A gentle “tilt” inside Natário bubble a **whisper of artificial gravity** (0.1 g-0.5 g)

A Natário bubble is *not* limited to perfect weightlessness; you can dial in a **whisper of artificial gravity** (< 0.1 g) by superposing an ultra-small linear β-gradient. The extra curvature is so tiny that it hardly dents the exotic-mass budget and leaves the Ford–Roman quantum-inequality margin essentially unchanged. In short, “almost-geodesic” is still QI-legal—just don’t aim for full Earth-gravity, or the energy bill (and the Ford–Roman police) will catch up quickly.

Where does the “down-side” of a Natário bubble come from?

Think of the bubble’s thin Casimir shell as a programmable *phase array*.
When the shift-vector β is **perfectly symmetric**, β is the same everywhere inside and drops by exactly one unit across the wall, so every interior point rides a geodesic and you feel weightless.

To create a preferred “down” direction we *spoiler* the symmetry just a hair:

Across the wall β still must drop to zero, but now the drop is a tiny bit **steeper** on the “down” side and **shallower** on the “up” side:

$$\left|\nabla\beta\right|_{\text{wall, down}} = 2.000\,000\,000\,000\,002\ \mathrm{m}^{-1}, \qquad \left|\nabla\beta\right|_{\text{wall, up}} = 1.999\,999\,999\,999\,998\ \mathrm{m}^{-1}.$$

That 1-in-10^{16} tweak is what buys you all the cabin-wide weight you want.

you can “tilt” a Natário bubble so the crew feels a **very small, uniform pull** (≲ 0.1 g) without blowing either the **Ford–Roman quantum-inequality (QI)** or the lattice’s structural limits. The trick is to introduce a *tiny* linear gradient in the shift-vector β across the cabin, thousands of times weaker than the steep β-drop that already exists in the 0.5 m Casimir wall. Because the stress–energy scales with $(\nabla\beta)^2$, the extra exotic-mass cost is only parts-per-million of the current budget and fits comfortably inside the Mark-1’s QI head-room (ζ ≈ 0.84) .

1. Definition of the “tilted” shift

$$\beta(z) \;=\; \beta_0 \;+\; \varepsilon\,\frac{z}{L}, \qquad \varepsilon \ll 1.$$

2. Expansion (the divergence of β)
   Since β only varies in the z-direction,

$$\theta \;=\; \nabla\!\cdot\!\beta \;=\; \frac{d\beta}{dz} \;=\; \frac{\varepsilon}{L}\,.$$

3. Proper acceleration of a comoving observer
   In linearized gravity one finds

$$a \;\approx\; c^2\,\theta \;=\; c^2\,\frac{\varepsilon}{L} \quad\Longrightarrow\quad \varepsilon \;=\; \frac{a\,L}{c^2}.$$

Uniform “tilt” field in a Natário bubble

$$a \;=\; c^2\,\frac{\varepsilon}{L} \;\Longrightarrow\; \varepsilon \;=\; \frac{a\,L}{c^2}$$

For half-gravity inside a 20 m-tall habitat:

$$a = 0.5\,g_{\oplus} = 0.5 \times 9.8 = 4.9\ \mathrm{m\ s}^{-2}$$

$$L = 20\ \mathrm{m}$$

$$c^2 = (3.0 \times 10^8)^2 = 9.0 \times 10^{16}\ \mathrm{m^2 s^{-2}}$$

1. Tiny slope

$$\varepsilon \;=\; \frac{a\,L}{c^2} \;=\; \frac{(0.5\times 9.8)\times 20}{(3.0\times 10^8)^2} \;\approx\; 1.1\times 10^{-15}\,.$$

2. Cabin gradient

$$|\nabla\beta|_{\text{cabin}} = \frac{\varepsilon}{L} \approx \frac{1.1\times 10^{-15}}{20} = 5.4\times 10^{-17}\,\mathrm{m}^{-1}.$$

3. Wall gradient (across a 0.5 m wall)

$$|\nabla\beta|_{\text{wall}} \sim \frac{1}{0.5\,\mathrm{m}} = 2\,\mathrm{m}^{-1}.$$

4. Ratio of gradients

$$\frac{|\nabla\beta|_{\text{cabin}}}{|\nabla\beta|_{\text{wall}}} \approx \frac{5.4\times 10^{-17}}{2} = 2.7\times 10^{-17}.$$

5. Fractional stress–energy

$$\frac{\Delta\rho}{\rho_{\text{wall}}} \sim \left(\frac{|\nabla\beta|_{\text{cabin}}}{|\nabla\beta|_{\text{wall}}}\right)^2 \approx (2.7\times 10^{-17})^2 = 7.4\times 10^{-34}.$$

That addition is **thirty-three orders of magnitude** below the existing exotic-mass density, so the Ford–Roman quantum-inequality margin stays unchanged to every meaningful decimal place.

Practical limits

- **Comfort ceiling** – Even 0.1 g lets fluids settle and gives a consistent sense of “down” without over-loading bones or needing spin rigs.
- **QI ceiling** – You can, in principle, push to ~0.5 g before the ε^2 term eats the entire $\zeta \approx 0.16$ safety margin.
- **Structural ceiling** – Wall hoop-stress is dominated by the 2 m^{-1} gradient; adding a 10^{-12} m^{-1} cabin term changes σ by < 10^{-24}—unmeasurable.
- **Navigation caution** – The “down” direction is fixed to bubble geometry, not to the planet. If you rotate the ship, gravity vector rotates with it.