0.2],[0.5,-1],'r--
','Linewidth',1.5)
hold on;
plot([-0.2,-0.2],[0.5,-1],'r--
','Linewidth',1.5)

xticks([-0.5 -0.2 0 0.2 0.5 1 1.5
2])
xlabel('e')
ylabel("e'")
grid on
title('¹¡©M¨t²Î¤§¬Û¥--±-y¸ñ¹Ï ');
legend([a1 a2 a3
a4],'part1','part2','part3','part4'
);
%----------------------------------
------
function dxdt = vdp1(t,x) % -e0 < e
< e0
dxdt = [x(2);-x(2)-4*x(1)]
end
function dxdt = vdp2(t,x) % e >= e0
dxdt = [x(2);-x(2)-0.8]
end
function dxdt = vdp3(t,x) % e <= -
e0
dxdt = [x(2);-x(2)+0.8]
end
```