lista 4

1 intuitiva: é mais provável acertar apenas 10% das questões.

cálculo: (i) 80%: $X \sim Bin(30, 0{,}5) \Rightarrow \binom{30}{24} \cdot 0{,}5^{24} \cdot 0{,}5^{6} = 0{,}00055$

(ii) 10%: $\binom{30}{3} \cdot 0{,}5^{3} \cdot 0{,}5^{27} = 0{,}0000038$

logo, Acertar 80% é mais provável pelo cálculo.

2 $E(x) = 6$

$Var(x) = 2{,}4$

$P[(0 \leq x-1 \leq 8)^c] = ?$

(i) $\begin{cases} E(x) = np = 6 \\ Var(x) = np(1-p) = 2{,}4 \end{cases} \Rightarrow 6 \cdot (1-p) = 2{,}4 \Rightarrow 1-p = 0{,}4$

$\Rightarrow p = 0{,}6$

$\Rightarrow np = 6 \Rightarrow n \cdot 0{,}6 = 6$

$n = 10$

(ii) $X \sim Bin(10, 0{,}6)$

- $P[(0 \leq x-1 \leq 8)^c] = P[(1 \leq x \leq 9)^c] = P[x<1] + P[x>9]$

$= P[x=0] + P[x=10] = \binom{10}{0} 0{,}6^0 \cdot 0{,}4^{10} + \binom{10}{10} \cdot 0{,}6^{10} \cdot 0{,}4^0 = 0{,}4^{10} + 0{,}6^{10}$

$= 0{,}0062$

③ ① $X \sim Bin(3,p)$

⑪ $\binom{3}{2} p^2 \cdot (1-p) = 12 \cdot \binom{3}{3} p^3$

$3 \cdot p^2 \cdot (1-p) = 12 \cdot p^3$

$3 \cdot (p^2 - p^3) = 12 p^3$

$3p^2 - 3p^3 = 12 p^3$

$3p^2 = 15p^3 \Rightarrow 15p = 3 \Rightarrow p = 0{,}2$

④

①

$$\binom{5}{3} p^3 (1-p)^2 + \binom{5}{4} p^4 (1-p) + \binom{5}{5} p^5 > \binom{3}{2} p^2 (1-p) + \binom{3}{3} p^3$$

$$10 \cdot p^3 \cdot (1-p)^2 + 5p^4(1-p) + p^5 > 3p^2(1-p) + p^3$$

$$10p^3(1 - 2p + p^2) + 5p^4 - 5p^5 + p^5 > 3p^2 - 3p^3 + p^3$$

$$10p^3 - 20p^4 + 10p^5 + 5p^4 - 4p^5 > 3p^2 - \cancel{3}2p^3$$

$$6p^5 - 15p^4 + 10p^3 + 2p^3 - 3p^2 > 0 \Rightarrow 6p^5 - 15p^4 + 12p^3 - 3p^2 > 0$$

$\Rightarrow p^2(6p^3 - 15p^2 + 12p - 3) > 0 \Rightarrow$ como $p^2 > 0$, temos que

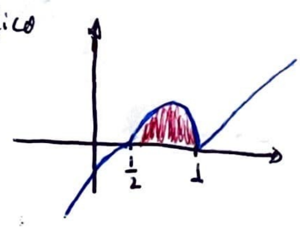
$6p^3 - 15p^2 + 12p - 3 > 0 \overset{\div 3}{\Rightarrow} 2p^3 - 5p^2 + 4p - 1 \,\overline{\overline{>}}\, 0 \Rightarrow$ vemos que $p = 1$ é raiz, com isso, por Briott Ruffini:

$$\Rightarrow \begin{array}{c|ccc|c} 1 & 2 & -5 & 4 & -1 \\ \hline & 2 & -3 & 1 & 0 \end{array} \Rightarrow (p-1)\underbrace{(2p^2 - 3p + 1)}\,\overline{\overline{>}}\, 0$$

• gráfico

$\Delta = 9 - 4 \cdot 2 \cdot 1 = 1$

$\Rightarrow \frac{3 \pm 1}{4}$ $p_1 = 1$, $p_2 = 1/2$

$\Rightarrow$ 

$\Rightarrow$ então para $p^2(6p^3 - 15p^2 + 12p - 3) > 0$,

$\frac{1}{2} < p < 1$

⑤

a)

$$\frac{p}{1-p} \cdot \frac{n-k}{k+1} \cdot P(X=k) = \frac{p}{1-p} \cdot \frac{n-k}{k+1} \cdot \binom{n}{k} \cdot p^k (1-p)^{n-k}$$

$$= \frac{p}{1-p} \cdot \frac{n-k}{k+1} \cdot \frac{n!}{k!\,(n-k)!} \cdot p^k \cdot (1-p)^{n-k} = p^{k+1} \cdot (1-p)^{n-k-1} \cdot \frac{n!}{(k+1)!} \cdot \frac{\cancel{n-k}}{\cancel{(n-k)} \cdot (n-}$$

$$= p^{k+1} \cdot (1-p)^{n-(k+1)} \cdot \frac{n!}{(k+1)! \cdot (n-(k+1))!} = \binom{n}{k+1} \cdot p^{k+1} \cdot (1-p)^{n-(k+1)} = P(X=k+1)$$

b) $X \sim Bin(6, 0{,}4)$

① $\cdot P(X=0) = \binom{6}{0} 0{,}4^0\, 0{,}6^6 = 0{,}6^6 = 0{,}0467$

- $P(X=1) = 0{,}0467 \cdot \frac{0{,}4}{0{,}6} \cdot \frac{6-0}{0+1} = 0{,}0467 \cdot \frac{0{,}4}{0{,}6} \cdot 6 = 0{,}1868$
- $P(X=2) = 0{,}1868 \cdot \frac{0{,}4}{0{,}6} \cdot \frac{6-1}{2} = 0{,}3113$
- $P(X=3) = 0{,}3113 \cdot \frac{0{,}4}{0{,}6} \cdot \frac{6-2}{3} = 0{,}2767$
- $P(X=4) = 0{,}2767 \cdot \frac{0{,}4}{0{,}6} \cdot \frac{6-3}{4} = 0{,}1384$
- $P(X=5) = 0{,}1384 \cdot \frac{0{,}4}{0{,}6} \cdot \frac{6-4}{5} = 0{,}0369$
- $P(X=6) = 0{,}0369 \cdot \frac{0{,}4}{0{,}6} \cdot \frac{6-5}{6} = 0{,}0041$

(6) $X \sim Bin(10\,,\ 0{,}01)$

- $P(X>1) =$

$= 1 - P(X \leq 1) = 1 - \left(P(X=0) + P(X=1) \right)$

$= 1 - \left[\binom{10}{0} 0{,}01^0 \cdot 0{,}99^{10} + \binom{10}{1} 0{,}01 \cdot 0{,}99^9 \right]$

$= 1 - \left[\underbrace{0{,}99^{10}}_{0{,}904} + \underbrace{10 \cdot 0{,}01 \cdot 0{,}99^9}_{0{,}091} \right] = 1 - 0{,}995$

$= 0{,}005$

(7)

$X \sim Poisson(1)$

a) $P(X=0) = \dfrac{e^{-1} \cdot 1^0}{0!} = e^{-1}$

b) $P(X \leq 2) = P(X=0) + P(X=1) + P(X=2) = e^{-1} + \dfrac{e^{-1} \cdot 1^1}{1} + \dfrac{e^{-1} \cdot 1^2}{2!}$

$= e^{-1} + e^{-1} + \dfrac{e^{-1}}{2}$

$= 2e^{-1} + \dfrac{e^{-1}}{2} = \dfrac{5e^{-1}}{2}$

c) $P(X \geq 2) =$

$1 - P(X \leq 1) = 1 - 2e^{-1}$

(8) a) (i) $P(X=1) = P(X=2) \Rightarrow \dfrac{e^{-\lambda} \cdot \lambda}{1!} = \dfrac{e^{-\lambda} \cdot \lambda^2}{2!} \Rightarrow e^{-\lambda} = \dfrac{e^{-\lambda} \cdot \lambda}{2} \Rightarrow \lambda = 2$

(ii) $P(X<4) = P(X=0) + P(X=1) + P(X=2) + P(X=3) = \dfrac{e^{-2} \cdot 2^0}{0!} + \dfrac{e^{-2} \cdot 2^1}{1} + \dfrac{e^{-2} \cdot 2^2}{2} + \dfrac{e^{-2} \cdot 2^3}{6}$

$= e^{-2} + 2e^{-2} \cdot 2 + \dfrac{e^{-2} \cdot 4}{3} = 5e^{-2} + \dfrac{4e^{-2}}{3} = \dfrac{19e^{-2}}{3}$

b)

(I) $\cdot P(x=1) = e^{-\lambda} \cdot \lambda = 0{,}1 \rightarrow e^{-\lambda} = 0{,}1/\lambda$

$\cdot P(x=2) = \dfrac{e^{-\lambda} \cdot \lambda^2}{2} = 0{,}2 \rightarrow e^{-\lambda} \cdot \lambda^2 = 0{,}4 \Rightarrow \dfrac{0{,}1}{\lambda} \cdot \lambda^2 = 0{,}4$

$\lambda = 4$

(II) $P(x=3) = \dfrac{e^{-4} \cdot 4^3}{6} = \dfrac{e^{-4} \cdot 32}{3}$

(9) $X \sim Poisson(2)$

a) $P(X \geq 1) = 1 - P(X<1) = 1 - P(X=0) = 1 - \dfrac{e^{-2} \cdot 2^0}{0!} = 1 - e^{-2} = 0{,}8647$

b)

$\bullet \; 1 - \binom{5}{0} \cdot 0{,}8647^0 \cdot 0{,}1353^5 = 0{,}9999$

c) $Y \sim Bin(5, 0{,}8647)$

(10) (I) $X \sim Bin(10, 0{,}2) \Rightarrow P(X \leq 1) = P(X=0) + P(X=1) = \binom{10}{0} 0{,}2^0 \, 0{,}8^{10} + \binom{10}{1} 0{,}2^1 \, 0{,}8^9$

$= 0{,}8^{10} + 10 \cdot 0{,}2 \cdot 0{,}8^9$

$= 0{,}1073 + 0{,}2684 = 0{,}3757$

(11) Aproximação Poisson:

(11)

$$P(X \leq k) = \sum_{x=0}^{k} p\cdot(1-p)^x = p(1-p)^0 + p(1-p)^1 + p(1-p)^2 + \dots p(1-p)^k$$

$$= p + p\left((1-p)^1 + (1-p)^2 + \dots (1-p)^k\right) = p + p\left(\sum_{x=1}^{k}(1-p)^x\right)$$

$$= p + p\left(\frac{(1-p).(1-(1-p)^k)}{1-(1-p)}\right) = p + \cancel{p}\left(\frac{(1-p)-(1-p)^{k+1}}{\cancel{p}}\right)$$

$$= \cancel{p} + 1 \cancel{-p} - (1-p)^{k+1} = 1-(1-p)^{k+1}$$

(12) $X \sim Geo(0,7)$

a) $P(X<5) = P(X\leq 4) = 1-(1-0,7)^5 = 1-0,3^5 = 0,998$

b) $P(2<X<4) = P(X=3) = 0,7 . 0,3^3 = 0,0189$

c) $P(X>8 \mid X>3) = P(X\geq 5) = 1-P(X\leq 4) = 1-(1-0,3^5) =$ $0,3^5$

d) Mostrar que $P(X>m+n \mid X>m) = P(X\geq n)$

(I) Antes de começar sabemos que $P(X\leq k) = 1-(1-p)^{k+1}$, com isso temos que

$$1-P(X\leq k) = 1-(1-(1-p)^{k+1}) = (1-p)^{k+1} = P(X\geq k+1)$$

(II) $$P(X>m+n \mid X>m) = \frac{P(X>m+n \cap X>m)}{P(X>m)} = \frac{P(X>m+n)}{P(X>m)} = \frac{1-P(X\leq m+n)^{(I)}}{1-P(X\leq m)^{(I)}}$$

$$= \frac{(1-p)^{m+n+1}}{(1-p)^{m+1}} = \frac{(1-p)^n.\cancel{(1-p)^{m+1}}}{\cancel{(1-p)^{m+1}}} = (1-p)^n = P(X\geq n)$$

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$X \sim Hgeo(3, 12, 4)$

a) $P(X \geq 2) = P(x=2) + P(x=3) = \dfrac{\binom{3}{2}\binom{9}{2}}{\binom{12}{4}} + \dfrac{\binom{3}{3}\binom{9}{1}}{\binom{12}{4}}$

$$= \frac{108 + 9}{495} = 0{,}2364$$

b) $P(x \leq 1) = P(x=0) + P(x=1) = \dfrac{\binom{3}{0}\binom{9}{4}}{\binom{12}{4}} + \dfrac{\binom{3}{1}\binom{9}{3}}{\binom{12}{4}}$

obs: Poderia fazer complementar letra A

$$= \frac{126 + 252}{495} = 0{,}7636$$

c) $P(x < 4) = P(x \leq 3) = 1$

(14) ~~[illegible]~~

$X \sim BN(4, 1/6)$

a) $P(X=6) = \binom{6+4-1}{3} \cdot \left(\frac{1}{6}\right)^4 \cdot \left(1-\frac{1}{6}\right)^6$

$= \binom{9}{3} \cdot \left(\frac{1}{6}\right)^4 \cdot \left(5/6\right)^6 = 0,022$

b) • $E(x) = \frac{r}{p} = \frac{4}{1/6} = 24$

• $VAR(x) = \frac{4 \cdot 5/6}{(1/6)^2} = \frac{4 \cdot 5 \cdot 6 \cdot 6}{6} = 120$

(15)

a) • $X \sim Bin(3, 1/6)$ • $P(x) = \binom{3}{x}\left(\frac{1}{6}\right)^x \cdot \left(\frac{5}{6}\right)^{3-x}$, $x = 0, 1, 2, 3$

b) X : nº de repetições

$X \in \{0, 1, 2\}$

• $P(X=0) = \frac{6}{6} \cdot \frac{5}{6} \cdot \frac{4}{6} = \frac{20}{36}$

• $P(x=1) = \frac{6}{6} \cdot \frac{1}{6} \cdot \frac{5}{6} \cdot 3 = \frac{15}{36}$

• $P(x=2) = \frac{6}{6} \cdot \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36}$

c) • $X \sim Geo(1/6)$

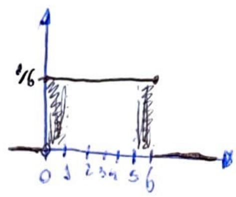
• $P(x) = \frac{1}{6} \cdot (5/6)^{x-1}$, $x = 1,2,3,\dots$

d) • $X \sim Hgeo(3,6,3)$

• $P(x) = \dfrac{\binom{3}{x}\binom{3}{3-x}}{\binom{6}{3}}$, $x = 0,1,2,3$

16 • $X \sim U[0,6]$

• $P(x<1) + P(x>5) = \frac{1}{6} + \frac{1}{6} = \frac{2}{6} = 1/3$



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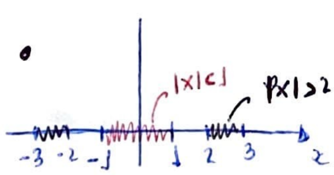
a) $3 \cdot \frac{1}{\alpha-(-\alpha)} = \frac{3}{4} \Rightarrow \frac{3}{2\alpha} = \frac{3}{4} \Rightarrow 2\alpha = 4 \Rightarrow \alpha = 2$

$X \sim U[-2,2]$

b) • $P(|x|<1) = P(|x|>2)$

•



→ $\alpha = 3$

• $X \sim U[-3,3]$

(18) Vou fazer dois casos, porque não sei ~~qual~~ o que a questão quer:

(I) $E[(x-1/2)^2] = E[x^2 - 2x \cdot \frac{1}{2} + \frac{1}{4}] = E[x^2 - x + \frac{1}{4}] = E(x^2) - E(x) + \frac{1}{4}$, vamos ac... $E(x)$ e $E(x^2)$

$\Rightarrow E(x) = 0$ e $Var(x) = E(x^2) - E(x)^2 \Rightarrow \frac{1}{12} = E(x^2) - 0^2 \Rightarrow E(x^2) = 1/12.$

$\Rightarrow$ voltando, $E[(x-1/2)^2] = E(x^2) - E(x) + 1/4 = \frac{1}{12} - 0 + \frac{1}{4} = 1/3$

(II) $[E(x-1/2)]^2 = [E(x) - \frac{1}{2}]^2 = [0 - \frac{1}{2}]^2 = (-\frac{1}{2})^2 = 1/4$

(19)

$$P(X>s+t \mid X>s) = \frac{P(X>s+t \cap X>s)}{P(X>s)} = \frac{P(X>s+t)}{P(X>s)} = \frac{e^{-\lambda(s+t)}}{e^{-\lambda s}}$$

$$= \frac{\cancel{e^{-\lambda s}} \cdot e^{-\lambda t}}{\cancel{e^{-\lambda s}}} = e^{-\lambda t} = P(X>t)$$

(20) $X \sim Exp(1/60)$

a) $P(X>70) = e^{-\frac{1}{60} \cdot 70} = e^{-7/6} = 0,3114$

b) $P(X<70 \mid X>50) = \frac{P(50<X<70)}{P(X>50)} = \frac{e^{-50/60} - e^{-70/60}}{e^{-50/60}} = \frac{e^{-5/6} - e^{-7/6}}{e^{-5/6}}$

$= 0,2835$

c) $P(X>m) = e^{-\frac{m}{60}} = 1/2$

$\Rightarrow \ln e^{-m/60} = \ln 1/2 \Rightarrow -\frac{m}{60} \cdot \cancel{\ln e} = \ln 1/2 \Rightarrow -m = 60 \cdot \ln 1/2$

$\Rightarrow -m = -41,5888$

$\Rightarrow m = 41,5888$

(21)

- $P(t>15/t>10) = P(t>10+5/t>10) = P(t>5) = e^{-\frac{1}{20}\cdot 5} = e^{-1/4}$

$= 0,779$

(22)

a) $P(X<115) = P\left(\frac{X-100}{10} < \frac{115-100}{10}\right) = P(Z<1,5) = 0,9332$

b) $P(X\geq 80) = P\left(\frac{X-100}{10} \geq \frac{80-100}{10}\right) = P(Z\geq -2) = 1 - P(Z<-2)$

$= 1 - 0,0228 = 0,9772$

c) $P(|X-100|\leq 10) = P(90<X<110) = P\left(\frac{90-100}{10} < \frac{X-100}{10} < \frac{110-100}{10}\right)$

$= P(-1<Z<1) = 0,8413 - 0,1587 = 0,6826$

d) $P(100-\alpha \leq X \leq 100+\alpha) = 0,95 \Rightarrow P\left(\frac{100-\alpha-100}{10} \leq \frac{X-100}{10} \leq \frac{100+\alpha-100}{10}\right) = 0,95$

$P\left(-\frac{\alpha}{10} \leq Z \leq \frac{\alpha}{10}\right) = 0,95 \rightarrow \frac{\alpha}{10} = 1,96 \rightarrow \alpha = 19,6$

(23)

- $X \sim N(500, 50^2)$

- $P(X>600) = P\left(\frac{X-500}{50} > \frac{600-500}{50}\right) = P(Z>2) = P(Z<-2) = 0,0228$

(24)

- $X \sim N(170, 5^2)$

a) (1) $P(X>165) = P\left(Z > \frac{165-170}{5}\right) = P(Z>-1) = 1 - P(Z<-1) = 1 - 0,1587 = 0,8413$

(1) Y: nº de Alunos com mais de 1,65 $\rightarrow Y \sim Bin(10\,000, 0,8413)$

$\Rightarrow E(Y) = n\cdot p = 10\,000 \cdot 0,8413 = 8413$ Alunos

b) - $P(-\alpha < Z < \alpha) = 0,75$

$\Rightarrow P(Z<\alpha) - P(Z<-\alpha) = P(Z<\alpha) - (1-P(Z<\alpha)) = 2\cdot P(Z<\alpha) - 1 = 0,75 \Rightarrow P(Z<\alpha) = 0,875 \Rightarrow$ Pela tabela $\alpha \cong 1,15$

$\Rightarrow$ $Z = \dfrac{X - 170}{5}$ $\Rightarrow$ (i) $1{,}15 = \dfrac{X - 170}{5}$ $\Rightarrow$ $X = 175{,}75$

(ii) $-1{,}15 = \dfrac{X_1 - 170}{5}$ $\Rightarrow$ $X_1 = 164{,}25$

R $\rightarrow$ $164{,}25 < X < 175{,}75$

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- $D_1 \sim N(42, 6^2)$
- $D_2 \sim N(45, 3^2)$

(i) 45 hrs:

D_1 - $P(X > 45) = P(Z > \dfrac{45 - 42}{6}) = P(Z > 0{,}5)$

$= P(Z < -0{,}5)$

$= 0{,}3085$

D_2 - $P(X > 45) = P(Z > \dfrac{45 - 45}{6}) = P(Z > 0) = 0{,}5$

(ii) 49 hrs

D_1 - $P(X > 49) = P(Z > 1{,}16) = P(Z < -1{,}16) = 0{,}1230$

D_2 - $P(X > 49) = P(Z > 1{,}33) = P(Z < -1{,}33) = 0{,}0918$

R $\rightarrow$ 45 horas: D_2

49 horas: D_1

(26)

a) $P(|X|>26) = P(|Z|>2) = 2 \cdot P(Z<-2) = 2 \cdot 0{,}0228 = 0{,}0456$

b) $0{,}0456 \cdot 0{,}9544^9 = 0{,}0299$

c) geometrica de parâmetro $p = 0{,}0456$

(27)

a) $M_x(t) = E(e^{tx}) = \sum_{k=0}^{+\infty} e^{tx} \cdot p(1-p)^k = p \cdot \sum_{k=0}^{+\infty} [e^t \cdot (1-p)]^x$

$\Rightarrow$ Para $E(e^{tx})$ existir a série $\sum_{k=0}^{+\infty} [e^t(1-p)]^x$ tem que ser convergente, para isso $|e^t(1-p)| < 1$, com isso $\Rightarrow$ $E(e^{tx}) = p \cdot \sum_{k=0}^{+\infty} [e^t \cdot (1-p)]^x = p \cdot \frac{1}{1-e^t(1-p)}$

$$= \frac{p}{1-(1-p)e^t}$$

b) $M_x(t) = E(e^{xt}) = \int_{-\infty}^{+\infty} e^{tx} \cdot \frac{1}{b-a} dx = \frac{1}{b-a} \cdot \int_a^b e^{tx} dx = \frac{1}{b-a} \cdot \left(\frac{e^{tx}}{t} \Big|_a^b \right)$

$$= \frac{1}{b-a} \cdot \left(\frac{e^{tb} - e^{at}}{t} \right) = \frac{e^{tb} - e^{at}}{t(b-a)}$$

c) $M_x(t) = E(e^{tx}) = \int_{-\infty}^{+\infty} e^{tx} \cdot \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{\frac{-(x-\mu)^2}{2\sigma^2}} dx = \frac{1}{\sigma\sqrt{2\pi}} \cdot \int_{-\infty}^{+\infty} e^{tx} \cdot e^{-\frac{1}{2} \cdot \left(\frac{x-\mu}{\sigma}\right)^2} dx$,

$\frac{x-\mu}{\sigma} = w$, $x = \sigma w + \mu$, $dx = \sigma dw$ $\Rightarrow$ $\frac{1}{\sigma\sqrt{2\pi}} \cdot \int_{-\infty}^{+\infty} e^{t(\sigma w + \mu)} \cdot e^{-\frac{1}{2} \cdot w^2} \sigma dw = \frac{\sigma}{\sigma\sqrt{2\pi}} \int_{-\infty}^{+\infty} e^{t(\sigma w + \mu) - \frac{1}{2} w^2} dw$

$$= \frac{1}{\sqrt{2\pi}} \cdot \int_{-\infty}^{+\infty} e^{t\sigma w + t\mu - \frac{1}{2}w^2} dw = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} e^{t\mu} \cdot e^{t\sigma w - \frac{1}{2}w^2} dw$$

$$= \frac{e^{t\mu}}{\sqrt{2\pi}} \cdot \int_{-\infty}^{+\infty} e^{t.\sigma w - \frac{1}{2}w^2} dw = \frac{e^{t\mu}}{\sqrt{2\pi}} \cdot \int_{-\infty}^{+\infty} e^{-1/2(w^2 - 2t\sigma w)} dw$$

$$= \frac{e^{t\mu}}{\sqrt{2\pi}} \cdot \int_{-\infty}^{+\infty} e^{-1/2([w - t\sigma]^2 - t^2\sigma^2)} dw = \frac{e^{t\mu}}{\sqrt{2\pi}} \cdot e^{t^2\sigma^2/2} \int_{-\infty}^{+\infty} e^{-1/2 \cdot (w - t\sigma)^2} dw ,$$

$y = w - t\sigma$

$dy = dw$

$$\Rightarrow \frac{e^{t\mu}}{\sqrt{2\pi}} \cdot e^{t^2\sigma^2/2} \cdot \int_{-\infty}^{+\infty} e^{-(\frac{y^2}{2})} dy$$

$$= e^{t\mu} \cdot e^{t^2\sigma^2/2} \cdot \frac{1}{\sqrt{2\pi}} \cdot \int_{-\infty}^{+\infty} e^{-(y^2/2)} dy ,$$

como sabemos que $\frac{1}{\sqrt{2\pi}} \cdot \int_{-\infty}^{+\infty} e^{-(y^2/2)} dy = 1$, pois é a densidade da distribuição ~~normal~~ normal $N(0,1)$.

com isso,

$$M_{X}(t) = e^{t.\mu + \frac{t^2\sigma^2}{2}}$$

(28) ① $X \sim Bin(10, 0{,}5) \rightarrow M_X(t) = (0{,}5 + 0{,}5 \cdot e^t)^{10}$

②

$$M_Y(t) = M_{3X-1}(t) = e^{-t} \cdot M_X(3t) = e^{-t} \cdot (0{,}5 + 0{,}5 \cdot e^{3t})^{10}$$

$$-E(Y) = M_Y'(t)\big|_{t=0} = -e^{-t} \cdot (0{,}5+0{,}5e^{3t})^{10} + e^{-t} \cdot 10 \cdot (0{,}5+0{,}5e^{3t})^{9} \cdot 0{,}5 \cdot 3 \cdot e^{3t}$$

$$\rightarrow t=0:\ -1 \cdot (0{,}5+0{,}5)^{10} + 1 \cdot 10\,(0{,}5+0{,}5)^{9} \cdot 1{,}5 =$$

$$= -1 + 15 = 14$$

(29) ① $X \sim Poisson(2) \rightarrow M_X(t) = e^{2(e^t - 1)}$

②

$$M_Y(t) = M_{-\frac{1}{3}X + \frac{2}{3}}(t) = e^{2t/3} \cdot M_X(-\tfrac{1}{3}t) = e^{2t/3} \cdot e^{2(e^{-t/3} - 1)}$$

$$-E(Y) = M_Y'(t)\big|_{t=0} = e^{2t/3} \cdot \frac{2}{3} \cdot e^{2(e^{-t/3}-1)} + e^{2t/3} \cdot e^{2(e^{-t/3}-1)} \cdot 2 \cdot e^{-t/3} \cdot -$$

$$\rightarrow t=0:\ 1 \cdot \frac{2}{3} \cdot \overset{1}{\cancel{e^0}} + 1 \cdot \overset{1}{\cancel{e^0}} \cdot 2 \cdot \overset{1}{\cancel{e^0}} \cdot -\frac{1}{3} = \frac{2}{3} + 2 \cdot \left(-\frac{1}{3}\right)$$

$$= \frac{2}{3} - \frac{2}{3} = 0$$

(30) $\cdot\, U = X+Y \quad \cdot\, X \sim N(0,1) \quad Y \sim N(0,1)$

$$\cdot\, M_U(t) = M_{X+Y}(t) = M_X(t) \cdot M_Y(t) = e^{t^2/2} \cdot e^{t^2/2}$$

$$= \left[e^{t^2/2}\right]^2 = e^{\frac{2 \cdot t^2}{2}} = e^{t^2}$$

$\rightsquigarrow$ U segue o modelo $N(0,2)$

(31)

$$\cdot\, M_X(t) = \frac{1}{8}(e^t+1)^3 = \left(e^t \cdot \frac{1}{2} + \frac{1}{2}\right)^3 = \left(\left(1-\frac{1}{2}\right) + \frac{1}{2} \cdot e^t\right)^3$$

$\hookrightarrow$ X segue a distribuição $Bin(3, 0{,}5)$

(32) (i)

$$X_1 \sim N(\mu, \sigma^2) \rightsquigarrow M_{X_1}(t) = e^{t\mu + \sigma^2 t^2/2}$$

$$X_2 \sim N(2\mu, \sigma^2) \rightsquigarrow M_{X_2}(t) = e^{2t\mu + \sigma^2 t^2/2}$$

$$X_3 \sim N(3\mu, \sigma^2) \rightsquigarrow M_{X_3}(t) = e^{3t\mu + \sigma^2 t^2/2}$$

$$X_4 \sim N(4\mu, \sigma^2) \rightsquigarrow M_{X_4}(t) = e^{4t\mu + \sigma^2 t^2/2}$$

(ii) $Y = \sum_{j=1}^{4} (-1)^j \cdot X_j = -X_1 + X_2 - X_3 + X_4$

(iii) $M_Y(t) = M_{(-X_1)+(X_2)+(-X_3)+(X_4)}(t) = M_{X_1}(-t) \cdot M_{X_2}(t) \cdot M_{X_3}(-t) \cdot M_{X_4}(t)$

$$= e^{-t\mu + \sigma^2 t^2/2} \cdot e^{2t\mu + \sigma^2 t^2/2} \cdot e^{-3t\mu + \sigma^2 t^2/2} \cdot e^{4t\mu + \sigma^2 t^2/2} = e^{4 \cdot \frac{\sigma^2 t^2}{2} + (-t\mu + 2t\mu - 3t\mu + 4t\mu)}$$

$$= e^{2\sigma^2 t^2 + 2\mu t}$$

(iv)

• $E(Y) = M_Y'(t)\Big|_{t=0} = e^{2\sigma^2t^2+2\mu t} \cdot (2 \cdot 2\sigma^2 t + 2\mu)$

$t=0: \; e^0 \cdot (0 + 2\mu) = 2\mu$

• $E(Y^2) = M_Y''(t) = e^{2\sigma^2t^2+2\mu t} \cdot (4\sigma^2 t + 2\mu) \cdot (4\sigma^2 t + 2\mu)$

$+ e^{2\sigma^2t^2+2\mu t} \cdot 4\sigma^2$

$t=0: \; 2\mu \cdot 2\mu + 4\sigma^2 = 4\sigma^2 + 4\mu^2$

$\rightarrow Var(Y) = E(Y^2) - E(Y)^2 = 4\sigma^2 + 4\mu^2 - 4\mu^2 = 4\sigma^2$

R: $E(Y) = 2\mu$

$Var(Y) = 4\sigma^2$

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(i)

$X_1 \sim Gama(\alpha_1, \beta) \rightarrow M_{X_1}(t) = \left(\frac{\beta}{\beta - t}\right)^{\alpha_1}$

$X_2 \sim Gama(\alpha_2, \beta) \rightarrow M_{X_2}(t) = \left(\frac{\beta}{\beta - t}\right)^{\alpha_2}$

(ii) $M_Y(t) = M_{X_1+X_2}(t) = M_{X_1}(t) \cdot M_{X_2}(t) = \left(\frac{\beta}{\beta-t}\right)^{\alpha_1} \cdot \left(\frac{\beta}{\beta-t}\right)^{\alpha_2} = \left(\frac{\beta}{\beta-t}\right)^{\alpha_1+\alpha_2}$

$\Rightarrow$ Y segue distribuição Gama $(\alpha_1+\alpha_2, \beta)$

(34)

① $Y = X+1 \rightarrow M_Y(t) = M_{X+1}(t) = e^t . M_X(t) = e^t . \dfrac{p}{1-(1-p)e^t}$

$$= \frac{e^t . p}{1-(1-p)e^t}$$

logo Y segue distribu

Geo(p), mas Y

é o nº de ensaios at

o primeiro sucesso.

$Y \sim Geo(p)$, $Y = 1, 2, \ldots$

⑪ $Z = \sqrt{X}$

35 Função distribuição

1 $\cdot\ Y = |X|$

$\cdot\ F_Y(y) = P(Y < y) = P(|X| < y) = P(-y < X < y)$

$= F_X(y) - F_X(-y)$

$\to$ derivando:

$f_Y(y) = f_X(y) - [-f_X(-y)] = f_X(y) + f_X(-y),\ y \geq 0$

1.1 Jacobiano

$\cdot\ Y = |X|,$

• Não existe uma única inversa, logo separamos em duas regiões $x \in (-\infty, 0)$ e $x \in (0, +\infty)$.

$\cdot\ y \geq 0$

$\cdot\ x \in (-\infty, 0) \therefore -x = y \to \boxed{x = -y}$

$\cdot\ \boxed{\frac{dx}{dy} = -1}$

$\cdot\ x \in (0, +\infty) \therefore \boxed{x = y}$

$\cdot\ \boxed{\frac{dx}{dy} = 1}$

$\cdot\ f_Y(y) = f_X(-y) \cdot |-1| + f_X(y) \cdot |1|$

$= f_X(y) + f_X(-y),\ y \geq 0$

36 Pag 151 (magalhaes) X possui FD $F(x)$, $Y = F_x(x)$

$$F_Y(y) = P(Y < y) = P(F_x(x) < y) = P(X < F_x^{-1}(y)) = F_x(F_x^{-1}(y)) = y \,,\; 0 < y < 1$$

$$\Rightarrow F_Y(y) = \begin{cases} 0, & y \leq 0 \\ y, & 0 < y < 1 \\ 1, & y \geq 1 \end{cases} \Rightarrow Y \sim U(0,1)$$

7) ① $X \sim N(\mu, \sigma^2)$

$Y \sim N(\mu, \sigma^2)$

37) ① $X \sim N(\mu, \sigma^2)$

$Y \sim N(\mu, \sigma^2)$

②

$$M_W(t) = M_{2X+(-3Y)}(t) = M_X(2t) \cdot M_Y(-3t)$$

$$= e^{2t\mu + \frac{\sigma^2(2t)^2}{2}} \cdot e^{-3t\mu + \frac{\sigma^2(-3t)^2}{2}} = e^{2t\mu + \frac{4\sigma^2t^2}{2}} \cdot e^{-3t\mu + \frac{9\sigma^2 t^2}{2}}$$

$$= e^{\frac{13\sigma^2t^2}{2} + (-\mu t)} = e^{-\mu t + \frac{13\sigma^2t^2}{2}}$$

$W \sim N(-\mu, 13\sigma^2)$

38) $f_X(x) = \frac{1}{2} \cdot e^{-\frac{1}{2}x}$

$\to Y = \sqrt{x}$

$\hookrightarrow x = y^2$ (inversa) $\to \frac{dx}{dy} = 2y$

$$f_Y(y) = f_X(y^2) \cdot |2y| = \frac{1}{2} \cdot e^{-\frac{y^2}{2}} \cdot 2y = y e^{-\frac{1}{2} \cdot y^2}, \quad y \geq 0$$

$\Rightarrow Y \sim \text{Weibull}(2, 1/2)$

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- $f_x(x) = \frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{x^2}{2}}$

- $Y = e^x$

- $\boxed{\ln y = x \ , \ y > 0}$

- $\boxed{\frac{dx}{dy} = \frac{1}{y}}$

- $f_y(y) = f_x(\ln y) \cdot \left|\frac{1}{y}\right| = \frac{1}{\sqrt{2\pi}} e^{-\frac{(\ln y)^2}{2}} \cdot \frac{1}{y} = \frac{1}{y\sqrt{2\pi}} \cdot e^{-\frac{(\ln y)^2}{2}}$

distribuição l

de parametros (