1. Stress–Energy Tensor (General Relativity)

$$G_{\mu\nu} = 8\pi G\, T_{\mu\nu}$$

- $T_{\mu\nu}$: Stress–energy tensor (source of spacetime curvature)

2. Static Casimir Energy Density

For two parallel plates separated by distance a:

$$u_{\text{Casimir}} = \frac{E_{\text{Casimir}}}{V} = -\frac{\pi^2 \hbar c}{720 a^4}$$

- u_{Casimir}: Energy density (negative)
- a: Plate separation

3. Dynamical Casimir Geometry (with Q, Fractional Stroke, Gamma, Stored Energy, Power Loss, Time Scale Separation)

- **Stored energy in cavity (per cycle):**

$$U = \langle E \rangle_{\text{cycle}} = (\text{cycle-averaged energy at 15 GHz})$$

- **Q factor amplification:**

$$U_{\text{Q}} = Q \cdot U_{\text{static}}$$

- **Fractional stroke (duty cycle):**

$$d = \frac{t_{\text{burst}}}{t_{\text{cycle}}}$$

- **Geometric amplification (Van den Broeck):**

$$U_{\text{geo}} = \gamma \cdot U_{\text{Q}}$$

- **Power loss (per cavity):**

$$P_{\text{loss}} = \frac{U_{\text{geo}}}{Q/\omega}$$

where $\omega = 2\pi \times 15\,\text{GHz}$

- **Time scale separation ratio:**

$$\frac{\tau_{\text{pulse}}}{\tau_{\text{LC}}} \ll 1$$

(Pulse period much shorter than light-crossing time)

4. Per-Cavity and Per-Tile Quantities

- **Per-tile negative energy:**

$$E_{\text{tile}} = U_{\text{geo}} \cdot d$$

- **Tiles on hull surface (tessellation):**

$$N_{\text{tiles}} = \frac{A_{\text{hull}}}{A_{\text{tile}}}$$

where A_{hull} is the hull area, A_{tile} is the area per tile.

5. Total Negative Energy (Exotic Mass) with T_{00}

- **Total negative energy:**

$$M_{-} = \int_{\text{wall}} T_{00}\, d^3x$$

For a thin shell:

$$M_{-} \sim -\frac{A}{8\pi G \delta}$$

where A is the area, δ is the wall thickness.

6. Van den Broeck Metric Structure (Spherical Symmetry)

$$ds^2 = -dt^2 + \Omega^2(r)\,[dr^2 + r^2 d\Omega^2] - 2\,\beta(r)\,dt\,dr$$

- $\Omega(r)$: Conformal factor (large in pocket, 1 outside)
- $\beta(r)$: Shift function (nonzero in wall)

7. Metric with Artificial Gravity (Tilted Shift Vector)

$$ds^2 = -dt^2 + \Omega^2(x)\left[dx^i - \left(\beta_0^i + \epsilon\,\frac{z}{L}\,\hat{z}\right)dt\right]^2$$

- $\epsilon \ll 1$: Tiny slope for artificial gravity
- L: Height of habitat

8. Stress–Energy General Form (Static, Spherically Symmetric Case)

$$T_{00} \sim -\frac{1}{8\pi G}\left[(\nabla\beta)^2 + (\nabla\cdot\beta)^2 + \frac{2}{\Omega}(\nabla\Omega)\cdot(\nabla\beta) + \frac{1}{\Omega^2}(\nabla\Omega)^2 + \frac{1}{\Omega}\nabla^2\Omega\right]$$