Prob I

Lista 10 - P2

① a)

X	0	1	2	3
P(x)	$\frac{1}{56}$	$\frac{15}{56}$	$\frac{30}{56}$	$\frac{10}{56}$

$\frac{3}{8}\cdot\frac{2}{7}\cdot\frac{1}{6}$ $\quad \left(\frac{5.3.2}{8.7.6}\right).3 \quad \left(\frac{5.4.3}{8.7.6}\right).3 \quad \frac{5}{8}\frac{4}{7}\frac{3}{6}$

b)

X	0	1	2	3
P(x)	0,05	0,26	0,44	0,24

$\frac{3.3.3}{8.8.8} \quad \left(\frac{5.3.3}{8.8.8}\right).3 \quad \left(\frac{5.5.3}{8.8.8}\right).3 \quad \frac{5.5.5}{8.8.8}$

② (i) $X \in \{-3,-2,-1,0,1,2,3\}$

- V - Vermelha
- B - Branco
- Preto - P

(ii) • -3: $P(x) = \frac{\overbrace{3.2.1}^{VVV}}{11.10.9} = \frac{6}{990}$ • -2: $P(x) = \frac{\overbrace{3.2.5}^{VVP}.3}{11.10.9} = \frac{90}{990}$

• -1: $P(x) = \frac{\overbrace{3.5.4}^{VPP}.3}{11.10.9} + \frac{\overbrace{3.2.3}^{VVB}.3}{11.10.9} = \frac{234}{990}$

• 0: $P(x) = \frac{\overbrace{5.4.3}^{PPP}}{11.10.9} + \frac{\overbrace{3.5.3}^{VPB}.3!}{11.10.9} = \frac{330}{990}$ • 1: $\frac{\overbrace{3.5.4.3}^{BPP}}{11.10.9} + \frac{\overbrace{3.2.3.3}^{BBV}}{11.10.9} = \frac{234}{990}$

• 2: $P(x) = \frac{\overbrace{3.2.5}^{BBP}.3}{11.10.9} = \frac{90}{990}$ • 3: $P(x) = \frac{3.2.1}{11.10.9} = \frac{6}{990}$

(iii) fp:

X	-3	-2	-1	0	1	2	3
P(x)	$\frac{6}{990}$	$\frac{90}{990}$	$\frac{234}{990}$	$\frac{330}{990}$	$\frac{234}{990}$	$\frac{90}{990}$	$\frac{6}{990}$

(iv) A Prob de ganharmos algum dinheiro é $\frac{330}{990} = 0,333$

3

a) i)

$$\Omega = \begin{Bmatrix} (1,1); (1,2); (1,3) \\ (2,1); (2,2); (2,3) \\ (3,1); (3,2); (3,3) \end{Bmatrix}$$

ii) $\begin{cases} X: \text{Maior número} \\ Y: \text{Menor número} \end{cases}$

•

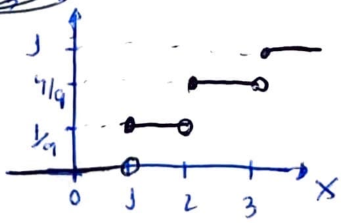
X	1	2	3
$P(x)$	$\frac{1}{9}$	$\frac{3}{9}$	$\frac{5}{9}$

•

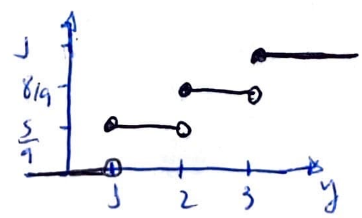
Y	1	2	3
$P(y)$	$\frac{5}{9}$	$\frac{3}{9}$	$\frac{1}{9}$

iii) FD:

X: $F_X(x) = \begin{cases} 0, & x<1 \\ 1/9, & 1\leq x<2 \\ 4/9, & 2\leq x<3 \\ 1, & x\geq 3 \end{cases}$



Y: $F_Y(y) = \begin{cases} 0, & x<1 \\ \frac{5}{9}, & 1\leq x<2 \\ 8/9, & 2\leq x<3 \\ 1, & x\geq 3 \end{cases}$



b) • $P(X>2) = 1-P(X\leq 2) = 1-F(2) = 1-\frac{4}{9} = 5/9$

• $P(Y\leq 1) = F(1) = 5/9$

c) • $P(X\leq Y) = P(X=1; Y=1) + P(X=2; Y=2) + P(X=3; Y=3) = \frac{1}{9}+\frac{1}{9}+\frac{1}{9} = 3/9 = 1/3$

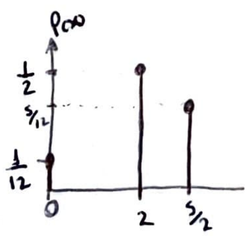
• $P(X<Y) \Rightarrow$ com Y é o menor valor não tem como Y ser exclusivamente maior que X, logo, $P(X<Y)=0$

4

i) • Função de Probabilidade (FP):

X	0	2	5/2
$P(x)$	$\frac{1}{12}$	$\frac{1}{2}$	$5/12$

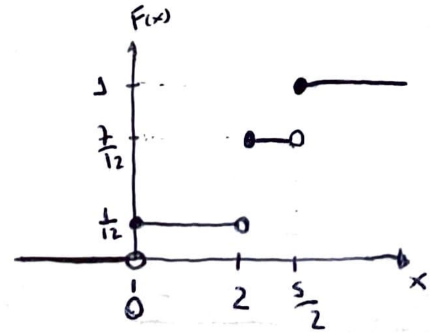
• Gráfico FP:



ii) • FD:

$$F(x) = \begin{cases} 0, & x<0 \\ \frac{1}{12}, & 0\leq x<2 \\ \frac{7}{12}, & 2\leq x< 5/2 \\ 1, & x\geq 5/2 \end{cases}$$

• Gráfico FD:



(5)
(I) $\begin{cases} \cdot\ g(x) \geq 0 \quad \forall x \\ \cdot\ \sum_i g(x_i) = 1 \end{cases}$

(II) $-\cancel{a} + b + b + \cancel{a} + \cancel{a} + b + b - \cancel{a} = 1$

$4b = 1$

$b = 1/4$

(III) $\cdot\ a \geq 0$

$\cdot\ -a + \frac{1}{4} \geq 0 \to a \leq 1/4$

$\cdot\ a + \frac{1}{4} \geq 0 \to a \geq -\frac{1}{4}$

interserção dos 3: $0 \leq a \leq \frac{1}{4}$

R $\to$ $0 \leq a \leq 1/4$

e

$b = 1/4$

(6) $\cdot\ P(x) = k(1-\theta)^{x-1}$, $0 < \theta < 1$, $x = 1, 2, \dots$

(I) $\cdot\ 1 - \theta > 0 \to (1-\theta)^{x-1} > 0 \to k \cdot (1-\theta)^{x-1} \geq 0 \to k \geq 0.$

(II) $\cdot\ \sum P(x) = 1 \to k \cdot (1-\theta)^0 + k(1-\theta)^1 + k(1-\theta)^2 + \dots = 1 \Rightarrow$

$\Rightarrow k\left((1-\theta)^0 + (1-\theta)^1 + (1-\theta)^2 + \dots + (1-\theta)^n \right) = 1$

$\Rightarrow k \cdot \sum_{n=0}^{\infty} (1-\theta)^n = 1$, Perceber que $\sum_{n=0}^{\infty} (1-\theta)^n$ é uma série Geométrica de razão $(1-\theta)$ e como $|1-\theta| < 1$, $\sum_{n=0}^{\infty} (1-\theta)^n = \frac{1}{1-(1-\theta)}$

$\Rightarrow$ voltando, $k \cdot \frac{1}{1-(1-\theta)} = 1 \Rightarrow \frac{k}{\theta} = 1 \Rightarrow k = \theta$

7

a) • $\frac{1}{2^x} \geq 0$, $\forall x$ com $x = 1, 2, \dots$ ✓

• $\sum_{x=1}^{\infty} \frac{1}{2^x} = \sum_{x=1}^{\infty} \left(\frac{1}{2}\right)^x = \lim_{x \to +\infty} \frac{(1/2)^{x+1} - 1/2}{\frac{1}{2} - 1} = \frac{0 - 1/2}{\frac{1}{2} - 1}$

$(1/2)^{x+1} \to 0$

$= \frac{\frac{1}{2}}{1 - \frac{1}{2}} = \frac{1/2}{1/2} = 1$ ✓

R → $P(x)$ é função de Probabilidade (FP)

b) • $\frac{|x|}{20} \geq 0$, $\forall x$, $x = \pm 1, \pm 2, \pm 3, \pm 4$ ✓

• $\sum_{x=-4}^{4} \frac{|x|}{20} = \frac{4}{20} + \frac{3}{20} + \frac{2}{20} + \frac{1}{20} + \frac{1}{20} + \frac{2}{20} + \frac{3}{20} + \frac{4}{20} = \frac{20}{20} = 1$ ✓

R → $P(x)$ é FP

c) • $\frac{x-1}{2^x} \geq 0$, $\forall x$, $x = 2, 3, \dots$

• $\sum_{x=2}^{+\infty} \frac{x-1}{2^x} = 1 \Rightarrow \sum_{x=2}^{+\infty} \frac{x}{2^x} - \sum_{x=2}^{+\infty} \frac{1}{2^x} = 1$

→ vamos trabalhar separadamente:

① $\sum_{x=2}^{+\infty} \frac{x}{2^x}$: • sabemos que $\sum_{x=2}^{+\infty} r^x = \frac{r^2}{1-r}$, $|r| < 1$,

com isso vamos derivar em relação a r dos dois lados:

$\sum_{x=2}^{+\infty} x \cdot r^{x-1} = \frac{r(2-r)}{(1-r)^2}$ ⇒ multiplicando r dos dois lados ⇒ $\sum_{x=2}^{+\infty} x \cdot r^{x-1} \cdot r = \frac{r^2(2-r)}{(1-r)^2}$

⇒

$$\Rightarrow \sum_{x=2}^{+\infty} x \cdot r^x = \frac{r^2 \cdot (2-r)}{(1-r)^2} \Rightarrow$$ como $|r| < 1$, r pode ser $\frac{1}{2}$, com isso,

$$\Rightarrow \sum_{x=2}^{+\infty} x \left(\frac{1}{2}\right)^x = \frac{\left(\frac{1}{2}\right)^2 \cdot \left(2 - \frac{1}{2}\right)}{\left(1-\frac{1}{2}\right)^2} \Rightarrow \sum_{x=2}^{+\infty} \frac{x}{2^x} = 2 - \frac{1}{2} = 3/2$$

(II) $\cdot \sum_{x=2}^{+\infty} \frac{1}{2^x}$: como vimos, $\sum_{x=2}^{+\infty} r^x = \frac{r^2}{1-r}$, $|r| < 1$

com isso fazendo $r = 1/2$;

$$\sum_{x=2}^{+\infty} \frac{1}{2^x} = \frac{(1/2)^2}{1-\frac{1}{2}} = \frac{\frac{1}{2} \cdot \cancel{\frac{1}{2}}}{\cancel{1/2}} = 1/2$$

(III) então:

$$\sum_{x=2}^{+\infty} \frac{x-1}{2^x} = \sum_{x=2}^{+\infty} \frac{x}{2^x} - \sum_{x=2}^{+\infty} \frac{1}{2^x} = \frac{3}{2} - \frac{1}{2} = 1$$

R: $\frac{x-1}{2^x}$ é função de Probabilidade

⑧ ① $c=1$; $\begin{cases} a-2b=0 \Rightarrow a=2b \xrightarrow{\text{II}} a=2\cdot\frac{1}{3}=2/3 \\ a+b\cancel{2}-b=1 \Rightarrow a+b=1 \end{cases}$

$\xrightarrow{\text{I}} 2b+b=1 \to \underbrace{b=1/3}$

$\rightarrow \underbrace{a=\frac{2}{3};\ b=1/3\ ;c=1}$

⑪ ~~F(x) possui densidade~~

F(x) possui densidade pois é contínua e derivável por partes

$\rightarrow \underset{\text{densidade}}{f(x)} = F'(x) = \begin{cases} 0, & x<0 \\ \frac{2}{3}, & 0\leq x<1 \\ 1/3, & 1\leq x<2 \\ 0, & x\geq 2 \end{cases}$

⑨

① $g(x)\geq 0 \Rightarrow a>0$

⑪ $\int_{-\infty}^{+\infty} \frac{1}{a}\cdot e^{-(x-b)/c}dx=1 \Rightarrow \int_{b}^{+\infty} \frac{1}{a}\cdot e^{-(x-b)/c}dx = \frac{1}{a}\cdot\int_{b}^{+\infty} e^{-(x-b)/c}dx$

$=\frac{1}{a}\cdot -c\cdot e^{-(x-b)/c}\Big|_{b}^{+\infty} = \frac{1}{a}\left(-c\cdot \underbrace{e^{-\infty/c}}_{\to 0,\ c>0} - \left[-c\cdot \underbrace{e^{-(\cancel{b-b})/c}}_{1}\right]\right)$

$=\frac{1}{a}\cdot\left(0-[-c]\right) = \frac{1}{a}\cdot c \Rightarrow \frac{c}{a}=1 \Rightarrow c=a$

R → $a=c>0,\ b\in\mathbb{R}$

(10) a) $\int_{-1}^{0} cx^2\,dx = 1 \Rightarrow c\cdot\int_{-1}^{0} x^2\,dx = c\cdot\frac{x^3}{3}\Big|_{-1}^{0} = c\left(0-(-1/3)\right)$

$\Rightarrow \frac{c}{3} = 1 \Rightarrow c = 3$

b) (I) calculando FD:

$$F(x) = \begin{cases} 0, & x < -1 \\ x^3+1, & -1 \le x \le 0 \\ 1, & x > 0 \end{cases}$$

$$\int_{-1}^{x} 3t^2\,dt = 3\int_{-1}^{x} t^2\,dt = 3\cdot\frac{t^3}{3}\Big|_{-1}^{x} = 3\cdot\left(\frac{x^3}{3}+\frac{1}{3}\right) = x^3+1$$

(II) $P(X>b \mid X<b/2) = \frac{P(b<X<b/2)}{P(X<b/2)} = \frac{F(b/2)-F(b)}{F(b/2)}$

$$= \frac{\left(\frac{b}{2}\right)^3+1-\left(b^3+1\right)}{\left(\frac{b}{2}\right)^3+1} = \frac{\frac{b^3}{8}+\cancel{1}-b^3-\cancel{1}}{\frac{b^3}{8}+1} = \frac{\frac{b^3-8b^3}{8}}{\frac{b^3+8}{8}} = \frac{-7b^3}{b^3+8}$$

c) $x^3+1 = \frac{1}{4} \Rightarrow x^3 = -\frac{3}{4} \rightarrow x = \sqrt[3]{-0{,}75} \rightarrow x = -0{,}9085$

(11) a) (I) $f(x) = \begin{cases} 0, & x<0 \\ 1-x, & 0 \le x \le 1 \\ x-1, & 1 < x \le 2 \\ 0, & x > 2 \end{cases}$

(II) FD:

- $x<0$: $F(x) = 0$
- $0 \le x \le 1$: $F(x) = \int_{0}^{x} 1-t\,dt = t-\frac{t^2}{2}\Big|_{0}^{x} = x-\frac{x^2}{2} = \frac{-x^2+2x}{2}$
- $1<x\le 2$: $F(x) = \underbrace{\int_{0}^{1} 1-t\,dt}_{1/2} + \int_{1}^{x}(t-1)\,dt =$

$= \frac{1}{2}+\left[\frac{t^2}{2}-t\right]\Big|_{1}^{x} = \frac{1}{2}+\frac{x^2}{2}-x+\frac{1}{2} = \frac{x^2-2x+2}{2}$

• $x > 2$: $F(x) = \int_0^1 1-x\,dx + \int_1^2 x-1\,dx + \underbrace{\int_2^{+\infty} 0\,dx}_{0} = \frac{1}{2} + \frac{1}{2} + 0 = 1$

$$\Rightarrow F(x) = \begin{cases} 0, & x < 0 \\ \frac{-x^2+2x}{2}, & 0 \leq x \leq 1 \\ \frac{x^2-2x+2}{2}, & 1 < x \leq 2 \\ 1, & x > 2 \end{cases}$$

b) $P(X > \frac{1}{2}) = 1 - P(X \leq 1/2) = 1 - F(1/2)$

$$= 1 - \left(\frac{-(1/2)^2 + \cancel{2} \cdot 1/\cancel{2}}{2} \right) = 1 - \left(\frac{3}{8}\right)$$

$$= 5/8$$

c) $P(X < 2/3 \,/\, X > 1/2) = \frac{P(\frac{1}{2} < X < 2/3)}{P(X > 1/2)} = \frac{F(2/3) - F(1/2)}{5/8}$

$$= \frac{\left(-\left(\frac{2}{3}\right)^2 + 2 \cdot \frac{2}{3}\right)\frac{1}{2} - \overbrace{3/8}^{F(1/2)}}{5/8} = \frac{\frac{1}{2} \cdot \left(-\frac{4}{9} + \frac{4}{3}\right) - 3/8}{5/8} =$$

$$= \frac{\frac{1}{2} \cdot \frac{8}{9} - 3/8}{5/8} = \frac{\frac{8}{18} - 3/8}{5/8} = 0{,}11$$

12)

a) I

$$f(x) = \begin{cases} \frac{a}{2} \cdot e^{-\lambda \cdot (x-u)}, & x \geq u \\ \frac{a}{2} \cdot e^{-\lambda (u-x)}, & x < u \end{cases} \quad ; \quad x \in \mathbb{R}, \lambda > 0$$

II

- $\frac{a}{2} \cdot e^{-\lambda |x-u|} \geq 0 \rightarrow a \geq 0$

- $\int_{-\infty}^{+\infty} f(x)\,dx = 1 \Rightarrow \int_{-\infty}^{u} \frac{a}{2} e^{-\lambda(u-x)}\,dx + \int_{u}^{+\infty} \frac{a}{2} \cdot e^{-\lambda(x-u)}\,dx =$

$$= \frac{a}{2} \cdot \int_{-\infty}^{u} e^{-\lambda \cdot (u-x)}\,dx + \frac{a}{2} \cdot \int_{u}^{+\infty} e^{-\lambda(x-u)}\,dx =$$

$$= \frac{a}{2} \cdot \left[\left. \frac{e^{-\lambda \cdot (u-x)}}{\lambda} \right|_{-\infty}^{u} + \left. \frac{e^{-\lambda \cdot (x-u)}}{-\lambda} \right|_{u}^{+\infty} \right] =$$

$$= \frac{a}{2} \cdot \left[\left(\frac{1}{\lambda} - 0 \right) + \left(0 - \frac{1}{-\lambda} \right) \right]$$

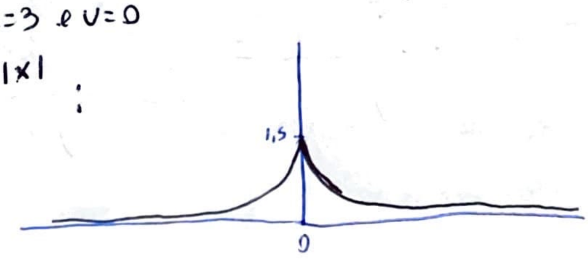
$$= \frac{a}{2} \cdot \left[\frac{1}{\lambda} + \frac{1}{\lambda} \right] = \frac{a}{2} \cdot \left[\frac{2}{\lambda} \right] = \frac{a}{\lambda} = 1$$

$\Rightarrow \underline{a = \lambda}$

R: $a = \lambda > 0$ e $u \in \mathbb{R}$

III gráfico com $\lambda = 3$ e $u = 0$

$f(x) = \frac{3}{2} \cdot e^{-3|x|}$:



b)

(I)

$$f(x) = \begin{cases} B, & -1 \leq x \leq 3 \\ 0, & cc \end{cases}$$

(II)

- $f(x) \geq 0, \forall x$

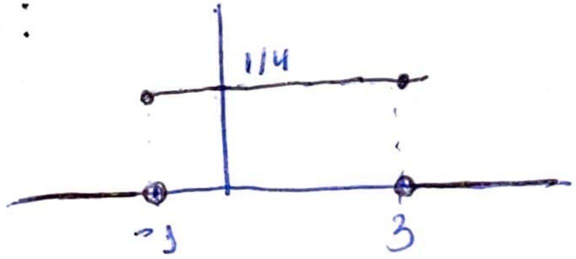
- $\int_{-\infty}^{+\infty} f(x)\,dx = 1 \Rightarrow \int_{-1}^{3} B\,dx = Bx \Big|_{-1}^{3}$

$3B - (-B) = 4B = 1 \Rightarrow \boxed{B = 1/4}$

R: $B \cdot \mathbb{I}_{[-1,3]}(x)$ é função densidade se $B = 1/4$

(III) gráfico:

$f(x) = \frac{1}{4} \cdot \mathbb{I}_{[-1,3]}$:



c) 1) $f(x) = \gamma \cdot e^{-x^2/2}$

Pesquisar sobre: • Função erro
• integral de Prob

1.1) • $f(x) \geq 0, \forall x \in \mathbb{R} \Rightarrow \gamma \geq 0$

• $\int_{-\infty}^{+\infty} \gamma \cdot e^{-x^2/2} dx = 1 \Rightarrow \gamma \cdot \underbrace{\int_{-\infty}^{+\infty} e^{-x^2/2}}_{\text{integral não elementar}} \Rightarrow \gamma \cdot \int_{-\infty}^{+\infty} e^{-(x/\sqrt{2})^2} dx$

$\Rightarrow \frac{x}{\sqrt{2}} = u$

$\frac{dx}{\sqrt{2}} = du \Rightarrow dx = \sqrt{2}du \Rightarrow \gamma \cdot \int_{-\infty} e^{-u^2} \cdot \sqrt{2} \cdot du$

$\Rightarrow \gamma \cdot \sqrt{2} \cdot \int_{-\infty}^{+\infty} e^{-u^2} = \gamma \cdot \sqrt{2} \cdot \left(\frac{\sqrt{\pi}}{2} \cdot erf(u) \Big|_{-\infty}^{+\infty} \right)$

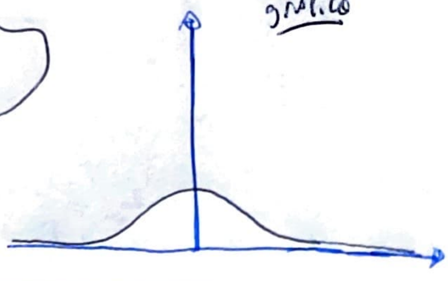
$= \gamma \cdot \sqrt{2} \cdot \left(\frac{\sqrt{\pi}}{2} \cdot erf\left(\frac{x}{\sqrt{2}}\right) \Big|_{-\infty}^{+\infty} \right) = \gamma \cdot \sqrt{2} \cdot \left(\frac{\sqrt{\pi}}{2} \cdot \underbrace{\cancel{erf(+\infty)}}_{1} - \frac{\sqrt{\pi}}{2} \underbrace{\cancel{erf(-\infty)}}_{-1} \right)$

$= \gamma \cdot \sqrt{2} \left(\frac{\sqrt{\pi}}{2} - \left(-\frac{\sqrt{\pi}}{2}\right) \right) = \gamma \cdot \sqrt{2} \cdot \left(2 \frac{\sqrt{\pi}}{2} \right)$

$= \gamma \cdot \sqrt{2} \cdot \sqrt{\pi} \Rightarrow \gamma \cdot \sqrt{2} \sqrt{\pi} = 1 \rightarrow \gamma = \frac{1}{\sqrt{2\pi}}$

R → $f(x)$ é função densidade de Probabilidade se $\gamma = \frac{1}{\sqrt{2\pi}}$

gráfico



d) $f(x) = \cos(x)$, $0 \le x \le \delta$

(II) • $f(x) \geq 0$, se $0 \le x \le 90^\circ$

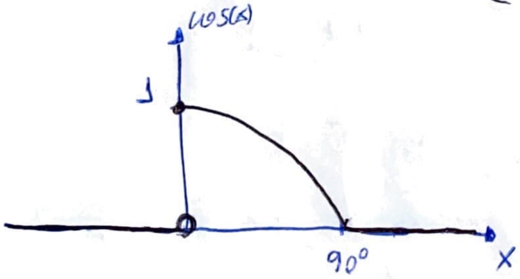
• $\displaystyle\int_0^{\delta} \cos(x)\,dx = \text{sen}(x)\Big|_0^{\delta} = \text{Sen}(\delta) - \text{Sen}(0) = 1$

$\Rightarrow \quad \text{Sen}(\delta) - 0 = 1$

$\text{Sen}(\delta) = 1 \rightarrow \delta = 90^\circ$

R→ $0 \le x \le 90^\circ$

(III) gráfico



(13)

a) • variável aleatória discreta

• fp:

X	-1	1/2	2
P(x)	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$

b) • $P(X \geq 0) = 1 - P(X<0) = 1 - F(0^-) = 1 - \frac{1}{2} = \frac{1}{2}$

• $P(X>0) = 1 - P(X \leq 0) = 1 - F(0) = 1 - F(0) = 1 - \frac{1}{2} = \frac{1}{2}$

c) • $P(X \geq -1) = 1 - P(X < -1) = 1 - F(-1^-) = 1 - 0 = 1$

• $P(X > -1) = 1 - P(X \leq -1) = 1 - F(-1) = 1 - \frac{1}{2} = 1/2$

→ A diferença em relação a (b) é que -1 é ~~valor~~ assumido pela V.A discreta e 0 não.

(14) ① FD

• $x < -1$: $\int_{-\infty}^{x} 0\,dx = 0$

• $-1 \leq x \leq 0$: $\underbrace{\int_{-\infty}^{-1} 0\,dx}_{0} + \int_{-1}^{x} -\frac{t}{2}\,dt = \int_{-1}^{x} -\frac{t}{2}\,dt = -\frac{1}{2}\int_{-1}^{x} t\,dt = -\frac{1}{2}\cdot\frac{t^2}{2}\Big|_{-1}^{x} = -\frac{1}{2}\cdot\left(\frac{x^2}{2}-\frac{1}{2}\right)$

$= -\frac{1}{2}\cdot\left(\frac{x^2-1}{2}\right) = \frac{-x^2+1}{4}$

• $0 < x \leq 1$: $\underbrace{\int_{-\infty}^{-1} 0\,dt}_{0} + \underbrace{\int_{-1}^{0} -\frac{t}{2}\,dt}_{1/4} + \int_{0}^{x} \frac{t}{2}\,dt = \frac{1}{4} + \frac{1}{2}\int_0^x t\,dt = \frac{1}{4} + \frac{1}{2}\cdot\frac{t^2}{2}\Big|_0^x$

$= \frac{1}{4} + \frac{1}{2}\cdot\left(\frac{x^2}{2} - 0\right) = \frac{1}{4} + \frac{x^2}{4} = \frac{x^2+1}{4}$

• $1 < x \leq 2$: $\underbrace{\int_{-\infty}^{-1} 0\,dt}_{0} + \underbrace{\int_{-1}^{0} -\frac{t}{2}\,dt}_{1/4} + \underbrace{\int_{0}^{1} \frac{t}{2}\,dt}_{1/4} + \int_{1}^{x} \frac{1}{2}\,dt = \frac{1}{2} + \frac{1}{2}\int_1^x 1\,dt = \frac{1}{2} + \frac{1}{2}\cdot t\Big|_1^x = \frac{1}{2} + \frac{1}{2}(x-1)$

$= \frac{1}{2} + \frac{x-1}{2} =$

$= \frac{x}{2}$

$\bullet\ x > 2:\ \underbrace{\int_{-\infty}^{-1} 0\,dt}_{0} + \underbrace{\int_{-1}^{0} -\frac{t}{2}\,dt}_{\frac{1}{4}} + \underbrace{\int_{0}^{1} \frac{t}{2}\,dt}_{\frac{1}{4}} + \underbrace{\int_{1}^{2} \frac{1}{2}\,dt}_{1/2} + \underbrace{\int_{2}^{+\infty} 0\,dt}_{0} = 1$

$$\rightarrow \quad F(x) = \begin{cases} 0, & x < -1 \\ \frac{-x^2+1}{4}, & -1 \leq x \leq 0 \\ \frac{x^2+1}{4}, & 0 < x \leq 1 \\ \frac{x}{2}, & 1 < x \leq 2 \\ 1, & x > 2 \end{cases}$$

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a) $F(+\infty) = 1 \Rightarrow \lim_{x \to +\infty} \frac{1}{c} \cdot (1 - e^{-1} + e^{-2} - \overset{\to 0}{e^{-2(x-1)}}) = 1$

$\Rightarrow \frac{1}{c} \cdot (1 - e^{-1} + e^{-2}) = 1 \Rightarrow \underline{c = 1 - e^{-1} + e^{-2}}$

b) • V.A contínua

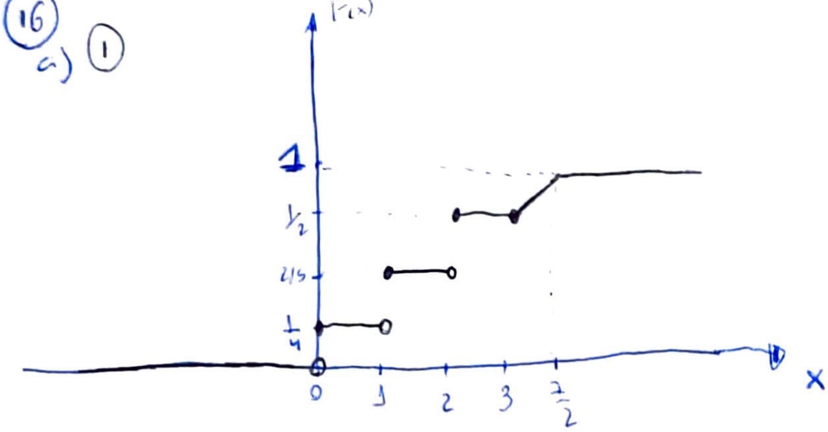
$$\bullet\ \underset{\text{densidade}}{f(x)} = F'(x) = \begin{cases} 0, & x < 1 \\ \frac{1}{c} \cdot e^{-(x-1)}, & 1 \leq x < 2 \\ \frac{2}{c} \cdot e^{-2(x-1)}, & x \geq 2 \end{cases}$$

c) $P(X \geq \frac{3}{2} \mid X < 4) = \frac{P(\frac{3}{2} \leq X < 4)}{P(X < 4)} = \frac{F(4) - F(3/2)}{F(4)} =$

$= \frac{1 - e^{-1} + e^{-2} - e^{-6} - 1 + e^{-1/2}}{1 - e^{-1} + e^{-2} - e^{-6}} = 0{,}4856$

(16)

a) (I)



(II)
- $F(x)$ ñ é decrescente ✓
- $F(x)$ é contínua à direita ✓
- $F(-\infty)=0$ e $F(+\infty)=1$ ✓

logo $F(x)$ é função de distribuição.

b) (I) Parte discreta: notar que $P(X=0)=\frac{1}{4}$, $P(X=1)=\frac{3}{20}$ e $P(X=2)=\frac{1}{10}$,

logo a parte discreta totaliza $\frac{1}{2}$ da probabilidade total.

(II) Parte contínua:

$$\begin{cases} \bullet\ P(X<0)=F(0^-)=0 \\ \bullet\ P(0<X<1)=F(0)-F(1^-)=\frac{1}{4}-\frac{1}{4}=0 \\ \bullet\ P(1<X<2)=F(2^-)-F(1)=2/5-2/5=0 \\ \bullet\ P(X>2)=1-P(X\leq 2)=1-F(2)=1-\frac{1}{2}=\frac{1}{2} \end{cases}$$

logo a parte contínua totaliza $\frac{1}{2}$ da Prob total.

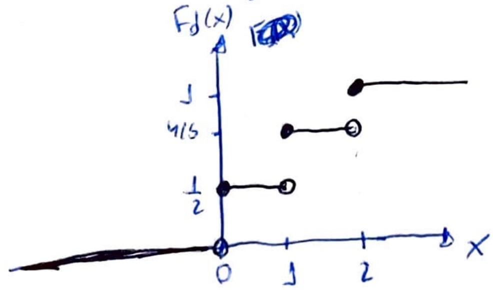
(III)
- como a parte discreta totaliza $\frac{1}{2}$ e a contínua $\frac{1}{2}$, não existe parte singular.
- Dessa forma X é do tipo mistura discreta com contínua:

$$F(x)=\alpha_d \cdot F_d(x)+\alpha_c \cdot F_c(x), \quad \alpha_d=\frac{1}{2} \text{ e } \alpha_c=\frac{1}{2}$$

(iv) Vamos Achar as FD discreta e contínua:

→ Discreta:

$$F_d(x) = \frac{1}{1/2} \cdot \begin{cases} 0, & x<0 \\ \frac{1}{4}, & 0 \le x < 1 \\ 2/5, & 1 \le x < 2 \\ 1/2, & x \geq 2 \end{cases} = \begin{cases} 0, & x<0 \\ \frac{1}{2}, & 0 \le x < 1 \\ \frac{4}{5}, & 1 \le x < 2 \\ 1, & x \geq 2 \end{cases}$$



→ Contínua:

(i) $$F'(x) = \begin{cases} 0, & x<3 \\ 1, & 3 \le x < 7/2 \\ 0, & x \geq 7/2 \end{cases}$$

(ii) • $x<3$: $\frac{1}{1/2} \cdot \int_{-\infty}^{x} 0\,dt = 2.0 = 0 \Rightarrow F_c(x) = 0$

• $3 \le x < 7/2$: $F_c(x) = 2 \cdot \left(\int_{-\infty}^{3} 0\,dt + \int_{3}^{x} 1\,dt \right) = 2.\left(0 + t\big|_3^x\right)$

$$= 2.(x-3) = 2x-6$$

• $x \geq 7/2$:

$$F_c(x) = 2 \cdot \left(\underbrace{\int_{-\infty}^{3} 0\,dt}_{0} + \underbrace{\int_{3}^{7/2} 1\,dt}_{0,5} + \underbrace{\int_{7/2}^{+\infty} 0\,dt}_{0} \right) = 2.0,5 = 1$$

→ $$F_c(x) = \begin{cases} 0, & x<3 \\ 2x-6, & 3 \le x < 7/2 \\ 1, & x \geq 7/2 \end{cases}$$ →