Attitude Estimation with Extended Kalman Filter

Petar Hlad Colic

CSAS
ETSETB - UPC

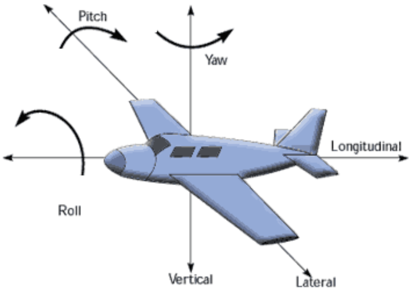
June 2022

Attitude Estimation

Attitude estimation is the process of estimating the orientation of an aerospace vehicle with respect to an intertial frame of reference.

Usually, in navigation, attitude is estimated with 3 angles:

- **Yaw**: angle respect to true north. Heading.
- **Pitch**: angle of the nose respect to the horizon. Angle of attack.
- **Roll**: angle of the wings respect to the horizon. Bank.



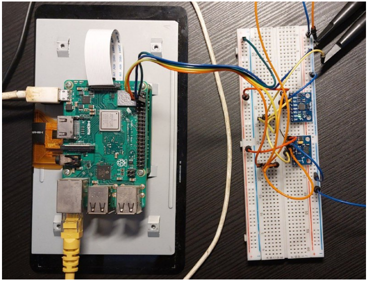
Sensors

To perform attitude estimation, sensors are needed:

- Accelerometer: gives direction of gravity.
- Gyroscope: gives rate of turn.
- Magnetometer: gives direction of magnetic north.

Setup

- Raspberry Pi 3
- MPU6050 Accel + Gyro
- ~~LSM3030DLHC Accel + Magneto~~
- Communicate through I2C using mpu6050 python library



Issues

- Magnetometer not working.
- Only pitch and roll.

Problem Formulation

Recall that the algorithm for Kalman filter is the same for every system.

Only need to define state vector x, measurement vector z, update function f, measurement function h and differential matrices F and H.

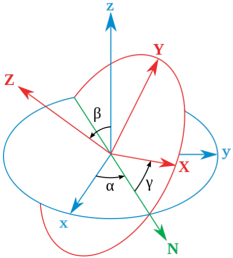
$$x_k = f(x_{k-1}, u_k) + v_k \tag{1}$$

$$y_k = h(x_k) + w_k \tag{2}$$

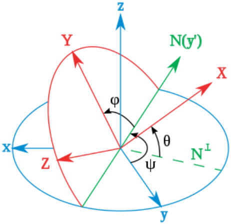
$$F = \left.\frac{\partial f(x)}{\partial x}\right|_{x=x_{k-1}}; \quad H = \left.\frac{\partial h(x)}{\partial x}\right|_{x=x_k} \tag{3}$$

Euler Angles

- Euler angles are used as a reference system to measure attitude.
- Tait-Bryan angles are used for navigation purposes. Same as Euler but the reference is different.
- Since we are not using magnetometer, we will just take into account pitch and roll (θ and ϕ resp).



Euler angles



Tait–Bryan angles

Problem matrices

Even though we are not measuring yaw (ψ), we still take it to acount in the system states since it does have an impact in giroscopic measures.

$$x_k = \begin{pmatrix} \varphi_k \\ \theta_k \\ \psi_k \end{pmatrix} \tag{4}$$

The accelerometer measurements will be transformed to attitude estimation through a simple transformation:

$$z = \begin{pmatrix} \theta \\ \varphi \end{pmatrix} = \begin{pmatrix} asin\left(\frac{a_x}{g}\right) \\ asin\left(-\frac{a_y}{g\cos\theta}\right) \end{pmatrix} \tag{5}$$

For that, the expression of H is very simple:

$$H = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \tag{6}$$

Title

We need a transformation T between the angular rates from gyroscope and the angular velocities of the fixed frame.

- $\dot{\varphi}$, $\dot{\theta}$ and $\dot{\psi}$ are the angular velocities of the fixed frame
- p, q and r are the angular rates from the gyroscopes.

$$\dot{x} = \begin{pmatrix} \dot{\varphi} \\ \dot{\theta} \\ \dot{\psi} \end{pmatrix} = \begin{pmatrix} 1 & \sin\varphi \tan\theta & \cos\varphi \tan\theta \\ 0 & \cos\varphi & -\sin\varphi \\ 0 & \frac{\sin\varphi}{\cos\theta} & \frac{\cos\varphi}{\cos\theta} \end{pmatrix} \begin{pmatrix} p \\ q \\ r \end{pmatrix} \tag{7}$$

Now the update function can be defined as:

$$f(x, p, q, r, dt) = x + T \begin{pmatrix} p \\ q \\ r \end{pmatrix} dt = x + \dot{x} dt \tag{8}$$

Problem matrices

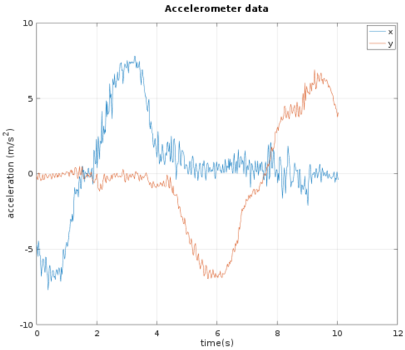
The differencial matrix is a bit trickier, since the derivatives are with respect to the euler angles and not the angle rates.

$$F = I + dt \begin{pmatrix} q\cos\varphi\tan\theta - r\sin\varphi\tan\theta & q\sin\varphi\sec^2\theta + r\cos\varphi\sec^2\theta & 0 \\ -q\sin\varphi - r\cos\varphi & 0 & 0 \\ q\cos\varphi\sec\theta - r\sin\varphi\sec\theta & q\sin\varphi\sec\theta\tan\theta + r\cos\varphi\sec\theta\tan\theta & 0 \end{pmatrix} \tag{9}$$

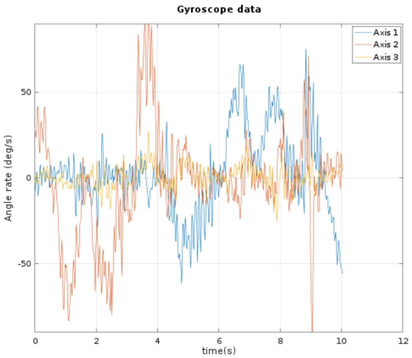
Kalman Filter

$$
\begin{aligned}
\hat{x}_k &= f(x_{k-1}, p, q, r, dt) \\
\hat{P}_k &= F_k P F_k' + Q \\
K_k &= \hat{P}_k H \left(H \hat{P}_k H' + R \right)^{-1} \\
x_k &= \hat{x}_k + K_k \left(z_k - H \hat{x}_k \right) \\
P_k &= \hat{P}_k - K H \hat{P}_k
\end{aligned} \tag{10}
$$

Measurements

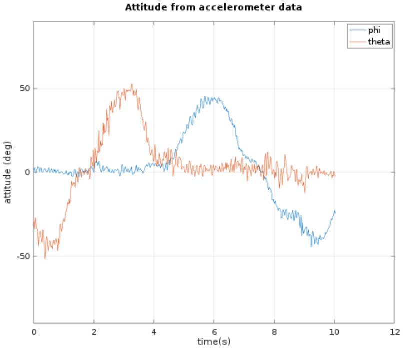


Accelerometer measurements

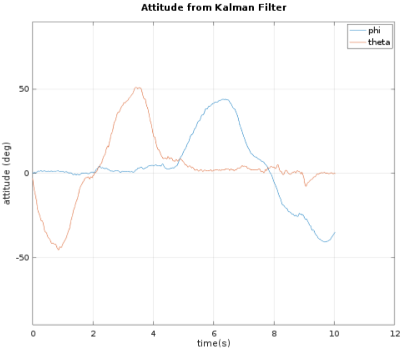


Gyroscope measurements.

Attitude without EKF



Attitude from accelerometer.



Attitude from Kalman Filter.

Conclusions

- Kalman Filter is avery good and powerful tool to work with dynamical systems
- There is a better version for attitude estimation using quaternions on a linear Kalman Filter, but the formulations is more complicated.
- To evaluate performance of setup, would need other tools to measure actual attitude.