UPC - Commutative Algebra

Discrete Morse Theory

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1 Introduction

2 Monomial Ideals

Monomial ideals are those ideals generated by monomials. For each monomial ideal there exists a unique minimal generating set of monomials.

2.1 Definitions

Definition 2.1. Let $G = (V(G), E(G))$ be a finite simple graph. The *edge ideal* associated to G is the monomial ideal

$$I(G) = \langle x_i x_j | \{x_i, x_j\} \in E(G) \rangle \subseteq R = k[x_1, \ldots, x_n]$$

The *cover ideal* is the monomial ideal

$$J(G) = \bigcap_{\{x_i, x_j\} \in E(G)} \langle x_i, x_j \rangle \subseteq R = k[x_1, \ldots, x_n]$$

As edge ideals are square-free, we can apply the theory of Stanley Reissner ideals and simplicial complexes. [6]

Definition 2.2. A simplicial complex on $V = \{x_1, \ldots, x_n\}$ is a subset Δ of the power set of V (i.e. $\Delta \subseteq \mathcal{P}(V)$) such that

1. if $F \in \Delta$, and $G \subseteq F$, them $G \in \Delta$.
2. $\{x_i\} \in \Delta$ for all i.

3 Results

In figure 1 we show all neighbourly Gale diagrams we found.

Figure 1: All neighbourly Gale Diagrams, some isomorphic.

Figure 2: The coordinates for all 3 different neighbourly 4-polytopes on 8 vertices up to combinatorial equivalence

([1])

References

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