TML Specification

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In this document, we will define the syntax of the TML, starting with an example. We next analyse the syntax and define execution of a valid TML program on a tape in a similar manner to the execution of a TM.

Consider the following TML program.

```
1  // checks whether a binary number is divisible by 2
2  alphabet = {0, 1}
3  module isDiv2 {
4      // move to the end
5      while 0, 1 {
6          move right
7      } if blank {
8          move left
9          // check last letter is 0
10         if 0 {
11             accept
12         } if 1, blank {
13             reject
14         }
15     }
16 }
```

A program in TML is used to execute on a tape, so the syntax used guides us in executing the program on a tape.

- A valid TML *program* is composed of an *alphabet*, followed by one or more *modules*. In the example above, the alphabet of the program is $\{0, 1\}$, and the program has a single module called `isEven`.
- A module contains one or more *blocks* (a specific sequence of commands). There are two types of blocks- *basic blocks* and *switch blocks*.
- A basic block consists of *basic commands* (*changeto*, *move* or *flow* command). A basic block consists of at least one basic command, but it is not necessary for a basic block to be composed of all the basic commands. If multiple commands are present in a basic block, they must be in the following order- *changeto*, *move* and *flow* command. In the program above, there is are many basic blocks, e.g. at lines 4, 6, 8-9 and 11-12. We do not say that line 8 is a basic block by itself; we want the basic block to be as long as possible.

- A *switch block* consists of cases (*if* or *while* commands), each of which corresponds to one or more letters. A switch block must contain precisely one case for each of the letter in the alphabet, including the `blank` letter. The first block within a case block cannot be another switch block. In the program above, there is a switch block at lines 3-14 and a nested switch block at lines 7-13.
- The body of an *if* command can be composed of multiple blocks. These blocks can be both basic blocks and switch blocks. We can see this at lines 5-13; the *if* block has a basic block at line 6 and then a switch block.
- The body of a *while* command must be composed of a single basic block. The basic block cannot have a *flow* command. This is because when we execute a *while* block, the next block to run is the switch block it is in; we cannot accept, reject or go to another module.
- A switch block must be the final block present; it cannot be followed by a basic block.

$$
\begin{aligned}
\textit{program} &= \textit{alphabet}\ \textit{module}^{+} \\
\textit{alphabet} &= \texttt{alphabet =}\ \{\ \textit{seq-val}\ \} \\
\textit{module} &= \texttt{module}\ \textit{id}\ \{\ \textit{block}^{+}\ \} \\
\textit{block} &= \textit{basic-block} \mid \textit{switch-block} \\
\textit{switch-block} &= \textit{case-block}^{+} \\
\textit{case-block} &= \textit{if-block} \mid \textit{while-block} \\
\textit{if-block} &= \texttt{if}\ \textit{seq-val}\ \{\textit{block}^{+}\} \\
\textit{while-block} &= \texttt{while}\ \textit{seq-val}\ \{\ \textit{core-com}^{+}\ \} \\
\textit{basic-block} &= (\textit{core-com} \mid \textit{flow-com})^{+} \\
\textit{core-com} &= \texttt{move}\ \textit{direction} \mid \texttt{changeto}\ \textit{value} \\
\textit{flow-com} &= \texttt{goto}\ \textit{id} \mid \textit{terminate} \\
\textit{terminate} &= \texttt{reject} \mid \texttt{accept} \\
\textit{direction} &= \texttt{left} \mid \texttt{right} \\
\textit{seq-val} &= (\textit{value}\texttt{,})^{*}\ \textit{value} \\
\textit{value} &= \texttt{blank} \mid \texttt{a} \mid \texttt{b} \mid \texttt{c} \mid \ \ldots\ \mid \texttt{z} \mid \texttt{0} \mid \texttt{1} \mid \ \ldots\ \mid \texttt{9} \\
\textit{id} &= (\texttt{a} \mid \texttt{b} \mid \texttt{c} \mid \ \ldots\ \mid \texttt{z} \mid \texttt{A} \mid \texttt{B} \mid \texttt{C} \mid \ \ldots\ \mid \texttt{Z})^{+}
\end{aligned}
$$

Figure 1: The EBNF of the TML.

The EBNF of the TML is Figure 1.

We will now consider how to execute a tape on a valid TML program. Let P be a TML program with alphabet Σ and let T be a tape on Σ. We execute P on T inductively, as follows:

- At any point during execution, we maintain 3 objects- a tape on Σ, a block of P and the tapehead index.
- At the start, the tape is T; the tapehead index is 0; and the block is the first block in the first module in P.
- At some point during the execution, assume that we have the tape S, tapehead index j, with tapehead value $T(j) = t$, and a block b. We define the next triple as follows:
  - if b is a *switch* block, we take the first block from the case corresponding to the tapehead value- because the program is valid, this is a basic block; we will now refer to this block as b.
  - if b has a *changeto* `val` command, the next tape T' is given by

$$T'(x) = \begin{cases} \texttt{val} & x = i \\ T(x) & \text{otherwise.} \end{cases}$$

    If the *changeto* command is missing, then the tapehead $T' = T$.
  - if b has a *move* `dir` command, the next tapehead index is given by:

$$i' = \begin{cases} i+1 & \texttt{dir} = \texttt{right} \\ i-1 & \texttt{dir} = \texttt{left}. \end{cases}$$

    If the *move* command is missing, then $i' = i - 1$.
  - we either terminate or determine the next block b' to execute (in decreasing precedence):
    - if the block is the body of a while case block, then the next block $b' = b$, i.e. we execute this switch block again (not necessarily the same case block);
    - if the block contains a terminating *flow* command, execution is terminated and we return the terminated state (`accept` or `reject`);
    - if the block contains a *goto* `mod` command, then b' is the first block of the module `mod`;
    - if the block is not the final block in the current module, then b' is next block in this module;
    - otherwise, execution is terminated and we return the state `reject`.

  If execution is not terminated, execution continues with the next triplet.

We will now illustrate this process. We execute the program `isEven` on the tape at Figure 2.

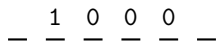
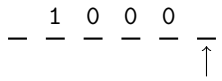


Figure 2: A TM tape on $\{0, 1\}$.

- Initially, the tape is the given tape; the current block is at lines 5-15; and the tapehead index is 0, with value 1.
- Since the tapehead value is `1`, the basic block to be executed is lines 5-7. Hence,
  - the tape remains unchanged;
  - the current block is still at lines 5-15; and
  - and the tapehead index becomes 1, with value `0`.
- The transition for `0` and `1` are the same with respect to the current block. This means that we keep moving to the right until we end up at a blank symbol. At that point, the following is the state of the tape:

  The arrow points at the tapehead value. We are still executing the same block, and the tape has not been altered.
- Now, since the tapehead value is `blank`, we move to the left. Moreover, the current block is at lines 10-14. The tape has still not been changed. The current value is now 0.
- The tapehead value is currently `0`. So,
  - the tape value changes to `blank`;
  - we have reached the `accept` command; and
  - the tapehead pointer move to the left (by default), to index 2.

  Hence, execution terminates, with result accept and the following tape state: