Lista de Exercícios 1

1. Calcule as integrais indefinidas a seguir.

(a) $\int (x^2 + x^{-2})\, dx$

(b) $\int (\sqrt{x^3} + \sqrt[3]{x^2})\, dx$

(c) $\int (4x^3 + 3x^2 + x - 1))\, dx$

(d) $\int \left(\frac{2}{x} + \frac{3}{x^2}\right) dx$

(e) $\int (u + 4)(2u + 1)\, du$

(f) $\int v(v^2 + 2)^2\, dv$

(g) $\int \frac{x^2 + 1}{x}\, dx$

(h) $\int \frac{x^3 - 2\sqrt{x}}{x}\, dx$

(i) $\int \left(x^2 + 1 + \frac{x}{x^2+1}\right) dx$

(j) $\int (\operatorname{sen}(x) + \cos(x))\, dx$

(k) $\int \left(\sqrt{x} + \frac{1}{x^2}\right) dx$

(l) $\int (x + 3e^x)\, dx$

(m) $\int (x^2 + \operatorname{sen}(x))\, dx$

(n) $\int \frac{e^x + e^{-x}}{2}\, dx$

(o) $\int \left(\frac{1}{x} + e^x\right) dx,\ x > 0$

(p) $\int \frac{\operatorname{sen}(2x)}{\cos(x)}\, dx$

2. Calcule as integrais indefinidas a seguir usando uma substituição conveniente.

(a) $\int \cos(3x)\, dx$

(b) $\int x(x^2 + 2)^{100}\, dx$

(c) $\int x^2 \sqrt{x^3 + 1}\, dx$

(d) $\int \frac{dt}{1 - 6t}$

(e) $\int x \operatorname{sen}(x^2)\, dx$

(f) $\int x^2 e^{x^3}\, dx$

(g) $\int (3t - 2)^{20}\, dt$

(h) $\int \frac{dx}{5 - 3x}$

(i) $\int \operatorname{sen}(\pi t)\, dt$

(j) $\int \cos(x)\operatorname{sen}^2(x)\, dx$

(k) $\int \frac{\operatorname{sen}(\sqrt{x})}{\sqrt{x}}\, dx$

(l) $\int \frac{z^2}{1 + z^3}\, dz$

(m) $\int x(2x + 1)^{19}\, dx$

(n) $\int \frac{\operatorname{sen}(x)}{1 + \cos(x)}\, dx$

(o) $\int e^{\cos(x)} \operatorname{sen}(x)\, dx$

(p) $\int \frac{\operatorname{sen}(t)}{1 + \cos^2(t)}\, dt$

3. Avalie a integral definida.

(a) $\int_0^1 \sqrt[3]{1 + 5x}\, dx$

(b) $\int_0^1 (3x - 1)^{100}\, dx$

(c) $\int_0^1 xe^{-x^2}\, dx$

(d) $\int_0^{\pi/2} \cos(x)\operatorname{sen}(\operatorname{sen}(x))\, dx$

(e) $\int_1^2 x\sqrt{x - 1}\, dx$

(f) $\int_0^1 \frac{e^x + 1}{e^x + x}\, dx$

4. Calcule as integrais trigonométricas a seguir.

(a) $\int \cos^2(2x)\, dx$

(b) $\int \operatorname{sen}^2(3x)\, dx$

(c) $\int \cos^4(x)\, dx$

(d) $\displaystyle\int \operatorname{tg}(x)\,dx$

(g) $\displaystyle\int \operatorname{sen}^2(x)\cos(x)\,dx$

(j) $\displaystyle\int \operatorname{tg}(x)\sec^2(x)\,dx$

(e) $\displaystyle\int \sec(x)\,dx$

(h) $\displaystyle\int \operatorname{sen}(x)\sqrt{\cos(x)}\,dx$

(k) $\displaystyle\int \operatorname{sen}^2(x)\cos^2(x)\,dx$

(f) $\displaystyle\int \operatorname{tg}(2x)\,dx$

(i) $\displaystyle\int \operatorname{sen}^3(x)\,dx$

5. Calcule as integrais indefinidas a seguir.

(a) $\displaystyle\int (3x-2)^3\,dx$

(e) $\displaystyle\int \operatorname{sen}^5(x)\cos(x)\,dx$

(i) $\displaystyle\int e^x\sqrt{1+e^x}\,dx$

(b) $\displaystyle\int \frac{1}{(3x-2)^2}\,dx$

(f) $\displaystyle\int \frac{5}{4x+3}\,dx$

(j) $\displaystyle\int \frac{1}{(x-1)^3}\,dx$

(c) $\displaystyle\int x\,\operatorname{sen}(x^2)\,dx$

(g) $\displaystyle\int \frac{3x}{5+6x^2}\,dx$

(k) $\displaystyle\int xe^{-x^2}\,dx$

(d) $\displaystyle\int x^3\cos(x^4)\,dx$

(h) $\displaystyle\int x\sqrt{1+3x^2}\,dx$

6. Calcule as integrais indefinidas a seguir usando a técnica de integração por partes quando conveniente.

(a) $\displaystyle\int xe^x\,dx$

(e) $\displaystyle\int \ln(x)\,dx$

(i) $\displaystyle\int e^x\cos(x)\,dx$

(b) $\displaystyle\int x\,\operatorname{sen}(x)\,dx$

(f) $\displaystyle\int x^2\ln(x)\,dx$

(j) $\displaystyle\int x^3\cos(x)\,dx$

(c) $\displaystyle\int x^2 e^x\,dx$

(g) $\displaystyle\int (\ln x)^2\,dx$

(k) $\displaystyle\int e^{-x}\cos(2x)\,dx$

(d) $\displaystyle\int x\ln(x)\,dx$

(h) $\displaystyle\int xe^{2x}\,dx$

(l) $\displaystyle\int x^2\,\operatorname{sen}(x)\,dx$

7. Em cada um dos itens abaixo, faça uma substituição e então use integração por partes para calcular a integral.

(a) $\displaystyle\int \cos\sqrt{x}\,dx$

(c) $\displaystyle\int x^3\cos(x^2)\,dx$

(b) $\displaystyle\int t^3 e^{-t^2}\,dt$

(d) $\displaystyle\int x\ln(1+x)\,dx$

8. Calcule as integrais indefinidas a seguir usando a técnica de integração por frações parciais quando conveniente.

(a) $\displaystyle\int \frac{2}{x-3}\,dx$

(e) $\displaystyle\int \frac{2x+3}{x+1}\,dx$

(i) $\displaystyle\int \frac{2x^3}{x^2+x}\,dx$

(b) $\displaystyle\int \left(x+\frac{3}{x-2}\right)\,dx$

(f) $\displaystyle\int \frac{x^2}{x+1}\,dx$

(j) $\displaystyle\int \frac{x^4+2x+1}{x^3-x^2-2x}\,dx$

(c) $\displaystyle\int \frac{x}{x+1}\,dx$

(g) $\displaystyle\int \frac{x+3}{x^2-3x+2}\,dx$

(k) $\displaystyle\int \frac{2x+1}{x^3-x^2-x+1}\,dx$

(d) $\displaystyle\int \frac{x+2}{x-1}\,dx$

(h) $\displaystyle\int \frac{x^2+2}{x^2-3x+2}\,dx$

(l) $\displaystyle\int \frac{5}{x^3+4x}\,dx$

Gabarito

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1. (a) $\dfrac{x^3}{3} - \dfrac{1}{x} + C$

   (b) $\dfrac{2x^{5/2}}{5} + \dfrac{3x^{5/3}}{5} + c$

   (c) $x^4 + x^3 + \dfrac{x^2}{2} - x + C$

   (d) $2\ln|x| - \dfrac{3}{x} + C$

   (e) $\dfrac{2u^3}{3} + \dfrac{9u^2}{2} + 4u + C$

   (f) $\dfrac{(v^2 + 2)^3}{6} + C$

   (g) $\dfrac{x^2}{2} + \ln|x| + C$

   (h) $\dfrac{x^3}{3} - 4\sqrt{x} + C$

   (i) $\dfrac{x^3}{3} + x + \dfrac{\ln x^2 + 1}{2} + C$

   (j) $-\cos(x) + \operatorname{sen}(x) + C$

   (k) $\dfrac{2\sqrt{x^3}}{3} - \dfrac{1}{x} + C$

   (l) $\dfrac{x^2}{2} + 3e^x + C$

   (m) $\dfrac{x^3}{3} - \cos(x) + C$

   (n) $\dfrac{e^x - e^{-x}}{2} + C$

   (o) $\ln x + e^x + C$

   (p) $-2\cos(x) + C$

2. (a) Fazendo $u = 3x$, temos $\dfrac{\operatorname{sen}(3x)}{3} + C$

   (b) Fazendo $u = x^2 + 2$, temos $\dfrac{(x^2 + 2)^{101}}{202} + C$

   (c) Fazendo $u = x^3 + 1$, temos $\dfrac{2}{9}\sqrt{(x^3 + 1)^2} + C$

   (d) Fazendo $u = 1 - 6t$, temos $-\dfrac{1}{6}\ln|1 - 6t| + C$

   (e) Fazendo $u = x^2$, temos $-\dfrac{\cos(x^2)}{2} + C$

   (f) Fazendo $u = x^3$, temos $\dfrac{e^{x^3}}{3} + C$

   (g) Fazendo $u = 3t - 2$, temos $\dfrac{(3t - 2)^{21}}{63} + C$

   (h) Fazendo $u = 5 - 3x$, temos $-\dfrac{\ln|5 - 3x|}{3} + C$

   (i) Fazendo $u = \pi t$, temos $-\dfrac{\cos(\pi t)}{\pi} + C$

   (j) Fazendo $u = \operatorname{sen} x$, temos $\dfrac{\operatorname{sen}^3(x)}{3} + C$

   (k) Fazendo $u = \sqrt{x}$, temos $-2\cos(\sqrt{x}) + C$

   (l) Fazendo $u = 1 + z^3$, temos $\dfrac{1}{3}\ln(1 + z^3) + C$

   (m) Fazendo $u = 2x + 1$, temos $\dfrac{1}{40}\left(x(2x + 1)^{20} - \dfrac{(2x + 1)^{21}}{42}\right) + C$

   (n) Fazendo $u = 1 + \cos x$, temos $-\ln 1 + \cos(x) + C$

3. (a) $\dfrac{9\sqrt[3]{6}}{10} - \dfrac{3}{20}$

   (b) $\dfrac{2^{101} + 1}{3}$

   (c) $\dfrac{1 - e^{-1}}{2}$

   (d) $1 - \cos(1)$

   (e) $\dfrac{16}{15}$

   (f) $\ln(e + 1)$

4.

(a) $\dfrac{x}{2} + \dfrac{\operatorname{sen}(4x)}{8} + C$

(b) $\dfrac{x}{2} - \dfrac{\operatorname{sen}(6x)}{12} + C$

(c) $\cos^3(x)\operatorname{sen}(x) + \dfrac{3x}{8} - \dfrac{3\operatorname{sen}(4x)}{32} + C$

(d) $-\ln|\cos(x)| + C$

(e) $\ln|\sec(x) + \operatorname{tg}(x)| + C$

(f) $-\dfrac{\ln|\cos(2x)|}{2} + C$

(g) $\dfrac{\operatorname{sen}^3(x)}{3} + C$

(h) $-\dfrac{2}{3}\sqrt{\cos^3(x)} + C$

(i) $-\cos(x) + \dfrac{\cos^3(x)}{3} + C$

(j) $\dfrac{\operatorname{tg}^2(x)}{2} + C$

(k) $\dfrac{x}{8} - \dfrac{\operatorname{sen}(4x)}{32} + C$

5. (a) $\dfrac{(3x-2)^4}{12} + C$

(b) $-\dfrac{1}{3(3x-2)} + C$

(c) $-\dfrac{\cos(x^2)}{2} + C$

(d) $\dfrac{\operatorname{sen}(x^4)}{4} + C$

(e) $\dfrac{\operatorname{sen}^6(x)}{6} + C$

(f) $\dfrac{5}{4}\ln|4x+3| + C$

(g) $\dfrac{1}{4}\ln|5+6x^2| + C$

(g) $\dfrac{1}{9}\sqrt{(1+3x^2)^3} + C$

(i) $\dfrac{2}{3}\sqrt{(1+e^x)^3} + C$

(j) $-\dfrac{1}{2(x-1)^2} + C$

(k) $-\dfrac{1}{2}e^{-x^2} + C$

6. (a) $e^x(x-1) + C$

(b) $-x\cos(x) + \operatorname{sen}(x) + C$

(c) $e^x(x^2 - 2x - 2) + C$

(d) $\dfrac{x^2}{4}(2\ln x - 1) + C$

(e) $x(\ln x - 1) + C$

(f) $\dfrac{x^3}{9}(3\ln x - 1) + C$

(g) $x\ln^2 x - 2x(\ln x - 1) + C$

(h) $\dfrac{e^{2x}}{4}(2x - 1) + C$

(i) $\dfrac{e^x}{2}(\operatorname{sen} x + \cos x) + C$

(j) $x^3\operatorname{sen}(x) + 3x^2\cos(x) - 6x\operatorname{sen}(x) - 6\cos(x) + C$

(k) $\dfrac{e^{-x}}{5}(2\operatorname{sen}(2x) - \cos(2x)) + C$

(l) $-x^2\cos(x) + 2x\operatorname{sen}(x) + 2\cos(x) + C$

7. (a) $2\left[\sqrt{x}\operatorname{sen}(\sqrt{x}) + \cos(\sqrt{x})\right] + C$

(b) $-\dfrac{e^{-t^2}}{2}[t^2 + 1] + C$

(c) $\dfrac{1}{2}\left[x^2\operatorname{sen}(x^2) + \cos(x^2)\right] + C$

(d) $\dfrac{(x+1)(x-1)\ln(x+1)}{2} - \dfrac{(x+1)(x-3)}{4} + C$

8. (a) $2\ln|x-3| + C$

(b) $\dfrac{x^2}{2} + 3\ln|x-2| + C$

(c) $x + 1 - \ln|x+1| + C$

(d) $x - 1 + 3\ln|x-1| + C$

(e) $2(x+1) + \ln|x+1| + C$

(f) $\dfrac{(x+1)^2}{2} - 2(x+1) + \ln|x+1| + C$

(g) $-4\ln|x-1| + 5\ln|x-2| + C$

(h) $x - 3\ln|x-1| + 6\ln|x-2| + C$

(i) $x^2 - 2x + 2\ln|x+1| + C$

(j) $\dfrac{x^2}{2} + x - \dfrac{1}{2}\ln|x| + \dfrac{21}{6}\ln|x-2| + C$

(k) $-\dfrac{1}{4}\ln|x+1| + \dfrac{1}{4}\ln|x+1| - \dfrac{3}{2(x-1)} + C$

(l) $\dfrac{5}{4}\ln|x| - \frac{5}{8}\ln|x^2+4| + C$