非線性控制第九章作業

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Consider the following system

$$\dot{y} + a_p y + c_p \sin y = b_p u$$

We're going to design adaptive control u in order to track the following linear model reference, with the system parameters being unknown:

$$\dot{y}_m + 4y_m = 4r$$

(a)

The control signal is written as

$$u = K_y y + K_f \sin y + K_r r$$

Assuming the parameters $a_p,\ b_p,\ c_p$ are known, we're going to acquire control law parameters $K_y^*,\ K_f^*,\ K_r^*$ which will ensure the transfer function of both $r \to y$ and $r \to y_m$ being identical.

Plugging u into the original system gives

$$\dot{y} + a_p y + c_p \sin y = b_p \left(K_y y + K_f \sin y + K_r r \right)$$

$$\Rightarrow \dot{y} + \left(a_p - b_p K_y \right) y + \left(c_p - b_p K_f \right) \sin y = b_p K_r r$$

We can then obtain the parameters $K_y^*,\ K_f^*,\ K_r^*$ by comparing this with the model reference:

$$\begin{cases} a_p - b_p K_y^* = 4 \\ c_p - b_p K_f^* = 0 \\ b_p K_r^* = 4 \end{cases} \Rightarrow \begin{cases} K_y^* = \dfrac{a_p - 4}{b_p} \\ K_f^* = \dfrac{c_p}{b_p} \\ K_r^* = \dfrac{4}{b_p} \end{cases}$$

(b)

Unlike in (a), we actually don't know the parameters a_p, b_p, c_p, therefore, we're going to change the control law into

$$u = \hat{K}_y(t)y + \hat{K}_f(t)\sin y + \hat{K}_r(t)r$$

where $\hat{K}_y(t)$, $\hat{K}_f(t)$, $\hat{K}_r(t)$ stand for the estimation of K_y^*, K_f^*, K_r^* , respectively.

Define the tracking error and parameters error as

$$e = y - y_m,\ \bar{K}_y = \hat{K}_y - K_y^*,\ \bar{K}_f = \hat{K}_f - K_f^*,\ \bar{K}_r = \hat{K}_r - K_r^*$$

We can then express the estimated parameters as follows:

$$\hat{K}_y(t) = \bar{K}_y + K_y^* = \bar{K}_y + \frac{a_p - 4}{b_p}$$

$$\hat{K}_f(t) = \bar{K}_f + K_f^* = \bar{K}_f + \frac{c_p}{b_p}$$

$$\hat{K}_r(t) = \bar{K}_r + K_r^* = \bar{K}_r + \frac{4}{b_p}$$

Plugging this into the system gives

$$\dot{y} + \left(a_p - b_p\left(\bar{K}_y + \frac{a_p - 4}{b_p}\right)\right)y + \left(c_p - b_p\left(\bar{K}_f + \frac{c_p}{b_p}\right)\right)\sin y = b_p\left(\bar{K}_r + \frac{4}{b_p}\right)r$$

$$\Rightarrow \dot{y} + \left(4 - b_p\bar{K}_y\right)y + \left(-b_p\bar{K}_f\right)\sin y = \left(b_p\bar{K}_r + 4\right)r$$

Afterwards, we apply the definition of tracking error:

$$e = y - y_m \Rightarrow \dot{e} = \dot{y} - \dot{y}_m = \dot{y} - \left(-4y_m + 4r\right)$$

$$\Rightarrow \dot{e} = \left(b_p\bar{K}_y - 4\right)y + b_p\bar{K}_f\sin y + \left(b_p\bar{K}_r + 4\right)r + 4y_m - 4r$$

$$\Rightarrow \dot{e} = -4\left(y - y_m\right) + b_p\bar{K}_y y + b_p\bar{K}_f\sin y + b_p\bar{K}_r r$$

$$\Rightarrow \dot{e} = -4e + b_p\left(\bar{K}_y y + \bar{K}_f\sin y + \bar{K}_r r\right)$$

The equation above shows the relationship between tracking error and parameters error. Now, we need to have three other differential equations since there are four time variables but only one equation.

We can do that by assuming

$$
\begin{aligned}
\dot{\bar{K}}_y &= f_1\left(e,\bar{K}_y,\bar{K}_f,\bar{K}_r\right) \\
\dot{\bar{K}}_f &= f_2\left(e,\bar{K}_y,\bar{K}_f,\bar{K}_r\right) \\
\dot{\bar{K}}_r &= f_3\left(e,\bar{K}_y,\bar{K}_f,\bar{K}_r\right)
\end{aligned}
$$

where f_1, f_2, f_3 are undecided parameters.

To make the tracking error $e(t)\to 0$, we need to apply Lyapunov stability theorem for choosing proper f_1, f_2, f_3.

For the Lyapunov function, we choose

$$
V\left(e,\bar{K}_y,\bar{K}_f,\bar{K}_r\right)=\frac{e^2}{2}+\frac{1}{2}\left|b_p\right|\left(\frac{\bar{K}_y^2}{\gamma_1}+\frac{\bar{K}_f^2}{\gamma_2}+\frac{\bar{K}_r^2}{\gamma_3}\right)\geq 0
$$

where $\gamma_i>0$ are free parameters (自由選定參數).

Then, taking the derivative of V gives

$$
\begin{aligned}
\dot{V} &= e\dot{e}+\left|b_p\right|\left(\frac{\bar{K}_y}{\gamma_1}\dot{\bar{K}}_y+\frac{\bar{K}_f}{\gamma_2}\dot{\bar{K}}_f+\frac{\bar{K}_r}{\gamma_3}\dot{\bar{K}}_r\right) \\
&= e\left[-4e+b_p\left(\bar{K}_y y+\bar{K}_f\sin y+\bar{K}_r r\right)\right]+\left|b_p\right|\left(\frac{\bar{K}_y}{\gamma_1}f_1+\frac{\bar{K}_f}{\gamma_2}f_2+\frac{\bar{K}_r}{\gamma_3}f_3\right)
\end{aligned}
$$

In order to satisfy the condition $\dot{V}\leq 0$, we can set f_1, f_2, f_3 as

$$
\begin{aligned}
f_1 &= -\operatorname{sgn}\left(b_p\right)\gamma_1 e y \\
f_2 &= -\operatorname{sgn}\left(b_p\right)\gamma_2 e\sin y \\
f_3 &= -\operatorname{sgn}\left(b_p\right)\gamma_3 e r
\end{aligned}
$$

Plugging this back into the equation gives

$$
\dot{V}=-4e^2\leq 0
$$

Therefore, we know that when $\dot{V}=0$, we can ensure $e=0$, but we can't promise that $\bar{K}_y$, $\bar{K}_f$, $\bar{K}_r$ will be as well. This means that this kind of gain-adjusting mode can achieve our goal of $e(t)=y(t)-y_m(t)\to 0$ but can't perfectly estimate the parameters K_y^*, K_f^*, K_r^*.

(c)

Let $a_p = b_p = 1,\ c_p = -1$, the system then becomes

$$\dot{y} = -y + \sin(y) + u,\ y(0) = 0$$

$$u = K_y y + K_f \sin y + K_r r$$

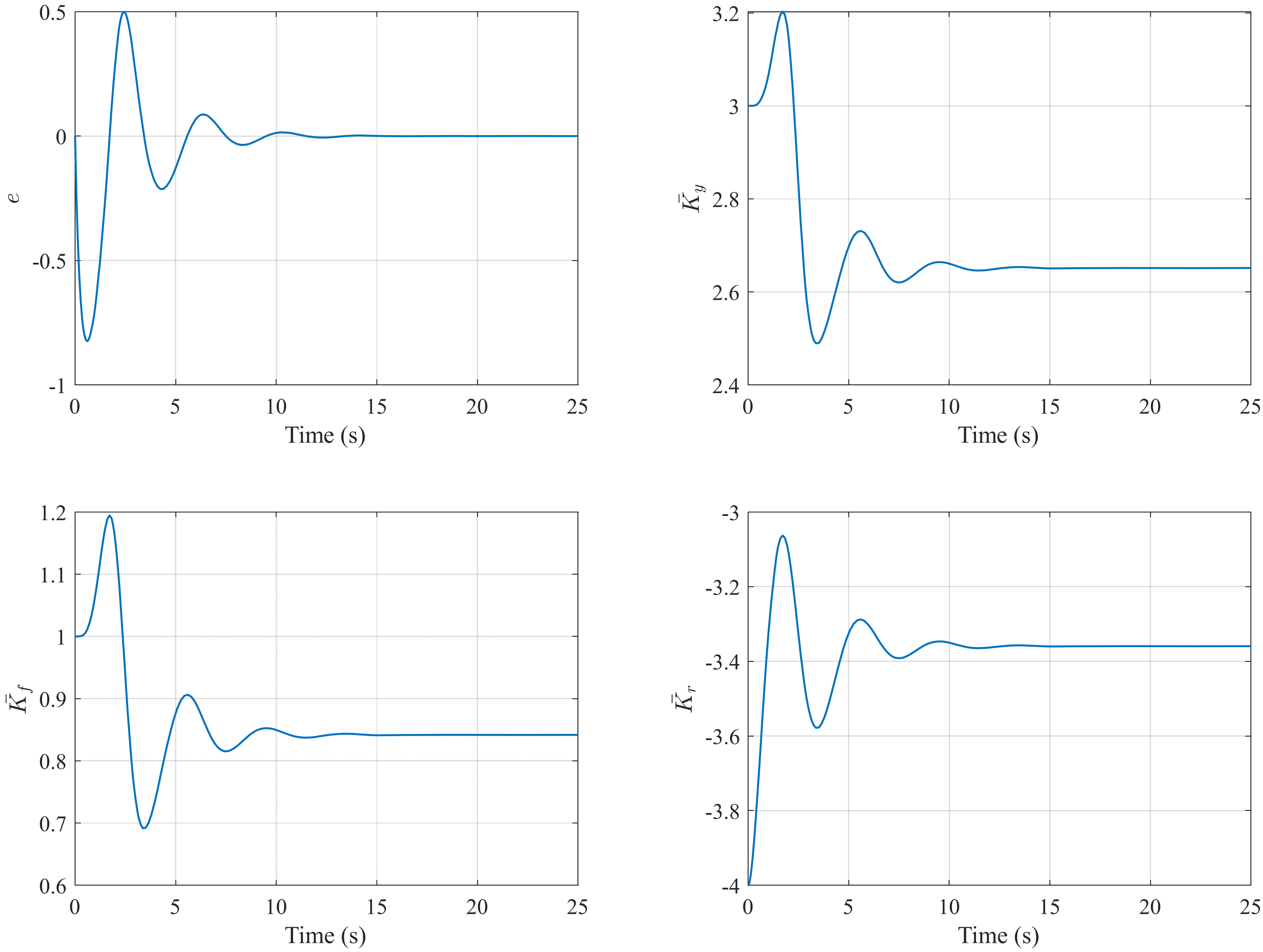
where $K_y = \dfrac{a_p - 4}{b_p} = -3,\ K_f = \dfrac{c_p}{b_p} = -1,\ K_r = \dfrac{4}{b_p} = 4$

Alongside with our first ODE, we also have the rest of the four ODE:

$$\begin{aligned}
\dot{y}_m &= -4y_m + 4r,\ y_m(0) = 0 \\
\dot{\hat{K}}_y &= -ey,\ \hat{K}_y(0) = 0 \\
\dot{\hat{K}}_f &= -e\cos(y),\ \hat{K}_f(0) = 0 \\
\dot{\hat{K}}_r &= -er,\ \hat{K}_r(0) = 0
\end{aligned}$$

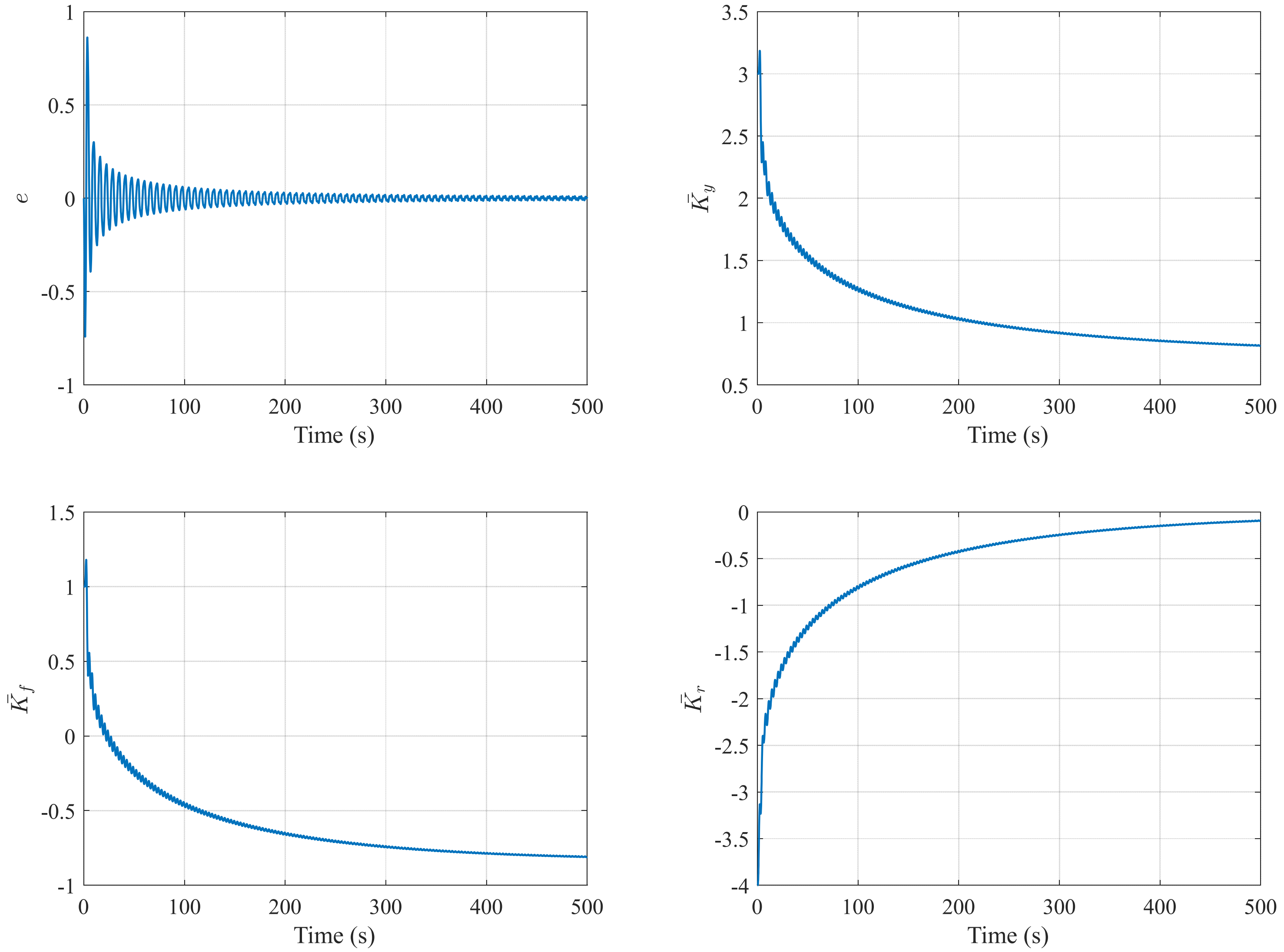
where $e = y - y_m$, and $\gamma_i = 1$ is chosen here.
Now, we're going to use Matlab to perform numerical simulation and attest our result with a couple of different input $r(t)$.

Firstly, $r(t)=1$ is applied.



From the figures, we can see that tracking error $e=y-y_m \to 0$ is successfully achieved. On the other hand, all the parameters error $\bar{K}_y$, $\bar{K}_f$, $\bar{K}_r$ doesn't converge to 0. There exists a difference between the real value of the parameters and our estimation of them, which means that we will never estimate the parameters perfectly when applying step input $r(t)=1$.

Secondly, we use $r(t)=\sin(t)$ instead to see how this will affect the error.

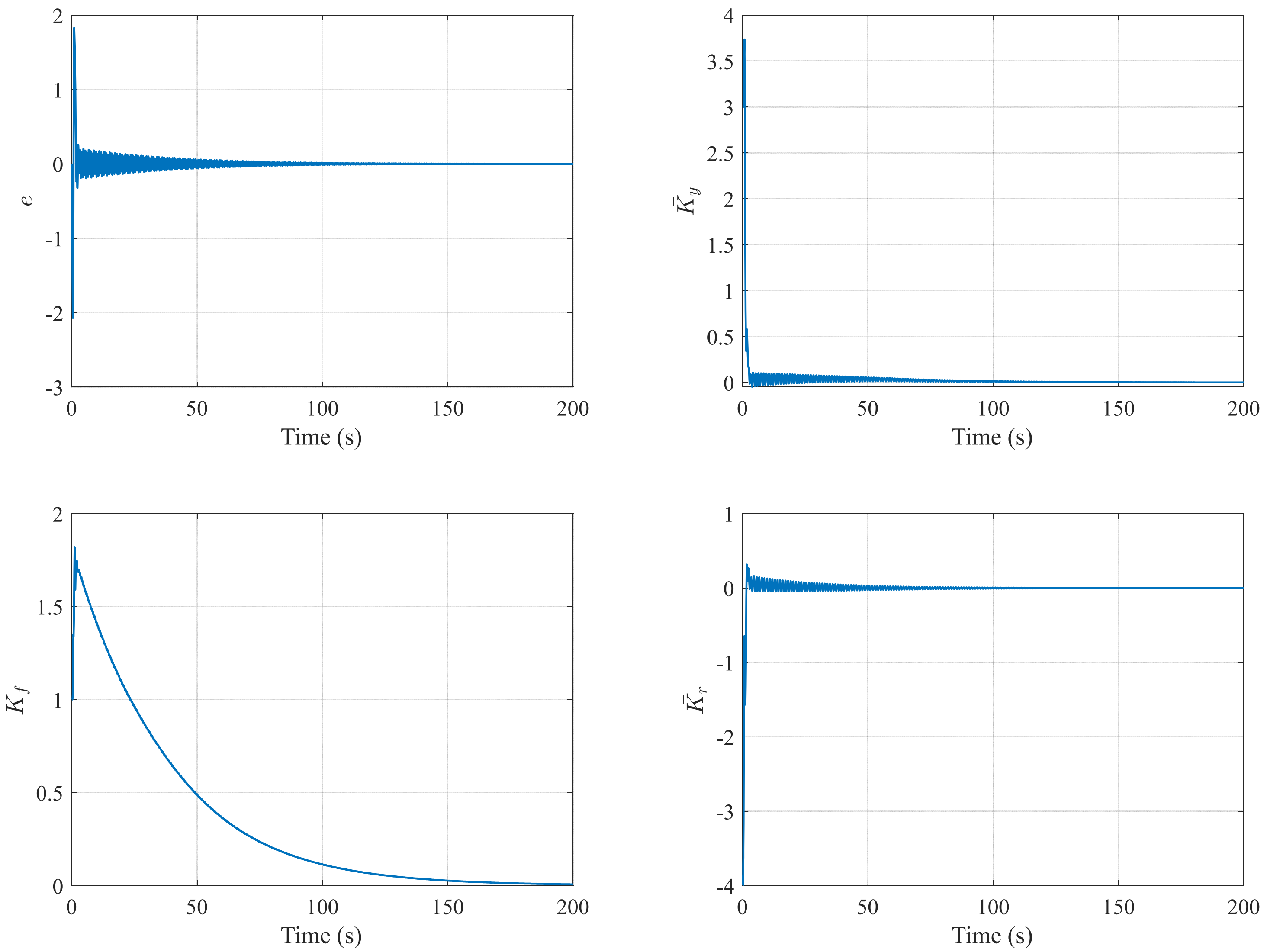


From the figures, we can see that when using $r(t)=\sin(t)$, although it takes much longer for tracking error to converge to 0, it still successfully achieves that at the end. What's more, all three of the parameters error $\bar{K}_y$, $\bar{K}_f$, $\bar{K}_r$ are a lot closer to 0. For $\bar{K}_r$, it can even estimate the parameter pretty correctly since the error is almost 0.

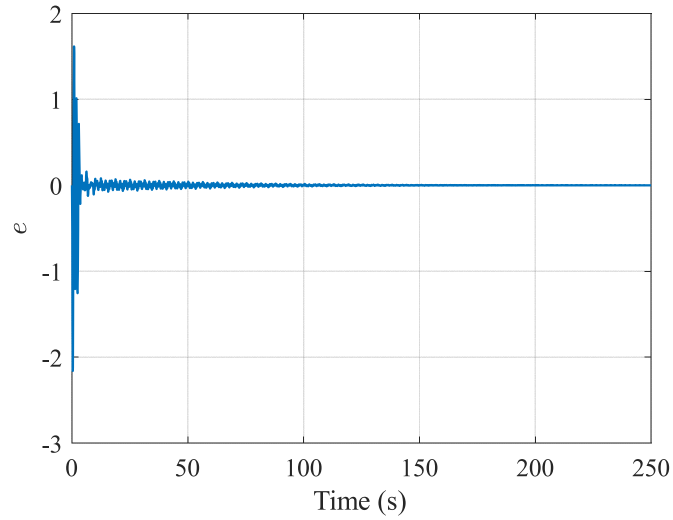
When we compare the two different input $r(t)=1$ and $r(t)=\sin(t)$, both tracking error will satisfy the condition $e \to 0$. In terms of parameters error $\bar{K}_y$, $\bar{K}_f$, $\bar{K}_r$, they won't be as close to 0 like tracking error does, but they do become closer to 0 when using $r(t)=\sin(t)$.

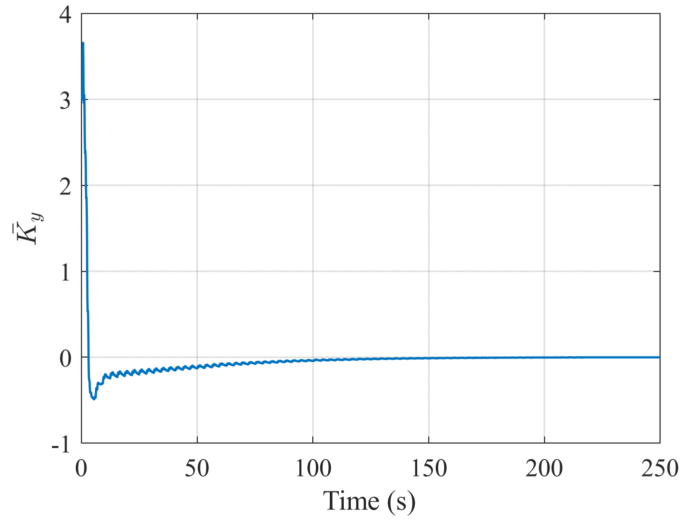
Before jumping into conclusion, I'm going to try out different input that's more "complicated" than $r(t)=\sin(t)$ such as changing the amplitude and frequency and even combine them together, hoping to estimate the error more quickly as well as accurately.

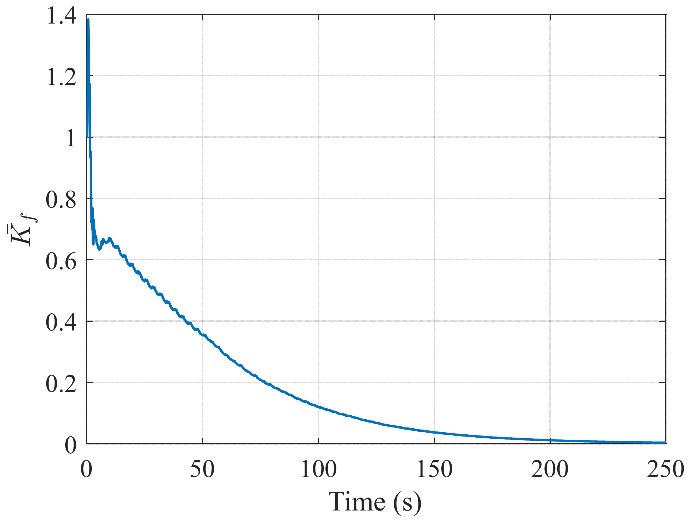
$r(t)=5\sin(t)$:

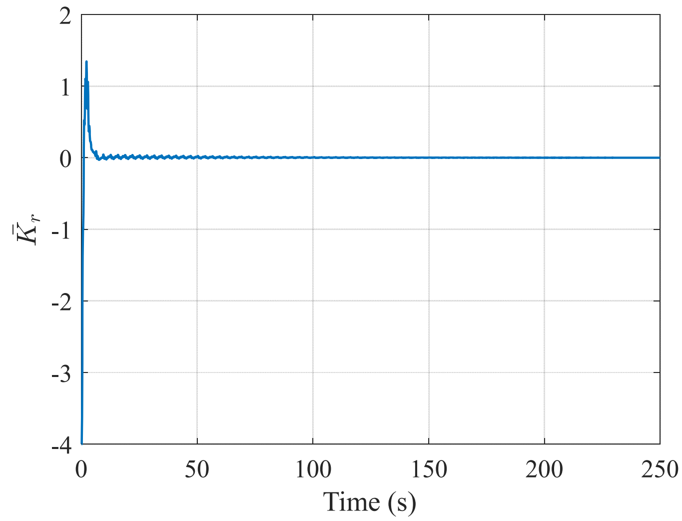


And also $r(t) = 4\sin(5t) + \sin(3t)$:









Finally, we can infer that when we're trying to estimate the parameters, we can do it better by using a more "complicated" signal to "excite" the system at higher frequency and therefore estimate the parameters more correctly. When we used the step input $r(t) = 1$, it's just too simple and quite boring actually, and as we saw in the figures, our estimation of parameters weren't even close.

When looking at the figures, we see that both $\bar{K}_y$ and $\bar{K}_r$ achieved 0 faster than $\bar{K}_f$ does. There are several possibilities for this situation. Anyhow, all three of them as well as tracking error will converge to 0 when using more complicated signals, as shown above, and complete our goal.