Lista 2 - Prob 1

(1)

i) $\Omega_I = \{ (1,1) \ldots (1,6)$

$(2,1) \ldots (2,6)$

$(3,1) \ldots (3,6)$

$(4,1) \ldots (4,6)$

$(5,1) \quad (5,6)$

$(6,1) \ldots (6,6) \}$

A: $\frac{18}{36} = \frac{1}{2}$

B: $\frac{18}{36} = 1/2$

C: $\frac{9}{36} = \frac{1}{4}$

ii) $\Omega_{II} = \{ (P,P), (P,i), (i,P), (i,i) \}$

A: $\frac{2}{4} = \frac{1}{2}$

B: $\frac{2}{4} = \frac{1}{2}$

C: $\frac{1}{4}$

R: Como A, B e C tem as mesmas Probabilidades considerando os dois espaços Amostrais, Ω_{II} tem vantagem por $\#\Omega_{II} = 4$ que é menor que o Ω_I com $\#\Omega_I = 36$

② $\cdot P(1) = f(1) = \frac{2}{9}$

$\cdot P(2) = f(2) = \frac{1}{9}$

$\cdot P(3) = f(3) = 0$

$\cdot P(4) = f(4) = \frac{1}{9}$

$\cdot P(5) = f(5) = \frac{2}{9}$

$\cdot P(6) = f(6) = \frac{3}{9}$

$- P(I) = \frac{2}{9} + 0 + \frac{2}{9} = \frac{4}{9}$

$- P(P) = \frac{1}{9} + \frac{1}{9} + \frac{3}{9} = \frac{5}{9}$

R → Não podemos Afirmar que tanto faz, pois a prob de sair Par é maior que sair impar

③

a) $P\left(\bigcap_{k=1}^{m} A_k\right) = 1 - P\left(\left[\bigcap_{k=1}^{m} A_k\right]^c\right) = 1 - P\left(\bigcup_{k=1}^{m} A_k^c\right) \geq 1 - \sum_{k=1}^{m} P(A_k^c)$

$\cdot P\left(\bigcup_k^m A_k^c\right) \leq \sum_k^m P(A_k^c)$

$-P\left(\bigcup_k^m A_k^c\right) \geq -\sum_k^m P(A_k^c)$

$1 - P\left(\bigcup_k^m A_k^c\right) \geq 1 - \sum_k^m P(A_k^c)$

≥ 0

b) ① $\cdot P(A_k) = 1 - P([A_k]^c) \geq 1 - \varepsilon \Rightarrow -P([A_k]^c) \geq -\varepsilon \overset{x-1}{\Rightarrow} \varepsilon \geq P([A_k]^c)$

⑪ $P\left(\bigcap_{k=1}^{m} A_k\right) \geq 1 - \underbrace{\sum_{k=1}^{m} P(A_k^c)}_{\text{como } \varepsilon \geq P(A_k^c) \geq 0} \geq 1 - m.\varepsilon$

3

a)

$$\bullet\ P\left(\bigcap_{k=1}^{n} A_k\right) = P\left(\left(\bigcup_{k=1}^{n} A_k^c\right)^c\right) = 1 - P\left(\bigcup_{k=1}^{n} A_k^c\right) \geq 1 - \sum_{k}^{n} P(A_k^c)$$

$$\ast\ P\left(\bigcup_{k}^{n} A_k^c\right) \leq \sum_{k}^{n} P(A_k^c)$$

multiplica por -1

$$-P\left(\bigcup_{k}^{n} A_k^c\right) \geq -\sum_{k}^{n} P(A_k^c)$$

soma 1 dos dois lados

$$1 - P\left(\bigcup_{k}^{n} A_k^c\right) \geq 1 - \sum_{k}^{n} P(A_k^c)$$

b) (I) $P(A_k) = 1 - P(A_k^c) \geq 1 - \varepsilon \Rightarrow -P(A_k^c) \geq -\varepsilon \Rightarrow \boxed{\varepsilon \geq P(A_k^c)}$

(II) como $\varepsilon \geq P(A_k^c)$ para $k = 1, 2, \ldots, n$, então $n.\varepsilon \geq \sum_{k=1}^{n} P(A_k^c) \Rightarrow$

$$\Rightarrow -n\varepsilon \leq -\sum_{k=1}^{n} P(A_k^c) \Rightarrow 1 - \sum_{k=1}^{n} P(A_k^c) \geq 1 - n.\varepsilon$$

(III) com isso:

$$P\left(\bigcap_{k=1}^{n} A_k\right) \geq 1 - \sum_{k=1}^{n} P(A_k^c) \geq 1 - n\varepsilon$$

c) Parecida com letra A

(4) a) • $P(A\cup B) = \underbrace{P(A\cap B^c)}_{P(A)=P(A\cap B^c)+P(A\cap B)} + P(A\cap B) + \underbrace{P(A^c\cap B)}_{P(B)=P(A^c\cap B)+P(A\cap B)}$

$= P(A) - P(A\cap B) + P(A\cap B) + P(B) - P(A\cap C) \Rightarrow P(A\cup B) = P(A) + P(B) - P(A\cap B)$

b) $P(A^c\cup B) = P(A\cap B) - P(A) + 1$

• $P(A^c\cup B) = \underbrace{P(A^c\cap B^c)}_{\underbrace{P[(A\cup B)^c]}_{1-P(A\cup B)}} + \underbrace{P(A^c\cap B)}_{P(B)-P(A\cap B)} + P(A\cap B)$

$= 1 - P(A\cup B) + P(B) - \cancel{P(A\cap B)} + \cancel{P(A\cap B)}$

$= 1 - P(A) - \cancel{P(B)} + P(A\cap B) + \cancel{P(B)} \Rightarrow P(A^c\cup B) = P(A\cap B) - P(A) + 1$

c) $P(A\cup B\cup C) = P((A\cup B)\cup C) = P(A\cup B) + P(C) - P((A\cup B)\cap C)$

$= P(A) + P(B) - P(A\cap B) + P(C) - P((A\cap C)\cup(B\cap C))$

$= P(A) + P(B) - P(A\cap B) + P(C) - [P(A\cap C) + P(B\cap C) - P(A\cap B\cap C)]$

$= P(A) + P(B) + P(C) - P(A\cap B) - P(A\cap C) - P(B\cap C) + P(A\cap B\cap C)$

⑤ a) $P(A \cap B)$

- $P(A \cap B) = 0$ //

b) $P(A \cup B)$

- $P(A \cup B) = \underbrace{P(A)}_{0,3} + \underbrace{P(B)}_{0,5} - \underbrace{P(A \cap B)}_{0} = 0,8$ //

c) $P(A \setminus B)$

- $P(A \setminus B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{0}{0,5} = 0$ //

d) $P(A^c \cap B^c)$

- $P(A^c \cap B^c) = P(A \cup B)^c = 1 - P(A \cup B) = 1 - 0,8 = 0,2$ //

⑥ (I) $x + 2x + 3x + 4x + 5x + 6x = 1 \Rightarrow 21x = 1 \Rightarrow x = 1/21$

(II)
1 - 1/21
2 - 2/21
3 - $3/21 = 1/7$
4 - 4/21
5 - 5/21
6 - $6/21 = 2/7$

(III) Par: $\frac{2}{21} + \frac{4}{21} + \frac{6}{21} = \frac{12}{21} = \frac{4}{7}$

impar: $\frac{1}{21} + \frac{3}{21} + \frac{5}{21} = \frac{9}{21} = \frac{3}{7}$

↳ Não seria justo.

(IV)

$\frac{4}{7} \cdot \alpha = \frac{3}{7} \cdot \beta \Rightarrow \frac{4}{3} = \frac{\beta}{\alpha} = \frac{12}{9}$

Aceitaria jogar se fosse Par - Ganha 9 reais

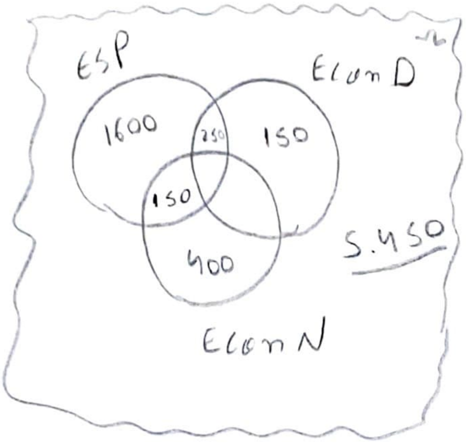
e impar - perde 12 //

(7)

a) $1 - \frac{5^4}{6^4} = 1 - \frac{625}{1296} = 1 - 0{,}48225 = \underline{0{,}51775}$ → mais Provável

b) $1 - \left(\frac{35}{36}\right)^{24} = 1 - (0{,}972)^{24} = 1 - 0{,}5058 = \underline{0{,}4942}$

(8)



a) $P(a) = \frac{2000}{8000} = \frac{2}{8} = \frac{1}{4} = 0{,}25$

b) $P(b) = \frac{7050}{8000} = 0{,}88$

c) $P(c) = P(ES \cup Econ) = P(esp) + P(Econ) - P(esp \cap Econ) = 0{,}25 + 0{,}11875 - 0{,}05$

$= 0{,}31875$

d) $P(c)^c = 1 - 0{,}31875 = 0{,}68125$

e) $P(Ec^c / Esp^c) = \frac{P(Ec^c \cap Esp^c)}{P(Esp^c)} = \frac{0{,}68125}{0{,}75} = 0{,}908$

(9) a)

$2^{\underline{o}} - 0{,}77$ ⟩ $\times 1{,}1$

$3^{\underline{o}} - 0{,}847$ ⟩ $\times 1{,}1$

$4^{\underline{o}} - 0{,}9317$ ⟩ $\times 1{,}1$

$5^{\underline{o}} - 1.0$ ⇝ No $5^{\underline{o}}$ exame

b) $0{,}3 \cdot 0{,}23 = 0{,}069$

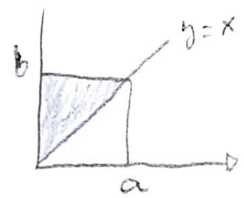
(10) $\cdot\, 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47 \rightarrow 15$

$$P(I/P) = \frac{P(I \cap P)}{P(P)} = \frac{\frac{14}{50}}{\frac{15}{50}} = \frac{14}{50} \cdot \frac{50}{15} = \frac{14}{15} = 0{,}9333$$

(11) $P_{(A)} = \frac{75}{120} = 0{,}625$

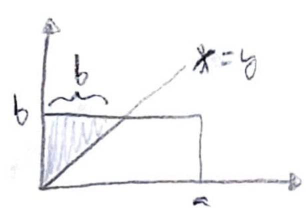
(12) a) $P(x \leq y)$

i) $a = b$



$$P(x \leq y) = \frac{1}{2}$$

ii) $a > b$



$$P(x \leq y) = \frac{\frac{b^2}{2}}{a.b} = \frac{b^2}{2ab} = \frac{b}{2a}$$

iii) $b > a$

$$P(x \leq y) = \frac{a.b - \frac{a^2}{2}}{b.a} = \frac{a(b - \frac{a}{2})}{b.a}$$

$$= \frac{b - \frac{a}{2}}{b} = \frac{2b - a}{2b}$$

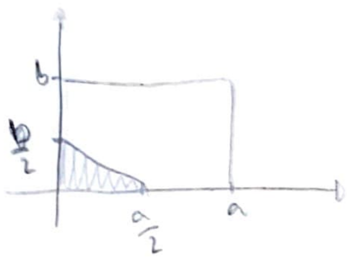
$$= 1 - \frac{a}{2b}$$

b) $P(bx + ay \leq \frac{ab}{2})$

(I) $x = 0$: $ay \leq \frac{ab}{2} \rightarrow y \leq b/2$

$y = 0$: $bx \leq \frac{ab}{2} \rightarrow x \leq \frac{a}{2}$

(II)

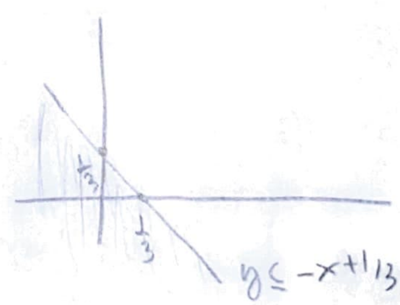


$$P(bx + ay \leq \frac{ab}{2}) = \frac{\frac{\frac{b}{2} \cdot \frac{a}{2}}{2}}{ab} = \frac{\frac{ab}{8}}{ab}$$

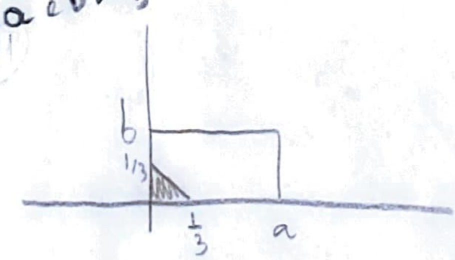
$$= \frac{ab}{8 \cdot ab} = \frac{1}{8}$$

c) $P(x+y \leq 1/3)$

(I) $x + y \leq \frac{1}{3} \rightarrow y \leq -x + \frac{1}{3}$

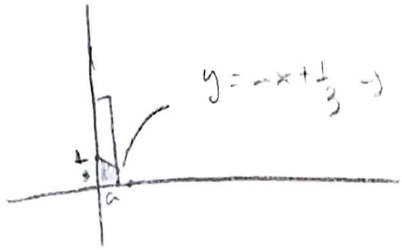


(II) $a < b > 1/3$



$$P(x+y \leq 1/3) = \frac{\left(\frac{\frac{1}{3} \cdot \frac{1}{3}}{2}\right)}{a \cdot b} = \frac{1}{18 \cdot ab}$$

(III) $a < \frac{1}{3}$ e $b > \frac{1}{3}$

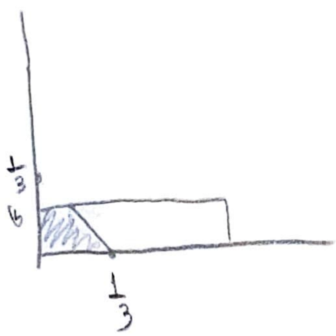


$y = -x + \frac{1}{3} \rightarrow y = -a + \frac{1}{3}$

$$P(x+y \leq \tfrac{1}{3}) = \frac{\frac{\left(-a + \frac{1}{3} + \frac{1}{3}\right) \cdot a}{2}}{ab} = \frac{\left(-a + \frac{2}{3}\right) \cdot \cancel{a}}{2\cancel{a}b}$$

$$= \frac{-a + \frac{2}{3}}{2b} = \frac{-3a + 2}{6b} = \frac{-a}{2b} + \frac{1}{3b}$$

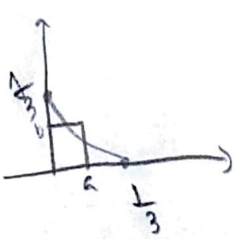
(IV) $a > \frac{1}{3}$, $b < \frac{1}{3}$



$$P(x+y \leq \tfrac{1}{3}) = \frac{\frac{\left(\frac{1}{3} + \frac{1}{3} - b\right) \cdot b}{2}}{ab}$$

$$= \frac{\frac{2}{3} - b}{2a} = \frac{-b}{2a} + \frac{1}{3a}$$

(V) $a < \frac{1}{3}$ e $b < \frac{1}{3}$



! fazer esse caso depois

separar em retangulo e trapezio

(13)

(I) 1ª branca : $\frac{b}{a+b} \cdot \frac{a}{a+b+c} = \frac{ab}{(a+b)(a+b+c)}$
2ª Azul

(II) 1ª Azul : $\frac{a}{a+b} \cdot \frac{a+c}{a+b+c} = \frac{a(a+c)}{(a+b)(a+b+c)}$
2ª Azul

$$(I)+(II) : P(A) = \frac{ab + a(a+c)}{(a+b)(a+b+c)} = \frac{a(b+a+c)}{(a+b)(a+b+c)}$$

$$= \frac{a}{a+b}$$

(14)

A - gostar 1º livro

B - gostar do 2º livro

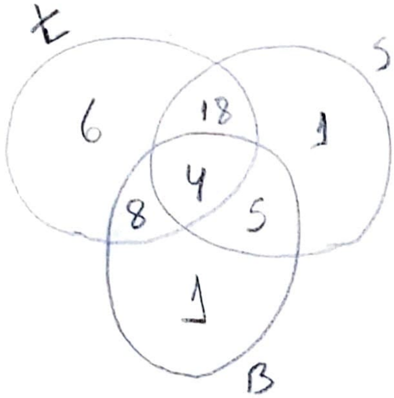
- $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0{,}5 + 0{,}4 - 0{,}3 = 0{,}6$

→ $P[(A \cup B)^c] = 1 - P(A \cup B) = 0{,}4$

(15)

$$P(A) = \frac{\binom{6}{1} \cdot \binom{5}{2}}{\binom{11}{3}} = \frac{6 \cdot 10}{165} = \frac{60}{165} = \frac{4}{11} = 0{,}363636$$

(16)



R→ $|t \cup S \cup B| = 43$

independência

(17) a) $P(A\cap B^c) = P(A) - \underbrace{P(A\cap B)}_{P(A)\cdot P(B)} = P(A) - P(A)\cdot P(B) = P(A)\cdot(1-P(B))$

$= P(A)\cdot P(B^c)$

b) $P(A^c\cap B) = P(B) - P(A\cap B) = P(B) - P(A)\cdot P(B) = P(B)\cdot(1-P(A))$

$= P(B)\cdot P(A^c)$

c) $P(A^c\cap B^c) = P[(A\cup B)^c] = 1 - P(A\cup B)$

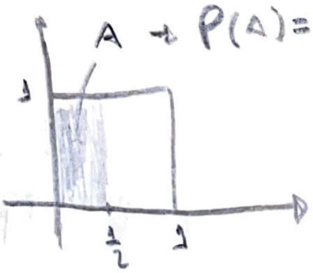
$= 1 - [P(A) + P(B) - P(A)\cdot P(B)] = 1 - P(A) - P(B) + P(A)\cdot P(B)$

$= (1-P(A)) - P(B)\cdot(1-P(A)) = (1-P(A))\cdot(1-P(B))$

$= P(A^c)\cdot P(B^c)$

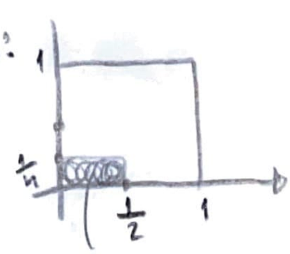
(19) I

A:



$A \to P(A) = \dfrac{\frac{1}{2}\cdot 1}{1} = \dfrac{1}{2}$

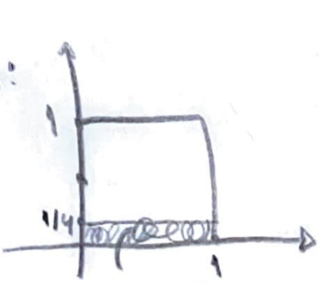
III $A\cap B$:



$A\cap B \to P(A\cap B) = \dfrac{\frac{1}{4}\cdot\frac{1}{2}}{1} = \dfrac{1}{8}$

II

B:



$B \to P(B) = \dfrac{\frac{1}{4}\cdot 1}{1} = \dfrac{1}{4}$

IV $P(A\cap B) = \dfrac{1}{8}$

$P(A)\cdot P(B) = \dfrac{1}{4}\cdot\dfrac{1}{2} = \dfrac{1}{8}$

$\to P(A\cap B) = P(A)\cdot P(B)$

18

$$P(C/A\cap B) = \frac{P(A\cap B\cap C)}{P(A\cap B)} = \frac{P(A)\,P(B)\,P(C)}{P(A)\,P(B)} = P(C)$$

21 a) considerando que $A_1^c, A_2^c, \dots A_n^c$ são independentes

$$\Rightarrow P\left(\bigcap_{i=1}^{n} A_i^c\right) = (1-P_1)\cdot(1-P_2)\cdot\ldots\cdot(1-P_n) =$$

$$= \prod_{i=1}^{n}(1-P_i)$$

b) $$P\left(\bigcup_{i=1}^{n} A_i\right) = \sum_{i=1}^{n} P_i - \sum_{i_1<i_2} P_{i_1}\cdot P_{i_2}$$

$$+\dots+(-1)^{m+1}\cdot\sum_{i_1<i_2<\dots<i_m} P_{i_1}\cdot P_{i_2}\cdot\ldots\cdot P_{i_m}$$

$$+\dots+(-1)^{n+1}\cdot P_1\cdot P_2\cdot P_3\cdots P_n$$

$$P\left(\bigcup_i^n A_i\right) = P\left(\bigcap_i^n A_i^c\right)^c$$

$$= 1 - P\left(\bigcap_i^n A_i^c\right)$$

$$= 1 - \prod_{i=1}^{n}(1-P_i)$$

Outra forma de fazer

d) $$P\left(\bigcap_{i=1}^{n} A_i\right) = P_1\cdot P_2\cdot P_3\cdot\ldots\cdot P_n$$

c) $$[P_1\cdot(1-P_2)(1-P_3)\dots(1-P_n)] + [(1-P_1)\cdot P_2\cdot(1-P_3)\cdot\ldots\cdot(1-P_n)] + \dots +$$

$$[(1-P_1)(1-P_2)(1-P_3)\dots P_n] = \sum_{i=1}^{n}\left[P_i\cdot\prod_{j\neq i}^{n}(1-P_j)\right]$$

(20) (I)

• $P(A) = P(B) = P(C) = x$ • $P(A \cap B) = P(A \cap C) = P(B \cap C) = x^2$

• $P(A \cap B \cap C) = 0$

(II) $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$

$P(A \cup B \cup C) = 3x - 3x^2 + 0$ → Para x ser o maior valor possível

$P(A \cup B \cup C) = 1$

→ $1 = 3x - 3x^2 + 0$ → $-3x^2 + 3x - 1 = 0$ (Agora eu quero o x que maximizar essa função)

→ deriva e iguala a 0

$-6x + 3 = 0$

$6x = 3$

$x = \frac{3}{6}$

$x = \frac{1}{2}$

Benito não soube fazer

NÃO tem resposta

Benito pulou.

Prob cond. e teor. de Bayes

(22)

$P(B) = 0{,}4 \quad P(A) = 0{,}7 \quad P(A \cap B) = 0{,}3$

$$P(A|B^c) = \frac{P(A \cap B^c)}{P(B^c)} = \frac{0{,}4}{0{,}6} = 0{,}666$$

• $P(A \cap B^c) = P(A) - P(A \cap B) = 0{,}7 - 0{,}3 = 0{,}4$

(23)

a) $P(A) = 1/3$

$P(B/A) = 3/5$

$$\rightarrow P(B|A) = \frac{P(B \cap A)}{P(A)} \Rightarrow \frac{3}{5} = \frac{P(B \cap A)}{1/3}$$

$$\Rightarrow \frac{3}{5} \cdot \frac{1}{3} = P(B \cap A)$$

$$\Rightarrow P(B \cap A) = \frac{1}{5}$$

válida

b) $P(A) = \frac{1}{2}$, $P(B|A) = 1$, $P(A|B) = \frac{1}{2}$

$$\rightarrow P(B|A) = \frac{P(B \cap A)}{P(A)} = 1 \Rightarrow P(B \cap A) = P(A) \Rightarrow P(B \cap A) = \frac{1}{2}$$

$$\rightarrow P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{1}{2} \Rightarrow 2 \cdot P(A \cap B) = P(B) \Rightarrow 2 \cdot \frac{1}{2} = P(B)$$

$$\Rightarrow P(B) = 1.$$

• como $P(B) = 1$, $B = \Omega$, logo como $A \subset \Omega$ então $A \subset B$

Ñ valida

(24) $P(H) = 0{,}4$ $\qquad P(F/H) = 0{,}5$

$P(M) = 0{,}6$ $\qquad P(F/M) = 0{,}3$

$$P(H/F) = \frac{P(H \cap F)}{P(F)} = \frac{P(F/H) \cdot P(H)}{P(F)} = \frac{0{,}5 \cdot 0{,}4}{0{,}5 \cdot 0{,}4 + 0{,}3 \cdot 0{,}6}$$

$$= \frac{0{,}2}{0{,}38} = 0{,}5263$$

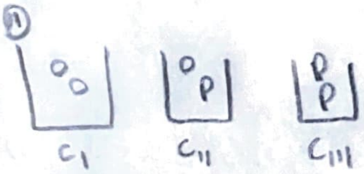
(25) $P(A) = \frac{1}{3}$ $\qquad P(D/A) = 0{,}03$

$P(B) = \frac{2}{3}$ $\qquad P(D/B) = 0{,}01$

$$P(A/D) = \frac{P(A \cap D)}{P(D)} = \frac{P(D/A) \cdot P(A)}{P(D/A) \cdot P(A) + P(D/B) \cdot P(B)} = \frac{0{,}03 \cdot \frac{1}{3}}{0{,}03 \cdot \frac{1}{3} + \frac{2}{3} \cdot 0{,}01}$$

$$= \frac{0{,}01}{0{,}0167} \cong 0{,}6$$

(26) (I)



$\rightarrow \cdot P(C_I) = P(C_{II}) = P(C_{III}) = 1/3$

$\cdot P(O/C_I) = 1 \qquad P(O/C_{II}) = \frac{1}{2} \qquad P(O/C_{III}) = 0$

(I) $P(O) = P(C_I) \cdot P(O/C_I) + P(C_{II}) \cdot P(O/C_{II}) + P(C_{III}) \cdot P(O/C_{III}) = \frac{1}{3} \cdot 1 + \frac{1}{3} \cdot \frac{1}{2} + \frac{1}{3} \cdot 0$

$= 1/2$

(II) o problema pode ser escrito como:

$$P(C_I/O) = \frac{P(C_I \cap O)}{P(O)} = \frac{P(C_I) \cdot P(O/C_I)}{P(O)} = \frac{\frac{1}{3}}{\frac{1}{2}} = \frac{1}{3} \cdot \frac{2}{1} = \frac{2}{3} = 0{,}666$$

(27) $P(+/D) = 0{,}95$ $\quad P(+/D^c) = 0{,}01$

$P(D) = 0{,}005$ $\quad P(D^c) = 0{,}995$

$$\cdot\ P(D|+) = \frac{P(D\cap +)}{P(+)} = \frac{P(+/D)\cdot P(D)}{P(+|D)\cdot P(D) + P(+|D^c)\cdot P(D^c)}$$

$$= \frac{0{,}95 \cdot 0{,}005}{0{,}95\cdot 0{,}005 + 0{,}01\cdot 0{,}995} = \frac{0{,}00475}{0{,}0147} = 0{,}3231$$

(28) (I)

A - Acertor

S - sabia resposta

NS - Não sabia a resposta

(II) $P(S) = P$; $P(NS) = 1 - P$

$\Rightarrow$ $P(A|S) = 1$ $\quad P(A|NS) = 1/n$

(III) $$P(A) = P(A|S)\cdot P(S) + P(A|NS)\cdot P(NS) = P\cdot 1 + (1-P)\cdot\frac{1}{n} = P + \frac{(1-P)}{n}$$

(IV) $$P(S/A) = \frac{P(S\cap A)}{P(A)} = \frac{P(A|S)\cdot P(S)}{P(A)} = \frac{P}{P + \frac{(1-P)}{n}} = \frac{1}{1 + \frac{(1-P)}{P\cdot n}}$$

i) $$\lim_{n\to\infty} \frac{1}{1 + \underbrace{\frac{(1-P)}{n\cdot P}}_{\to 0}} = \frac{1}{1} = 1$$

ii) $$\lim_{P\to 0} \frac{1}{1 + \underbrace{\frac{(1-P)}{P\cdot n}}_{\to\infty}} = \frac{1}{\infty} = 0$$