UPC - Discrete and algorithmic geometry

Problem sheet 2

Simon Van den Eynde
Petar Hlad Colic

November 26, 2016

1 Matousek

1.1 2

1.2 3

$$X^* = \{y \in \mathbb{R}^d : \langle x, y\rangle \leq 1, \forall x \in X\}$$
$$X^{**} = \{y \in \mathbb{R}^d : \langle x, y\rangle \leq 1, \forall x \in X^*\}$$

Now because $\forall x \in X, \forall y \in X^* : \langle x, y\rangle \leq 1 \implies x \in X^{**}$. Also clearly $0 \in X^{**}$. Since X^{**} closed and convex we find $conv(X \cup 0) \subset X^{**}$.
The separation theorem says that for a closed set Z: $conv(Z) = \bigcap$(all closed halfspaces that contain Z). So $conv(X \cup 0)$ is the intersection of all closed halfspace that contain 0 and X.

2 $C_4(7)$

2.1 (a)

We will first calculate the f-vector of $C_4(7)$.

- $f_0 = 7$
- $f_1 = \binom{7}{2} = 21$, because $C_4(7)$ is neighborly

- $f_3 = 14$, we counted this in class, using Gale's evenness criterium.

Define $h_k = \sum_{i \geq k}^{d} f_i (-1)^{(i-k)} \binom{k}{i}$, with $f_{-1} = f_d = 1$. Then the Dehn-Somerville equations learn us that $h_i = h_{d-i}$.
We find $h_0 = f_0 - f_1 + f_2 - f_3 + f_4 = 7 - 21 + f_2 - 14 + 1 = f_2 - 27$ and $h_4 = f_4 = 1$.
Now $h_0 = h_4 \implies f_3 = 28$.
So $f(C_4(7)) = (7, 21, 28, 14)$. And we find $f(C_4(7)^*) = (14, 28, 21, 7)$

3 24-cell