

Problem sheet 2

Simon Van den Eynde
Petar Hlad Colic

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1 Nonrational Pentagon

Given the incidences we have given, we should show that the inner pentagon is regular, then we can use <http://mathworld.wolfram.com/Pentagon.html> to show the construction cannot be realised with rational coordinates.

2 6.9: enumerate 4-polytopes with 7 vertices

Things we can use:

- a) Theorem 6.19 in Ziegler says that a Gale diagram represents a polytope iff every co-circuit has at least 2 positive elements.
- b) There is a cyclic symmetry $(+, +, -, -, -, +) = (+, -, -, -, +, -)$
- c) We should also consider coinciding points (I don't know if we should think about special points as well (pyramids)).

3 6.15: 2 different 2-neighborly 4-polytopes

In $C_4(7)$ there are no tetrahedra, in the second Gale diagram we can find some, for example since $CH(137)$ and $CH(2)$ intersect, 4568 is a facet, since it's 3-dimensional it's a simplex.

4 6.17: analyse gale diagram

Faces are: 235678

1268 1267 3457 3458

14568 14567 12347 12348

If we have the same circuits, we have the same intersection so both 7 and 8 should lie on the line 23 and on the line 56, since these lines are different and the intersection is 1 point, they lie on the same point.

Then 2356 lie in the same plane (I'm going to think a bit about this).

Octahedron cannot be prescribed and the oriented matroid is not rigid, I don't know yet.