Problem sheet 2

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1 Matousek

1.1 2

1.2 3

$$X^* = \{y \in \mathbb{R}^d : \langle x, y \rangle \leq 1, \forall x \in X\}$$
$$X^{**} = \{y \in \mathbb{R}^d : \langle x, y \rangle \leq 1, \forall x \in X^*\}$$

Now because $\forall x \in X, \forall y \in X^* : \langle x, y \rangle \leq 1 \implies x \in X^{**}$. Also clearly $0 \in X^{**}$. Since X^{**} closed and convex we find $conv(X \cup 0) \subset X^{**}$.
The separation theorem says that for a closed set X: $conv(X) = \bigcap$(all closed halfspaces that contain X)