非線性控制第八章作業

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Consider the following 2nd order nonlinear system:

$$\dot{x}_1 = x_2 + \theta_1 x_1 \sin x_x$$
$$\dot{x}_2 = \theta_2 x_2^2 + x_1 + u$$

where u is control signal; θ_1 and θ_2 are uncertain parameters with the condition

$$|\theta_1| \le a,\ |\theta_2| \le b$$

Our goal is to design sliding mode control that makes the system above satisfies Lyapunov stable.

Assume the sliding surface as

$$S = (1+a)x_1 + x_2$$

Remark. Sliding surface is the key to sliding control. It's a curve or a surface in the phase space which represents the final state for the control system.

(a)

We first take the derivative of S:

$$\begin{aligned}\dot{S} &= (1+a)\dot{x}_1 + \dot{x}_2 \\ &= (1+a)(x_2 + \theta_1 x_1 \sin x_2) + (\theta_2 x_2^2 + x_1 + u)\end{aligned}$$

Since there exist uncertain parameters

$$-a \le \theta_1 \le a \quad \text{and} \quad -b \le \theta_2 \le b,$$

we can express the control law u as

$$u = u_{eq} - \beta(x)\,\mathrm{sgn}(S)$$

where u_{eq} can be obtained by assuming $\theta_1 = 0,\ \theta_2 = 0$. The reason we do this is because u_{eq} is the "average" control law which maintains a balance system when the real parameters $\theta_{1,2}$ are at their average point $\theta_{1,2} = 0$.

Note that the definition of the sign function is

$$\operatorname{sgn}(S)=\begin{cases}1, & S>0\\ -1, & S<0\end{cases}$$

Therefore, we have

$$\dot{S}=(1+a)(x_2+0)+\left(0+x_1+u_{eq}\right)=0\Rightarrow u_{eq}=-x_1-(1+a)x_2$$

The control law then becomes

$$u=-x_1-(1+a)x_2-\beta(x)\operatorname{sgn}(S)$$

Substituting u into $\dot{S}$ gives

$$\begin{aligned}\dot{S}&=(1+a)(x_2+\theta_1 x_1\sin x_2)+\left(\theta_2 x_2^2+x_1-x_1-(1+a)x_2-\beta(x)\operatorname{sgn}(S)\right)\\&=(1+a)(\theta_1 x_1\sin x_2)+\theta_2 x_2^2-\beta(x)\operatorname{sgn}(S)\end{aligned}$$

Afterwards, to ensure the condition $S\dot{S}<0$ can be satisfied under this control law, we simplify the equation to obtain $S\dot{S}\le-\eta|S|$ where $\eta>0$.

$$\begin{aligned}\dot{S}S&=\left[(1+a)(\theta_1 x_1\sin x_2)+\theta_2 x_2^2-\beta(x)\operatorname{sgn}(S)\right]S\\&=\left[(1+a)(\theta_1 x_1\sin x_2)+\theta_2 x_2^2\right]S-\beta(x)|S|\\&\le\left[(1+a)(a x_1\sin x_2)+b x_2^2\right]|S|-\beta(x)|S|\\&\le\left[(1+a)(a|x_1|)+b x_2^2\right]|S|-\beta(x)|S|\end{aligned}$$

We then have

$$-\eta|S|=\left[(1+a)(a|x_1|)+b x_2^2\right]|S|-\beta(x)|S|$$

$$\Rightarrow\beta(x)=a(1+a)|x_1|+b x_2^2+\eta$$

Let $\eta=b_0$, we can successfully prove that the control law

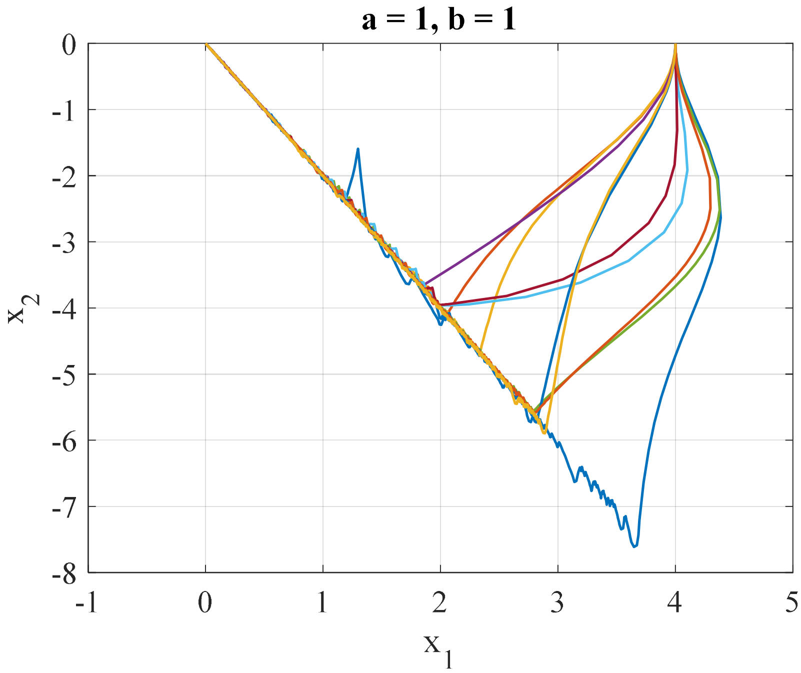
$$u=u_{eq}-\beta(x)\operatorname{sgn}(S)$$

where $u_{eq}=-x_1-(1+a)x_2$ and $\beta(x)=a(1+a)|x_1|+b x_2^2+b_0,\ b_0>0$ can ensure $S\dot{S}<0$ which will make this system Lyapunov stable.

(b)

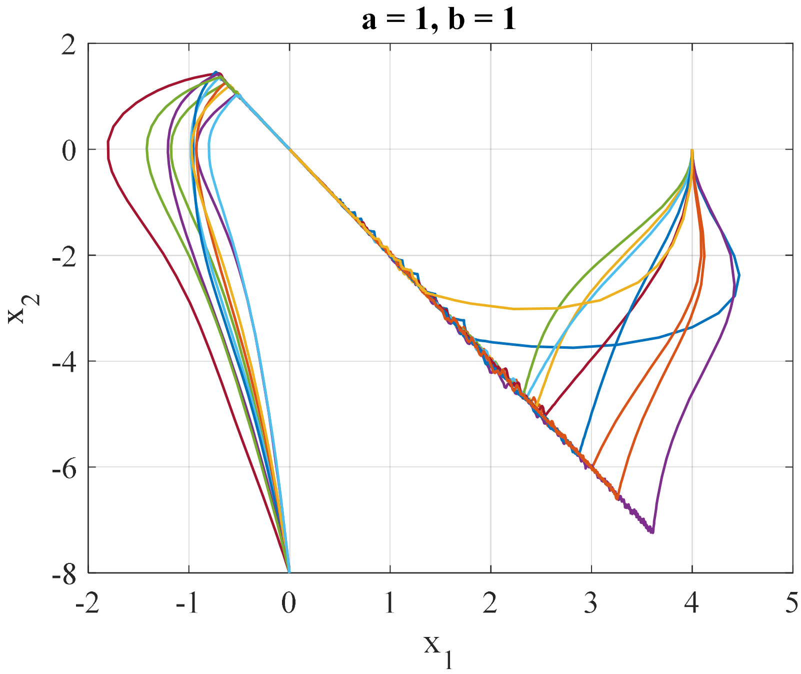
Now, we use Matlab to ensure the accuracy of the sliding mode control.

Let $a = b = 1$, so $\theta_{1,2}$ will be a random constant within the range of $[-1,1]$.



In the figure, with the initial point $(4,0)$, we can see that the phase space will eventually converge to a surface which is $S = 0$.

To make sure that the initial point won't affect the system to converge to the sliding surface, I also tried a different initial point $(0,-8)$ which lies on the left side of the surface.

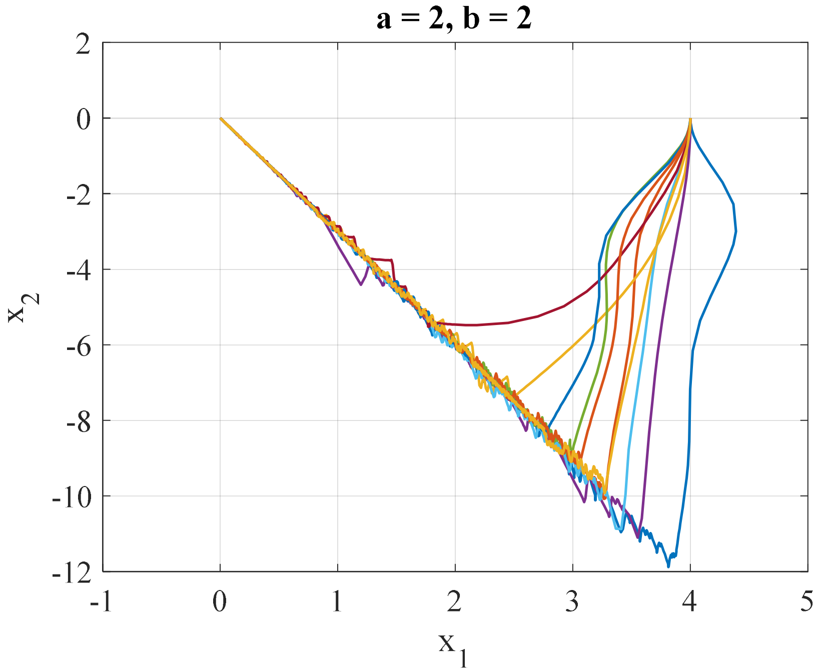


In this figure, we can see more clearly that it doesn't matter where the initial points are placed, they all ended up on the sliding surface.

However, note that in the two figures above, the system won't stay on the surface at all times once it gets to the surface, mainly due to the uncertain parameters. When the state kind of flows away from the surface because of the uncertainties, the control will once again pull it back to the surface and this situation goes on and on. That's why there exists the "vibration" or "jumping effect" near the surface and this situation becomes less obvious when it's closer to the origin.

(c)

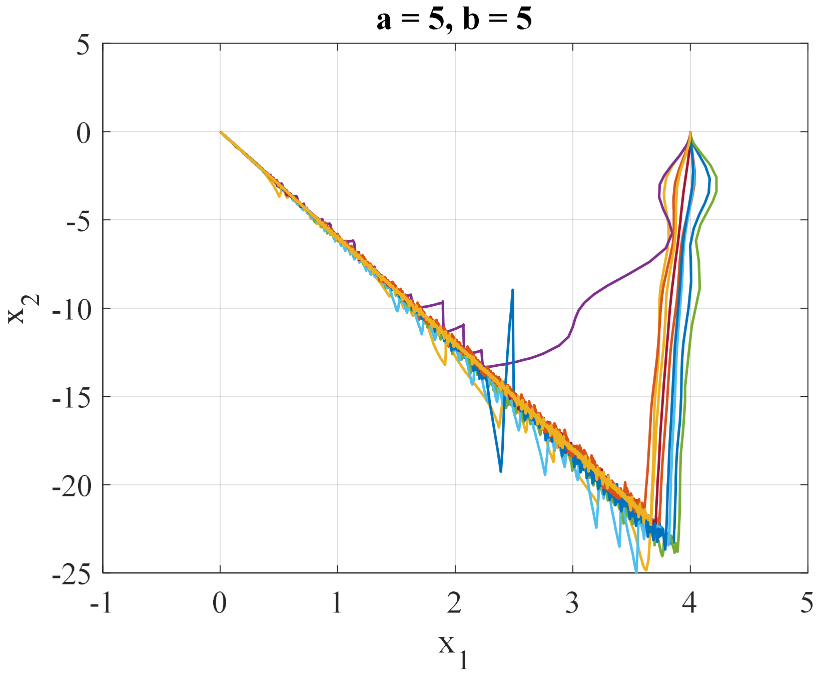
We now let $a = b = 2$ to observe how this will change the phase space of the system.



In the figure, the initial point $(4,0)$ is chosen once again, and we can see that the system will converge to the same sliding surface as before. We can easily notice that the vibration is more severe than when setting $a = b = 1$.

This is because when we apply $a = b = 2$ for the uncertain parameters $\theta_{1,2}$, we gave them "more space" which led to a more uncertain result here, especially after the system converges to the sliding surface.

With that being said, I decided to expand the range to $a = b = 5$ just to see whether my inference is correct and how much the range of the uncertain parameters actually affect the vibration/jumping effect.



In this figure, we can see that the value of range of the uncertain parameters a and b will certainly make a difference when it comes to causing the vibration/jumping effect near the sliding surface.

Aside from the range of the uncertain parameters, there are several other reasons that cause the vibration/jumping effect more obvious as well. These include the measurement error, calculation time delay and so on.