Lista 1 - Prob I

esqueci de fazer Venn

① a) $A \setminus (A \cap B) = A \setminus B$

$\bullet\ A \setminus (A \cap B) \leftrightarrow x \in A \wedge x \notin (A \cap B) \leftrightarrow$

$\leftrightarrow x \in A \wedge x \notin (x \in A \wedge x \in B) \leftrightarrow x \in A \wedge (x \notin A \vee x \notin B)$

$\leftrightarrow (x \in A \wedge x \notin A) \vee (x \in A \wedge x \notin B) \leftrightarrow \emptyset \vee (x \in A \wedge x \notin B)$

$\leftrightarrow (x \in A \wedge x \notin B) \leftrightarrow A \setminus B$

b) $(A \cup B) \setminus C = (A \setminus C) \cup (B \setminus C)$

$\bullet\ (A \cup B) \setminus C \leftrightarrow x \in (A \cup B) \wedge x \notin C$

$\leftrightarrow (x \in A \vee x \in B) \wedge x \notin C \leftrightarrow (x \in A \wedge x \notin C) \vee (x \in B \wedge x \notin C)$

$\leftrightarrow (A \setminus C) \cup (B \setminus C)$

c) $A \cup (B \setminus C) = (A \cup B) \setminus (C \setminus A)$

$\bullet\ A \cup (B \setminus C) \leftrightarrow x \in A \vee (x \in B \wedge x \notin C) \leftrightarrow (x \in A \vee x \in B) \wedge (x \in A \vee x \notin C)$

$\leftrightarrow (x \in A \vee x \in B) \wedge x \notin (x \in C \wedge x \notin A) \leftrightarrow (A \cup B) \setminus (C \setminus A)$

negação da negação na 2º sentença

d) $(A \cap B) \setminus B = \emptyset$

$$\bullet\ (A \cap B) \setminus B \leftrightarrow x \in A \wedge x \in B \wedge x \notin B \leftrightarrow x \in A \wedge \emptyset \leftrightarrow \emptyset$$

e) $A \setminus (A \setminus B) = A \cap B$

$$\bullet\ A \setminus (A \setminus B) \leftrightarrow x \in A \wedge x \notin (x \in A \wedge x \notin B)$$

$$\leftrightarrow x \in A \wedge (x \notin A \vee x \in B) \leftrightarrow (x \in A \wedge x \notin A) \vee (x \in A \wedge x \in B)$$

$$\leftrightarrow \emptyset \vee (x \in A \wedge x \in B) \leftrightarrow x \in A \wedge x \in B \leftrightarrow A \cap B$$

f) $[A \cup (B \cap C)] \cap \{[A^c \cup (B \cap C)] \cap (B \cap C)^c\} = \emptyset$

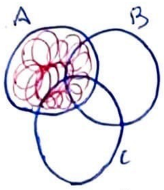
$$\bullet\ [x \in A \vee (x \in B \wedge x \in C)] \wedge \{[x \notin A \vee (x \in B \wedge x \in C)] \wedge (x \notin B \vee x \notin C)\}$$

$$\leftrightarrow [x \in A \vee (x \in B \wedge x \in C)] \wedge \{[x \notin A \wedge (x \notin B \vee x \notin C)] \vee \underbrace{[(x \in B \wedge x \in C) \wedge (x \notin B \vee x \notin C)]}_{\emptyset}$$

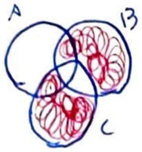
$$\leftrightarrow \underbrace{[x \in A \vee (x \in B \wedge x \in C)]}_{p} \wedge \underbrace{\{x \notin A \wedge (x \notin B \vee x \notin C)\}}_{\sim p} \leftrightarrow \emptyset$$

② $A \Delta (B \Delta C) = (A \Delta B) \Delta C$

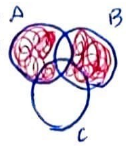
A



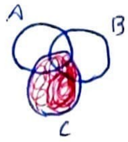
$B \Delta C = (B \setminus C) \cup (C \setminus B)$



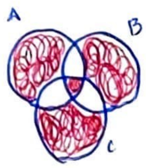
$(A \Delta B)$



C



$A \Delta (B \Delta C)$



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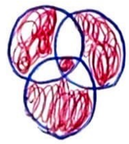
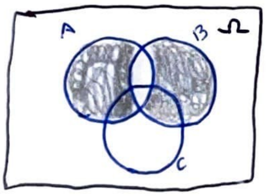
$(A \Delta B) \Delta C$



③ considerando "−" = "\"

a) $\underbrace{(A \setminus B) \cup (B \setminus A)} = \underbrace{(A \cup B) \setminus (A \cap B)}$



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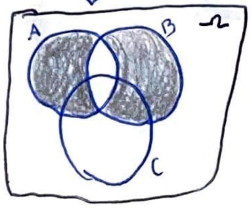
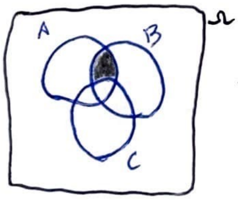


b) $\underbrace{A \cap (B \setminus C)} = \underbrace{(A \cap B) - (A \cap C)}$



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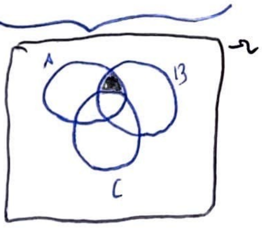
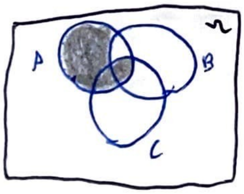


c) $\underbrace{A \setminus (B \setminus C)} = \underbrace{(A \setminus B) \cup (A \cap C)}$



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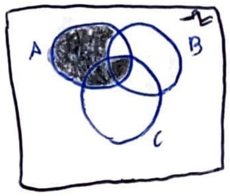
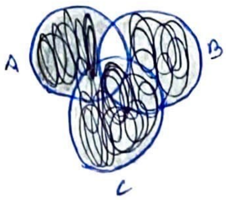


d) $\underbrace{(A \triangle B) \cup C} = \underbrace{(A \cup C) \triangle (B \setminus C)}$



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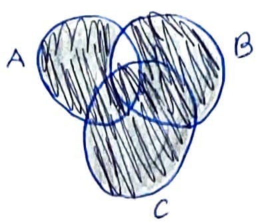

e) $\underbrace{(A \triangle B) \cap C} = \underbrace{(A \cap C) \triangle (B \cap C)}$

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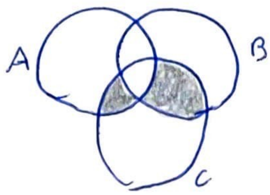

f) $\underbrace{(A \triangle B) \setminus C} = \underbrace{(A \setminus C) \triangle (B \setminus C)}$

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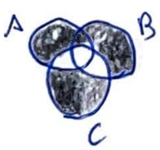
g) $\underbrace{(A \cup B) \triangle C} = \underbrace{(A \triangle C) \triangle (B \setminus A)}$



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h) $\underbrace{(A \cap B) \triangle C} = \underbrace{(A \triangle C) \triangle (A \setminus B)}$

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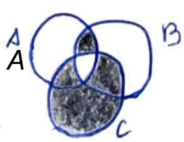

i) $\underbrace{(A \setminus B) \triangle C} = \underbrace{(A \triangle C) \triangle (A \cap B)}$

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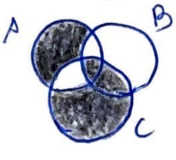
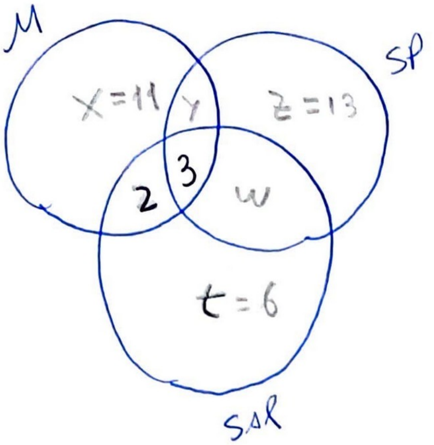


(4) total = 35 ; MANAUS - 16 ; SP - 16 ; SAL - 11 ; M/SAL - 5 , e desses 5 , 3 em SP também .



- $x + y = 11$
- $y + z + w = 13$
- $w + t = 6$
- $\underbrace{x + y}_{11} + z + \underbrace{w + t}_{6} = 30$

$\hookrightarrow 11 + z + 6 = 30$

$\boxed{z = 13}$

$y = 0 \; ; \; w = 0 \; ; \; t = 6 \; ; \; x = 11$

R → 29

(5) a) $\mathcal{A} = \{ \varnothing, \Omega \}$

b) $\mathcal{A} = \{ \varnothing, A, A^c, \Omega \}$

c) $\Omega = \{1,2,3,4,5,6\}$; $\overset{\text{conjunto das Partes}}{CP_\Omega} = \{ \varnothing, 1, 2, 3, 4, 5, 6,$
$(1,2), \ldots (5,6), (1,2,3), \ldots (6,5,4),$
$\vdots$
$(1,2,3,4,5,6) \}$

(I) $\Omega \in \mathcal{A} \Rightarrow \Omega = \{1,2,3,4,5,6\} \in CP_\Omega$ ✓

(II) $\{(5,6)\} \in CP_\Omega \Rightarrow \{(5,6)^c\} = \{(1,2,3,4)\} \in CP_\Omega$ ✓

(III) $\{\varnothing\} \cup \{1\}, \ldots, \cup \{1,2\} \ldots \cup \ldots \{1,2,3\} \ldots \cup \ldots \{1,2,3,4,5,6\} = \{1,2,3,4,5,6\} \in CP_\Omega$ ✓

R → logo CP_Ω é uma Algébra

6

a) i) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

• $A \times (B \cap C) \leftrightarrow x \in A \wedge (y \in B \wedge y \in C)$

$\leftrightarrow (x \in A \wedge y \in B) \wedge (x \in A \wedge y \in C) \leftrightarrow (A \times B) \cap (A \times C)$

ii) $A \times (B \cup C) = (A \times B) \cup (A \times C)$

• $A \times (B \cup C) \leftrightarrow x \in A \wedge (y \in B \vee y \in C) \leftrightarrow (x \in A \wedge y \in B) \vee (x \in A \wedge y \in$

$\leftrightarrow (A \times B) \cup (A \times C)$

b) $(A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D)$

• $(A \times B) \cap (C \times D) \leftrightarrow (x \in A \wedge y \in B) \wedge (x \in C \wedge y \in D)$

$\leftrightarrow (x \in A \wedge x \in C) \wedge (y \in B \wedge y \in D) \leftrightarrow (A \cap C) \times (B \cap D)$

c) $A \times \emptyset = \emptyset \times A = \emptyset$

• $A \times \emptyset \leftrightarrow x \in A \wedge y \in \emptyset \leftrightarrow \emptyset$

• $\emptyset \times A \leftrightarrow x \in \emptyset \wedge y \in A \leftrightarrow \emptyset$

(Produto cartesiano
Precisa de 2 coordenadas
Para o par ordenado existi

6

(7) $A_{10,4} = \dfrac{10!}{(10-4)!} = \dfrac{10.9.8.7.\cancel{6!}}{\cancel{6!}} = 5.040$

(8)

$\underset{5}{-}\ \underset{3}{-}\ \underset{0}{-}\ \underline{5}\ \underline{4} = 20$

$\underset{5}{-}\ \underset{3}{-}\ \underline{5}\ \underline{5}\ \underline{4} = 100$

$\underset{5}{-}\ \underset{4,6,7}{\underline{3}}\ \underline{6}\ \underline{5}\ \underline{4} = 360$

$\underset{6,7}{\underline{2}}\ \underline{7}\ \underline{6}\ \underline{5}\ \underline{4} = 1680$

R = 2160

(9)

$\underline{26}\ \underline{26}\ \underline{26}\ \underline{10}\ \underline{10}\ \underline{10}\ \underline{10} = 175.760.000$

(10)

a) $10! = 3.628.800$

b) $6! \cdot 4! = 720 \cdot 24 = 17.280$

(11)
- 4 russos
- 3 USA
- 2 GB
- 1 BR

$\Rightarrow \dfrac{10!}{4!\,3!\,2!} = \dfrac{10.9.\overset{4}{\cancel{8}}.7.\cancel{6}.5.\cancel{4!}}{\cancel{4!}\ \cancel{3}.\cancel{2}.\cancel{2}} = 12.600$

(12) ESTATISTICA

$\dfrac{11!}{3!\,2!\,2!\,2!} = \dfrac{11.10.9.8.7.\overset{3}{\cancel{6}}.5.\cancel{4}.\cancel{3!}}{\cancel{3!}\ \cancel{2}.\cancel{2}.\cancel{2}} = 831\,600$

$R \to (831.600 \times 2) - 1 = 1.663.199$

(13) 2^n

(14) (I) 1ª carta é um Rei de copas

$\underline{1}\ \underline{3}\ \underline{46} = 138$

(II) 1ª carta é uma dama de copas

$\underline{1}\ \underline{4}\ \underline{47} = 188$

$\Rightarrow R: 2.350$

(III) 1ª carta Não é rei e nem dama

$\underline{11}\ \underline{4}\ \underline{46} = 2024$

(15) $\underline{2} = 2$

$\underline{2}\ \underline{2} = 4$

$\underline{2}\ \underline{2}\ \underline{2} = 8$

$\underline{2}\ \underline{2}\ \underline{2}\ \underline{2} = 16$

R - 30

2

(16) • Cada estado pode ser representado por: A B C
AB AC BC } 6 formas

R → 6^{27}

17

a) $\Omega = \{ BC, BK, VB, VV \}$

branco colou colou vermelha

b) $\Omega = \{ 6, (\bar{6}, 6), (\bar{6}, \bar{6}, 6), \dots \}$

qualquer número sem ser 6.

c) $\Omega = \{ AA, BB, ACC, BCC, ACBB, BCAA, ACBA, BCAB \}$

d) $\Omega = \{ 0, 1, 2, 3, 4, \dots \}$

e) $\Omega = \{ (F,F,F,F), (F,F,F,M), (F,F,M,F), (F,F,M,M) \dots (M,M,M,M) \}$

f) $\Omega = \{ (\underbrace{S,S,\dots,S}_{250-S}), (\underbrace{S,S,\dots,S}_{249-S},\underbrace{N}_{1-N}), \dots (\underbrace{N,N,N,\dots,N}_{250-N}) \}$

g) $\Omega = \{ 3, 4, 5, 6, 7, 8, 9, 10 \}$

h) $\Omega = \{ AAA, AAV, AVA, AVV, VVV, VAV, VVA, VAA \}$

i) $\Omega = \{ 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12 \}$

j) $\Omega = \{ AA, AB, AC, AD, AE, BA, BB, BC, \dots tt1 \}$

k) $\Omega = \{AB, AC, AD, AE, BA, BC, BD, \ldots, ED\}$

l) $\Omega = \{AB, AC, AD, AE, BC, BD, BE, CD, CE, DE\}$

m) $\Omega = \{0°, 6°, 12°, \ldots, 354°\}$ $\rightarrow$ $\frac{360°}{60°} = 6°$ /2

n) $\Omega = \{0°, 1°, 2°, 3°, 4°, \ldots, 359°\}$

o) $\Omega = \{0, 1, 2, 3, 4, 5, 6, \ldots, 20\}$

p) $\Omega = \{C, KC, KKC, KKC, KKK\ldots C, \ldots\}$

(18) a) $A \cup B$ b) $A \cap B^c$ c) $A^c \cap B^c$ d) $(A \cap B^c)$

(19) a) $(A \cap B \cap C^c)$ b) $A^c \cap B^c \cap C^c$ c) $(A \cap B^c \cap C^c$

$\cup (A^c \cap B^c \cap$

$\cup (A \cap B^c \cap$

d) $(A \cap B \cap C^c) \cup (A \cap B^c \cap C) \cup (A \cap B^c \cap C^c)$

e) $\emptyset$

(20) a) $A \cap B^c \cap C^c$ b) $A \cap B \cap C^c$ c) $A \cap B \cap C$ d) $A \cup B \cup$

e) $A^c \cap B^c \cap C^c$ f) $(A \cap B^c \cap C^c) \cup (A^c \cap B \cap C^c) \cup (A^c$

g) $(A \cap B^c \cap C^c) \cup (A^c \cap B \cap C^c)$
$\cup (A^c \cap B^c \cap C) \cup (A^c \cap B^c \cap C^c)$ h) $(A \cap B \cap C^c) \cup (A \cap B^c \cap$

$\cup (A$

(2.1) ① Sejam $\mathcal{A}$ e $\mathcal{B}$ duas σ-Algebras sobre Ω

- Denotamos $\mathcal{F} = \mathcal{A} \cap \mathcal{B}$ a σ-Algebra interseção

(i) PD: $\Omega \in \mathcal{F}$

Prova:

$$\left.\begin{array}{l} \Omega \in \mathcal{A} \\ \Omega \in \mathcal{B} \end{array}\right\} \Rightarrow \Omega \in \mathcal{A} \cap \mathcal{B} = \mathcal{F}$$

(ii) PD: se $C \in \mathcal{F} \Rightarrow C^c \in \mathcal{F}$

Prova: se $C \in \mathcal{F} \rightarrow$

$$\left.\begin{array}{l} C \in \mathcal{A} \Rightarrow C^c \in \mathcal{A} \\ \text{e} \\ C \in \mathcal{B} \Rightarrow C^c \in \mathcal{B} \end{array}\right\} C^c \in \mathcal{A} \cap \mathcal{B} = \mathcal{F}$$

(iv)

PD: se $C_1, C_2, \dots \in \mathcal{F} \Rightarrow \left(\bigcup_i C_i\right) \in \mathcal{F}$

Prova: se $C_1, C_2, \dots \in \mathcal{F} \Rightarrow$

$$\left.\begin{array}{l} C_1, C_2, \dots \in \mathcal{A} \Rightarrow \left(\bigcup_i C_i\right) \in \mathcal{A} \\ \text{e} \\ C_1, C_2, \dots \in \mathcal{B} \Rightarrow \left(\bigcup_i C_i\right) \in \mathcal{B} \end{array}\right\} \left(\bigcup_i C_i\right) \in \mathcal{A} \cap \mathcal{B} = \mathcal{F}$$

ii) (I) $\Omega = \{0, 1, 2, 3, 4\}$ (II) $\mathcal{A} = \mathcal{P}(\Omega)$

(III)

$$P(n) = \frac{4^n \cdot 48^{4-n}}{52^4}, \quad n = 0, 1, 2, 3, 4$$

a) (I) $\Omega = \{ (x,y) \in \mathbb{R}^2 \,/\, 0 \leq x \leq 2 \text{ e } 0 \leq y \leq 2 \}$

(II) $\mathcal{A} = \mathcal{B}_{\mathbb{R}^2}$ (σ-Álgebra de Borel no plano)

(III)

$$P(A) = \frac{\text{Área}(A)}{\text{Área}(\Omega)}$$

22

b) I $\Omega = \{1, 2, 3, \dots\}$ II $\mathcal{A} = \mathcal{P}(\Omega)$

III $P(n) = \frac{4}{52} \cdot \left(\frac{48}{52}\right)^{n-1}, \ n \geq 1$

c) I $\Omega = \{(n_V, n_P, n_B) \mid n_V, n_P, n_B \in \mathbb{N}_0 \text{ e } n_V + n_P + n_B = 15\}$

V- vermelho
P- Preto
B- Branco

II $\mathcal{A} = \mathcal{P}(\Omega)$

III $P(A) = \left(\frac{5}{15}\right)^{V} \cdot \left(\frac{9}{15}\right)^{P} \cdot \left(\frac{1}{15}\right)^{B},$

sendo V, P, B, a qntd de bolas das cores vermelhas, Pretas e brancas retirados.

d) I $\Omega = \{(n_V = 5, n_P = 9, n_B = 1)\}$

II $\mathcal{A} = \{\Omega, \emptyset\}$ III $P(\Omega) = 1$

e) ① $\Omega: \{1,2,3,4,5,6\}$ ⑪ $\mathcal{A} = \mathcal{P}(\Omega)$

⑪ n: nº de retirados

$n=1: \frac{5}{10}$ $n=2: \frac{5}{10} \cdot \frac{5}{9}$ $n=3: \frac{5}{10} \cdot \frac{4}{9} \cdot \frac{5}{8}$

$n=4: \frac{5}{10} \cdot \frac{4}{9} \cdot \frac{3}{8} \cdot \frac{5}{7}$ $n=5: \frac{5}{10} \cdot \frac{4}{9} \cdot \frac{3}{8} \cdot \frac{2}{7} \cdot \frac{5}{6}$ $n=6: \frac{5}{10} \cdot \frac{4}{9} \cdot \frac{3}{8} \cdot \frac{2}{7} \cdot \frac{1}{6} \cdot \frac{5}{5}$

$$P(n) = \frac{\frac{5!}{[5-(n-1)]!}}{\frac{10!}{[10-(n-1)]!}} \cdot \frac{5}{10-(n-1)}, \quad n = 1,2,3,4,5,6$$

f) i) ① $\Omega: \{0,1,2,3,4\}$ ⑪ $\mathcal{A} = \mathcal{P}(\Omega)$

(III) n: nº de reis retirados

$n=0: \frac{48.47.46.45}{52.51.50.49}$ $n=1: \frac{4.48.47.46}{52.51.50.49}$ $n=2: \frac{4.3.48.47}{52.51.50.49}$

$n=3: \frac{4.3.2.48}{52.51.50.49}$ $n=4: \frac{4.3.2.1}{52.51.50.49}$

$$P(n) = \frac{\frac{4!}{(4-n)!} \cdot \frac{48!}{[48-(4-n)]!}}{52.51.50.49}, \quad n = 0,1,2,3,4$$