Prob I

Lista 3

(1) a) (I) $X \in \{36, 40, 44\}$

36	40	44
$\frac{36}{120}$	$\frac{40}{120}$	$\frac{44}{120}$

(II) $E[X] = 36 \cdot \frac{36}{120} + 40 \cdot \frac{40}{120} + 44 \cdot \frac{44}{120}$

$= 40{,}27$

b) A média é $\frac{120}{3} = 40$, A diferença se da pois quanto mais estudantes tiver dentro de um onibus, maior peso no calculo do valor esperado ele terá, e quanto menos ~~pessoas~~ estudantes, menos o peso.

(2)

(I) $f(x) = \begin{cases} 1+x, & -1 < x < 0 \\ 1-x, & 0 \leq x < 1 \end{cases}$

(II) $E[X] = \int_{-1}^{0} (1+x) \cdot x \, dx + \int_{0}^{1} x \cdot (1-x) \, dx$

$= \int_{-1}^{0} x + x^2 \, dx + \int_{0}^{1} x - x^2 \, dx = \left(\frac{x^2}{2} + \frac{x^3}{3}\right)\Big|_{-1}^{0} + \left(\frac{x^2}{2} - \frac{x^3}{3}\right)\Big|_{0}^{1}$

$= \left(-\frac{1}{6}\right) + \left(\frac{1}{6}\right) = 0$

tambem poderia fazer pela simetria

(3)

(I)

X	-2	1	2	4
$P(x)$	$\frac{1}{8}$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$

(ii) média: $E[x] = -2 \cdot \frac{1}{8} + 1 \cdot \frac{1}{2} + 2 \cdot \frac{1}{4} + 4 \cdot \frac{1}{8} = -\frac{1}{4} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2}$

$= 1,25$

(iii) variância: $VAR(x) = \underbrace{E[x^2]}_{\frac{4}{8} + \frac{1}{2} + \frac{4}{4} + \frac{16}{8} = 3} - \underbrace{E[x]^2}_{1,5625} = 1,4375$

(4)

a) (i)

X	-2	3
P(y)	P(x<0,9)	1-P(x<0,9

(ii) $P(x<0,9) = \int_0^{0,9} e^{-x} dx = -e^{-x}\Big|_0^{0,9}$

$= -e^{-0,9} - (-e^0) = -e^{0,9} + 1$

$= 0,5934$

(iii)

X	-2	3
P(x)	0,5934	0,4066

$\rightarrow E(x) = -2 \cdot 0,5934 + 3 \cdot 0,4066 = 0,033$ u.m.

(3) c) $VAR(x) = E(x^2) - E(x)^2 = 2 - 1^2 = 2 - 1 = 1$

b) Para comprovar, basta pegar alguns casos:

$E(x) = \int_0^{+\infty} x \cdot e^{-x} =$

$E(x^2) = \int_0^{+\infty} x^2 \cdot e^{-x} =$

$E(x^3) = \int_0^{+\infty} x^3 \cdot e^{-x} =$

Preguiça de fazer

(5) Como $\frac{1}{X}$ é uma função convexa, pela desigualdade de Jensen:

$$\frac{1}{E(x)} \leq E[1/x]$$, Podemos perceber que a igualdade é um caso particular.

(6) a) $$E(x) = \int_0^1 1-0\,dx + \int_1^4 1-\left(\frac{x^2}{18}+\frac{x}{18}-\frac{2}{18}\right)dx + \int_4^{+\infty} 1-1\,dx - \int_{-\infty}^{0} 0\,dx$$

$$= \int_0^1 1\,dx + \int_1^4 1-\frac{x^2}{18}-\frac{x}{18}+\frac{2}{18}\,dx + \int_4^{+\infty} 0\,dx - \int_{-\infty}^{0} 0\,dx$$

$$= x\Big|_0^1 + \left[x-\frac{x^3}{3.18}-\frac{x^2}{2.18}+\frac{2x}{18}\right]\Bigg|_1^4 + 0 - 0 = 2{,}75$$

b) $$f(x) = F'(x) = \begin{cases} \frac{1}{18}(2x+1), & 1 \leq x < 4 \\ 0, & cc \end{cases}$$

$$E(x) = \int_1^4 x \cdot \frac{1}{18}(2x+1)\,dx = \int_1^4 \frac{2x^2}{18}+\frac{x}{18}\,dx = \left[\frac{2x^3}{3.18}+\frac{x^2}{2.18}\right]\Bigg|_1^4 = 2{,}75$$

(7) $X\{2,3,4,\dots\}$

$P(X=2) = \frac{1}{2}\cdot\frac{1}{2}\cdot 2 = 2\cdot\frac{1}{4} = 2\cdot\frac{1}{2^2}$

$P(X=3) = \frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot 2 = 2\cdot\frac{1}{8} = 2\cdot\frac{1}{2^3} \Rightarrow P(X=k) = 2\cdot\frac{1}{2^k}$

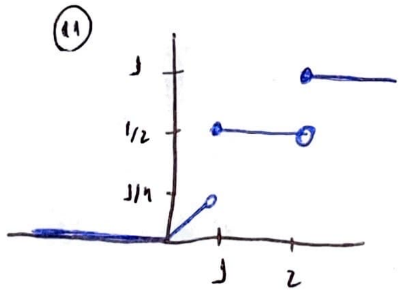
$P(X=4)\ \frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot 2 = 2\cdot\frac{1}{16} = 2\cdot\frac{1}{2^4}$

$$E(X) = \sum_{k=2}^{+\infty} k\cdot 2\cdot\frac{1}{2^k} = 2\cdot\underbrace{\sum_{k=2}^{+\infty} k\cdot\frac{1}{2^k}}_{\frac{r^2(2-r)}{(1-r)^2}} = 2\cdot\frac{3}{2} = 3$$

(8)

(8) (I) $F(x) = \begin{cases} 0, & x<0 \\ \frac{x}{4}, & 0\le x<1 \\ 1/2, & 1\le x<2 \\ 1, & x\ge 2 \end{cases}$

(II)



(III) Distribuição mista:

- discreta: $P(X=1) = 1/4$

  $P(X=2) = 1/2$

- contínua: $P(X<1) = 1/4$

  $P(1<x<2) = 0$

  $P(x>2) = 0$

(IV) $E(X) = E_d(X) + E_c(X)$

- $E_d[X] = 1\cdot\frac{1}{4} + 2\cdot\frac{1}{2} = 1{,}25$

$E_c(x)$: $f(x) = F'(x) = \begin{cases} 1/4, & 0\le x<1 \\ 0, & \text{c.c.} \end{cases} \Rightarrow E_c(X) = \int_0^1 x\cdot\frac{1}{4}\,dx = \frac{x^2}{8}\Big|_0^1 = \frac{1}{8} = 0{,}125$

$E(X) = 1{,}25 + 0{,}125 = 1{,}375$

(9)

$$E[(x-\alpha)^2] - [E(x)-\alpha]^2 = E[x^2 - 2x\alpha + \alpha^2] - [E(x)^2 - 2E(x)\cdot\alpha + \alpha^2]$$

$$= E(x^2) - \cancel{2E(x)\cdot\alpha} + \cancel{\alpha^2} - E(x)^2 + \cancel{2E(x)\alpha} - \cancel{\alpha^2} = E(x^2) - E(x)^2 = Var(x)$$

(10)

- $P(|x-\mu| < 2\sigma)$
- $1 - P(|x-\mu| \geq 2\sigma)$

$$\underbrace{1 - P(|x-\mu| \geq 2\sigma)}_{\leq \frac{Var(x)}{(2\sigma)^2} = \frac{1}{4}} \Rightarrow 1 - P(|x-\mu| \geq 2\sigma) \geq 1 - \frac{1}{4} = 3/4$$

$\frac{3}{4} \leq P(|x-\mu| < 2\sigma) \leq 1$

(11)

a) $P(x \geq 75) \leq \frac{50}{75} = \frac{2}{3}$,

logo, $P(x \geq 75) \leq 2/3$

b)

$$P(|x-\mu| < 10) = 1 - \underbrace{P(|x-\mu| \geq 10)}_{\leq \frac{25}{100} = \frac{1}{4}} \geq 1 - \frac{1}{4} = 3/4$$

$\frac{3}{4} \leq P(|x-\mu| < 10) \leq 1$

(12)

$\cdot\ M_{\alpha X+b}(t) = E[e^{t(\alpha X+b)}] = E[e^{\alpha X t+tb}] = E[e^{\alpha X t}\cdot e^{tb}]$

$= e^{tb}\cdot E[e^{(\alpha t)\cdot X}] = e^{tb}\cdot M_X(\alpha t)$

(13)

(i) $X = c$

$\hookrightarrow M_X(t) = E[e^{tX}] = E[e^{tc}] = e^{tc}$

(ii) $Y = 2X - 3$

$M_Y(t) = M_{2X-3}(t) = e^{-3t}\cdot M_X(2t) = e^{-3t}\cdot E[e^{2tX}]$

$= e^{-3t}\cdot E[e^{2tc}] = e^{-3t}\cdot e^{2tc}$

(14) (i) $P(x) = \binom{30}{x}\cdot\frac{1}{2^{30}},\ x = 1, 2, \ldots, 30$

(ii) FGM : $M_X(t) = \sum_{x=1}^{30} e^{tx}\cdot\binom{30}{x}\cdot\frac{1}{2^{30}} = \frac{1}{2^{30}}\cdot\sum_{x=1}^{30}\binom{30}{x}\cdot(e^t)^x\cdot 1^{30-x}$

$= \frac{1}{2^{30}}\cdot(e^t+1)^{30}$

(15) ① $f(x) = \frac{1}{2} \cdot e^{-|x|}$, $x \in \mathbb{R}$

(ii) FGM: $M_x(t) = \int_{-\infty}^{+\infty} e^{tx} \cdot \frac{1}{2} \cdot e^{-|x|} dx = \int_{-\infty}^{0} e^{tx} \cdot \frac{1}{2} \cdot e^{x} dx + \int_{0}^{+\infty} e^{tx} \cdot \frac{1}{2} \cdot e^{-x} dx$

$= \frac{1}{2} \cdot \int_{-\infty}^{0} e^{x(t+1)} dx + \frac{1}{2} \cdot \int_{0}^{+\infty} e^{x(t-1)} dx = \frac{1}{2} \cdot \left(\frac{e^{x(t+1)}}{t+1} \Big|_{-\infty}^{0} \right) + \frac{1}{2} \left(\frac{e^{x(t-1)}}{t-1} \Big|_{0}^{+\infty} \right)$

$= \frac{1}{2} \cdot \left(\frac{1}{t+1} - \frac{0}{t+1} \right) + \frac{1}{2} \left(\frac{0}{t-1} - \frac{1}{t-1} \right) = \frac{1}{2(t+1)} - \frac{1}{2(t-1)}$

$= -\frac{1}{t^2-1}$, com $|t| < 1$

• Momentos K-ésimo:

$K=1$: $M'_x(t)\Big|_{t=0} = \frac{-(-2t)}{(1-t^2)^2} = \frac{2t}{(1-t^2)^2} = 0$

$K=2$: $M''_x(t)\Big|_{t=0} = \frac{2(1-t^2)^{2} - 2t \cdot 2(\cancel{1-t^2}) \cdot -2t}{(\cancel{1-t^2}) \cdot (1-t^2)^3} = \frac{2(1-t^2) + 8t^2}{(1-t^2)^3} = \frac{6t^2+2}{(1-t^2)^3} = 2$

$K=3$: $M'''_x(t)\Big|_{t=0} = \left(\frac{6t^2+2}{(1-t^2)^3} \right)' = \frac{24t^3 + 24t}{(1-t^2)^4} = 0$

$K=4$: $M^{iv}_x(t)\Big|_{t=0} = \left(\frac{24t^3+24t}{(1-t^2)^4} \right)' = \frac{120t^4 + 240t^2 + 24}{(1-t^2)^5} = 24$

logo, ~~os k-ésimos momentos~~

os momentos de X, $E(X^k) = \begin{cases} k!\text{, se } k \text{ é par} \\ 0\text{, se } k \text{ é ímpar} \end{cases}$