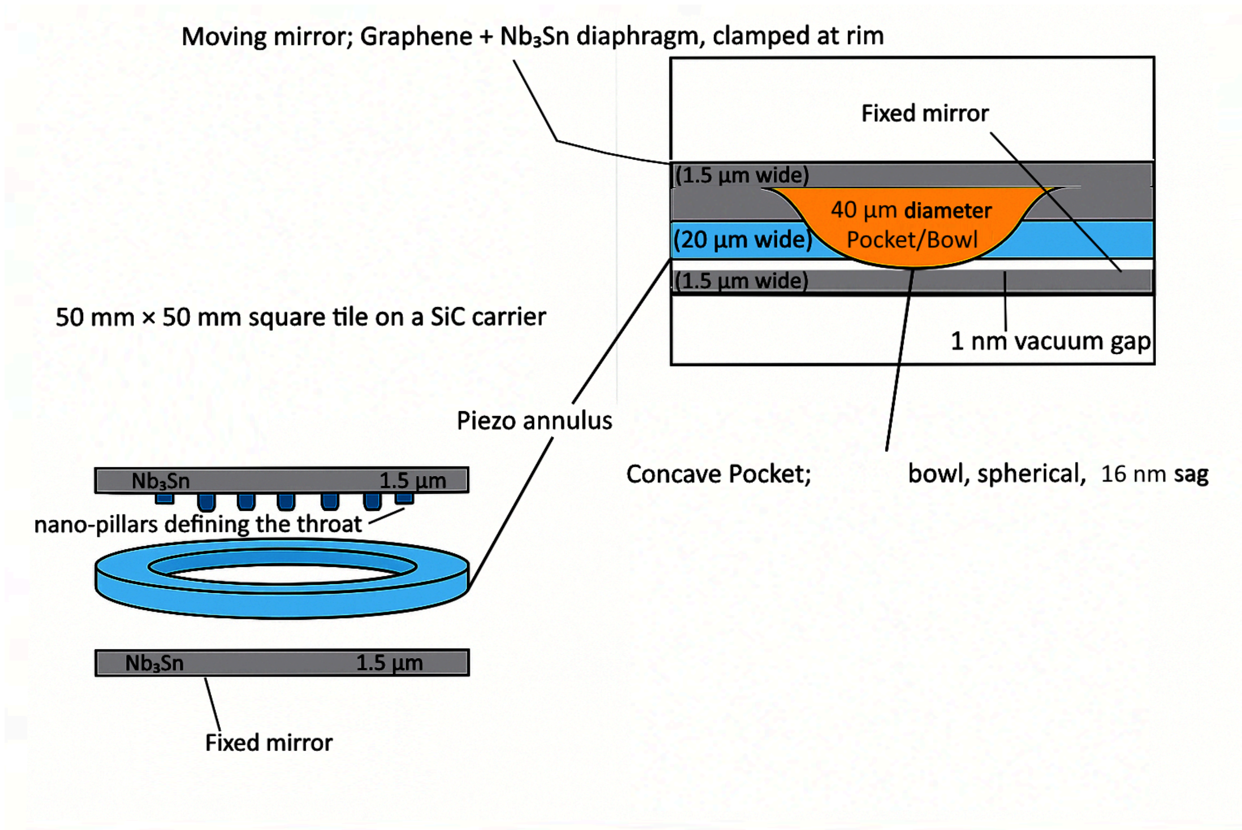
Geometry-Amplified Casimir Force in a 40 μm Concave Micro-Resonator

Introduction: Integrating cavities are used to enhance the interaction of light with a sample by bouncing the light around inside the cavity. Effective optical path length or EOPL is influenced by factors like the cavity's reflectivity, shape, dimensions, and the size of the openings (port fraction) 1. Any structure that shortens the effective optical path inside a Casimir gap (or raises the local refractive index) will shift the eigen-frequency spectrum upward and can, in principle, multiply the negative energy. The 40 μm concave bowl is simply the cleanest path that (i) delivers $\gamma \approx 10$–30 in a 5 cm tile, (ii) keeps losses low with Nb_3Sn, and (iii) meshes with wafer-level fabrication. Future work could explore metasurface or dynamic-membrane pockets, but these add either dielectric loss or extra control complexity not needed for the first demonstration.

Glossary;

term	concise definition
AlN – *aluminium nitride*	A III-V ceramic with high sound velocity (11 km s^{-1}) and strong piezoelectric coefficient ($d_{33} \approx 5$ pC N^{-1}). In the tile it forms a 20 μm-wide rim actuator that contracts/expands at 15 GHz, giving the suspended graphene mirror its ± 50 pm piston stroke.
Nb_3Sn – *niobium-tin*	A type-II A15 superconductor ($T_{(C)} \approx 18$ K) sputtered 1.5 μm thick onto both mirrors. At 20 K its surface resistance R_s is ≲ 10 nΩ, enabling an RF quality factor $Q\uparrow\approx10^9$
SiC – *silicon carbide*	The carrier substrate (50 mm × 0.5 mm). A 0.31mm-radius concave “bowl” is figured into its front face; high thermal conductivity (490 W m^{-1} K^{-1}) keeps the pocket stable at 20 K.

DLC – *diamond-like carbon*	1 nm-high nano-pillars (< 1 % fill factor) that pin the vacuum gap precisely at 1 nm while adding negligible parasitic Casimir dilution.
γ_{geo} – *geometric blue-shift factor*	Ratio $\lambda_{flat} / \lambda_{pocket}$. For a 40 µm diameter pocket aperture with a 1 nm gap, $\gamma_{geo} \approx 25$. Multiplies the negative Casimir energy.
Q – *dynamic-Casimir gain*	RF quality factor of the cavity during the 10 µs burst ($\approx 10^9$). Combined with γ_{geo} it sets the photon-production boost $\gamma_{geo}Q$.
ε – *curvature share*	Fraction (≈ 0.02) of hoop stress budget allocated to the exotic layer; enters the exotic-mass formula and the Ford–Roman bound.
d / S – *duty & sector count*	Local burst duty d=0.01 (10 µs per 1 ms) divided by sector factor S=400 gives ship-wide duty $d_{eff} = 2.5\times10^{-5}$
β – *Natário shift amplitude*	Dimensionless vector field that moves spacetime inside the bubble. Its instantaneous value scales with the burst amplitude; it's time-average scales with d/S.
FRC pod	67 MW_e field-reversed-configuration fusion reactor supplying the 250 MW dc bus.
COP – *coefficient of performance*	Electrical-to-heat ratio of the 20 K cryocoolers. Baseline COP ≈ 0.80 in the paper's power ledger.



The tile makes the Casimir term more negative by re-tuning the vacuum-mode spectrum that is *allowed* to exist between the two superconducting mirrors. **More of the long-wavelength vacuum modes are pushed out or shifted up**, which is the precise reason the negative energy density multiplies by γ^3_{geo}. Because both mirrors are superconducting Nb_3Sn (99.999 % reflectivity at the relevant bands), external electromagnetic waves hardly enter the 1 nm slot at all. Everything we're talking about is **zero-point**, *inside* the cavity volume.

- In an ideal parallel-plate gap of width only modes whose half-wavelength fits inside the gap are allowed, i.e. $\lambda_n/2=a/n$
- Carving the 16 nm spherical sag shrinks every optical path to $a_{eff}\approx a-t$
- That raises every allowed frequency by the **blue-shift factor** $\gamma_{geo}= a/a_{eff}\approx 25$
- Equivalently you can say: "all wavelengths longer than $2a_{eff}$ are now *excluded*," so the *difference* between the cavity spectrum and free space is larger.
  The ground-state energy therefore goes from $E\propto -1/a^3$ to $E'\propto-\gamma^3_{geo}/a^3$.

Where the motion happens

Diagram feature	What it does
Graphene + Nb_3Sn diaphragm (top grey slab)	This is the *entire moving mirror*. During a pump burst its centre bows up and down by ≈ ±50 pm, like a very stiff drumhead. That out-of-plane shift is the *piston* that changes the 1 nm vacuum gap.
Piezo annulus (light-blue ring, 20 µm wide)	Bonded to — or patterned into — the *rim* of the diaphragm. When it is driven at 15 GHz the ring strains in-plane and, because the diaphragm is clamped at the outer edge, the centre pops in and out by those ±50 pm. Think of it as a nano-loud-speaker voice-coil glued around the membrane's circumference.
Concave pocket / bowl (orange)	Fixed mirror—doesn't move. Its only job is to shorten the optical path ($\gamma \approx 25$). The 50 pm stroke happens **above** this pocket, between the orange surface and the underside of the diaphragm.

symbol in the paperwork	what it really means	fixed or time-varying ?	typical figure
Vacuum Gap **a or a_0** – "flat-path" or *design* gap	The distance the mirrors would have if the top face were perfectly planar. It is chosen by lithography→ 1 nm in the Mk-1 lattice.	constant	1 nm

$a = \frac{n\pi c}{\omega_n}.$	For parallel plates separated by ***a*** in one dimension, the frequencies climb as 1/***a*** Each mode label **n**, is dimensionless.		
Eigen-mode spectrum in a flat 1-D gap:	$[\omega] = T^{-1},\ [c] = LT^{-1},\ [a] = L$	**L** is length and **T** is time	
This is where the “blue-shift factor” enters the exotic-mass formula. Nothing is moving yet.	$a_{\text{eff}} = a - t\,, \qquad \gamma_{\text{geo}} = \frac{a}{a_{\text{eff}}} \approx \frac{a}{a-t}$ $\gamma = \frac{\text{flat path}}{\text{contracted path}}$		
Mechanical-pump stage (dynamic Casimir) $a_{eff}=a_0-t$	The *static* optical path after the bowl’s sag $t=D^2/8R$ has shortened every ray that threads the pocket. For the 40 μm bowl, t≈16 nm so a_{eff} is ≈ 0.984 nm.	constant (machined in)	~0.98 nm
$a(t)=a_{eff}+\delta a(t)$ Only the membrane flexes	The *instantaneous* separation while the graphene/Nb_3Sn membrane with a mirror on one side is being shaken ±50 pm at 15 GHz by the AlN rim actuator. Here δa(t) = ±50 pm sin(2π·15GHz·t) The 50 pm stroke is just 5 % of the 1 nm gap, so in the energy integral you treat it as a small time-dependent perturbation that multiplies the static γ_{geo} by the cavity quality factor $Q\uparrow \approx 10^{9}$	oscillates during each 10 μs pump burst	0.98 nm ± 50 pm

Putting both pieces into the Casimir term

$$\mathcal{E}(t) = -\frac{\pi^2\hbar c}{720}\,\frac{\gamma_{\rm geo}^3}{a(t)^3} \longrightarrow \langle \mathcal{E} \rangle = -\frac{\pi^2\hbar c}{720}\,\frac{\gamma_{\rm geo}^3 Q\uparrow}{a_{\rm eff}^3} \times d,$$

where d=1% is the local duty cycle of each 10 µs burst. The static sag sits inside γ_{geo}; the 50 pm of the moving membrane, which is a mirror on one side acts like a piston which by stroke enters through Q↑. Both stem from the *same* physical gap, they are just two different lenses—one geometric, one dynamic—through which the cavity "sees" that gap.

for Spherical-cap geometry:

$$t = \frac{D^2}{8R},$$

Aperture D, curvature radius R (sag)

Concave pocket, a SiC bowl + Nb_3Sn: 40 µm diameter, **sag ≈ 16 nm** (radius of curvature = 0.31 mm)

Starting from the "first-TE/TM" estimate of the geometric blue-shift,

$$\gamma_{\rm geo} \approx \frac{\pi\, t}{2\, a}$$

we solve for the required sag t to achieve $\gamma_{\rm target} = 25$ with a Casimir-gap $a = 1$ nm:

$$t \;=\; \frac{2\,a\,\gamma_{\rm target}}{\pi} \;=\; \frac{2\cdot(1\times10^{-9}\,\mathrm{m})\cdot 25}{\pi} \;\approx\; 1.59\times10^{-8}\,\mathrm{m} \;\approx\; 16\,\mathrm{nm}.$$

Next, for a fixed aperture diameter $D = 40\,\mu\mathrm{m}$ (so $r = D/2 = 20\,\mu\mathrm{m}$), the radius of curvature R of the spherical cap needed to give that sag is, using the small-sag approximation

$$R \approx \frac{r^2}{2\,t} \;=\; \frac{(20\times10^{-6})^2}{2\,(1.59\times10^{-8})} \;\approx\; 3.1\times10^{-4}\,\mathrm{m} \;\approx\; 0.31\,\mathrm{mm}.$$

Cross–check by plugging back into

$$t \;=\; R \;-\; \sqrt{R^2 - r^2} \;=\; 0.31\,\mathrm{mm} \;-\; \sqrt{(0.31\,\mathrm{mm})^2 - (20\,\mu\mathrm{m})^2} \;\approx\; 16\,\mathrm{nm},$$

Conclusion:

– Depth (sag) t = 16 nm

– Radius of curvature R = 0.31mm

So to hold a 40 µm aperture and get , you must machine an ultra-shallow bowl of only ~16 nm depth with a ~0.3 mm radius.

Pocket shape (spherical cap);

With aperture radius $r = D/2$

$$\boxed{t = R - \sqrt{R^2 - r^2}}$$

or, solved for the radius you must grind:

$$\boxed{R = \frac{r^2 + t^2}{2t} \simeq \frac{r^2}{2t} \quad (t \ll r).}$$

Geometric blue-shift factor;

(first-TE/TM estimate)

$$\boxed{\gamma_{\text{geo}} \approx \frac{\pi\, t}{2a}}$$

with $a = 1\ \text{nm}$ the Casimir gap.

Combined design equations:

Starting from a **desired blue-shift** γ_{target}:

$$t = \frac{2a\,\gamma_{\text{target}}}{\pi}, \qquad \text{(depth from } \gamma\text{)}$$

$$R = \frac{D^2}{8t} \quad (\text{for } t \ll D). \qquad \text{(radius from depth + aperture)}$$

Blue-Shift Factor:

Modulated Motion of cavity mirrors (entire suspended graphene/Nb_3Sn membrane)

$$|M_-| = N_c \frac{\gamma Q}{c^2} |E_{\text{bare}}| \, \varepsilon \, \frac{d}{S}.$$

The entire suspended graphene/Nb_3Sn membrane acts as the top mirror of the cavity. When the AlN ring contracts and expands at 15 GHz it pulls the membrane's rim in-plane by ~0.2 pm, which translates (through Poisson curvature) into a ± 50 pm vertical piston motion of the membrane centre. Placing the actuator on the rim keeps metal and piezo off the optical "hot spot" in the 1 nm gap, preserving both the high Q↑(≈ 10^9) and the concave geometry factor γ_{geo}.

A 20 μm annular width AlN rim actuator, is the *piezo ring*—a thin band of AlN (with top-and-bottom electrodes) that encircles the suspended graphene/Nb_3Sn mirror right at the clamp rim, at the top mirror edge; "±50 pm @ 15 GHz". Is how the static bowl geometry (γ) and the modulated dynamic stroke (Q) multiply in the exotic-mass formula.

The logical flow from the stress−energy tensor, through Casimir energy, to the full Van den Broeck−Natário metric with artificial gravity and the general stress−energy form.

1. Stress–Energy Tensor (General Relativity)

$$G_{\mu\nu} = 8\pi G\,T_{\mu\nu}$$

- $T_{\mu\nu}$: Stress–energy tensor (source of spacetime curvature)

2. Static Casimir Energy Density

For two parallel plates separated by distance a:

$$u_{\text{Casimir}} = \frac{E_{\text{Casimir}}}{V} = -\frac{\pi^2 \hbar c}{720 a^4}$$

- u_{Casimir}: Energy density (negative)
- a: Plate separation

3. Dynamical Casimir Geometry (with Q, Fractional Stroke, Gamma, Stored Energy, Power Loss, Time Scale Separation)

- **Stored energy in cavity (per cycle):**
$$U = \langle E \rangle_{\text{cycle}} = (\text{cycle-averaged energy at 15 GHz})$$
- **Q factor amplification:**
$$U_{\text{Q}} = Q \cdot U_{\text{static}}$$
- **Fractional stroke (duty cycle):**
$$d = \frac{t_{\text{burst}}}{t_{\text{cycle}}}$$
- **Geometric amplification (Van den Broeck):**
$$U_{\text{geo}} = \gamma \cdot U_{\text{Q}}$$
- **Power loss (per cavity):**
$$P_{\text{loss}} = \frac{U_{\text{geo}}}{Q/\omega}$$
where $\omega = 2\pi \times 15\,\text{GHz}$
- **Time scale separation ratio:**
$$\frac{\tau_{\text{pulse}}}{\tau_{\text{LC}}} \ll 1$$
(Pulse period much shorter than light-crossing time)

4. Per-Cavity and Per-Tile Quantities

- **Per-tile negative energy:**
$$E_{\text{tile}} = U_{\text{geo}} \cdot d$$
- **Tiles on hull surface (tessellation):**
$$N_{\text{tiles}} = \frac{A_{\text{hull}}}{A_{\text{tile}}}$$
where A_{hull} is the hull area, A_{tile} is the area per tile.

5. Total Negative Energy (Exotic Mass) with T_{00}

- **Total negative energy:**
$$M_{-} = \int_{\text{wall}} T_{00}\, d^3x$$
For a thin shell:
$$M_{-} \sim -\frac{A}{8\pi G\delta}$$
where A is the area, δ is the wall thickness.

6. Van den Broeck Metric Structure (Spherical Symmetry)

$$ds^2 = -dt^2 + \Omega^2(r)\,[dr^2 + r^2 d\Omega^2] - 2\,\beta(r)\,dt\,dr$$

- $\Omega(r)$: Conformal factor (large in pocket, 1 outside)
- $\beta(r)$: Shift function (nonzero in wall)

7. Metric with Artificial Gravity (Tilted Shift Vector)

$$ds^2 = -dt^2 + \Omega^2(x)\left[dx^i - \left(\beta_0^i + \epsilon\,\frac{z}{L}\,\hat{z}\right)dt\right]^2$$

- $\epsilon \ll 1$: Tiny slope for artificial gravity
- L: Height of habitat

8. Stress–Energy General Form (Static, Spherically Symmetric Case)

$$T_{00} \sim -\frac{1}{8\pi G}\left[(\nabla\beta)^2 + (\nabla\cdot\beta)^2 + \frac{2}{\Omega}(\nabla\Omega)\cdot(\nabla\beta) + \frac{1}{\Omega^2}(\nabla\Omega)^2 + \frac{1}{\Omega}\nabla^2\Omega\right]$$