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Observations on Ferrite Rod Antennas

Here are the results of the author's extensive research into winding ferrite rod antennas

Although the air-core loop antenna was known from the earliest days of radio, its modern variant, the ferrite loopstick, was made possible by the development of ferrite materials in the mid 1940s.[1] Coupled with the introduction of transistors, the loopstick made it possible to produce compact, truly portable AM broadcast band receivers.

Although the Amateur Radio literature contains ferrite loop antenna design articles, their focus is upon replication of the presented designs.[2] Two notable exceptions aside, many of the peculiarities of working with ferrite loop antennas are not well covered in the amateur literature.[3] Even the readily available engineering literature gives short shrift to ferrite loops.[4]

In working with ferrite loops while building receiving antennas for the frequency range of 10 kHz to 2 MHz, I found considerable divergence between practice and theory. This article attempts to capture my experiences in building and testing more than 50 ferrite rod antenna configurations.

These results are not oriented to designs that may be copied and replicated, but rather serve as a guide for those wishing to experiment with this small corner of antenna development. Nevertheless, sufficient detail is provided to permit replication of the more successful configurations.

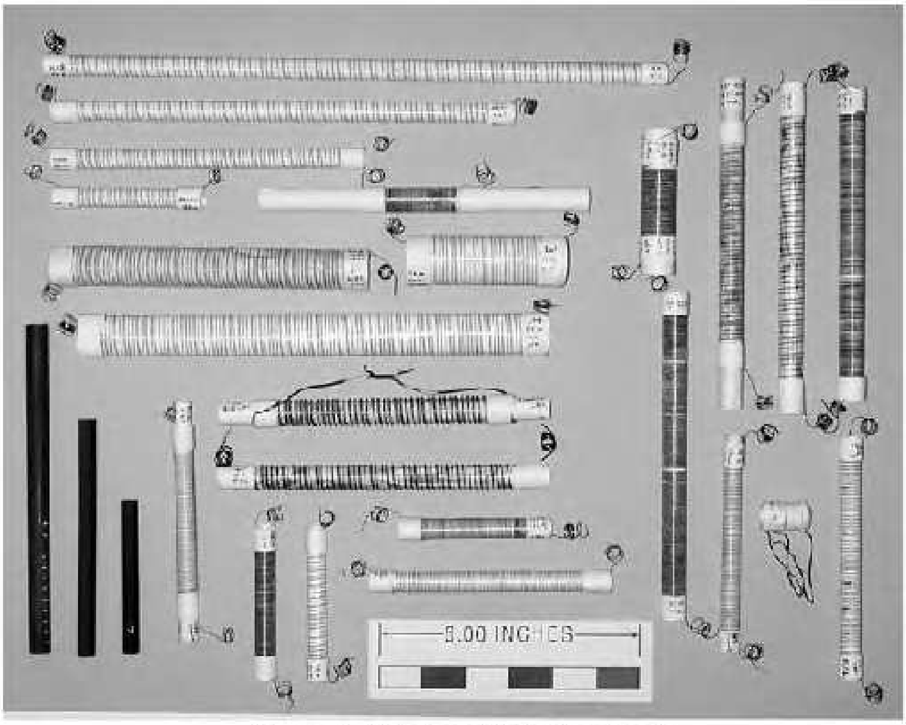

Figure 1 — Selection of test coils and rods.

Theoretical Considerations

By Faraday's law, we know the voltage induced into a loop of wire is equal to the rate of change of the magnetic flux through the loop. If the loop contains more than one turn, and is physically small, each turn's induced voltage, in essence, is in series with all other turns. Ignoring signs, since the amplitude of the received signal is our interest, we have, by Faraday's Law:[5]

$$V_{OC} = N\frac{d\Phi}{dt} \qquad \text{[Eq 1]}$$

where:

V_{OC} = Open circuit voltage (volts)

N = number of turns

Φ = magnetic flux (webers)

with $\frac{d\Phi}{dt}$

representing the rate of change of the flux with time.

If the loop is small compared with a wavelength, Φ may be assumed to be constant throughout the loop area at any instant of time. If so, the total flux, Φ, and the flux density, B, have a simple relationship:[6]

$$\Phi = BA\cos\phi \qquad \text{[Eq 2]}$$

where:

B = flux density (webers / square meter)

A = loop area (square meters)

ϕ = angle between the plane of the loop axis and the incoming flux

A radiated electromagnetic field, such as a radio wave, contains both electric and magnetic field components, related as follows:[7]

[1]Notes appear on page 34.

$E = cB$ [Eq 3]

where:

E = electric field (volts / meter)

c = the speed of light, approximately 300 × 10^6 meters / second in a vacuum

B = flux density (webers / square meter)

Solving Equation 3 for B and substituting it into Equation 2 allows us to express the open circuit voltage induced in a loop as a function of the electric field strength of the incoming electromagnetic signal.

$$V_{OC} = \frac{NA}{c}\frac{dE}{dt}\cos\phi \quad \text{[Eq 4]}$$

For radio waves of the type we are interested in receiving, both the electric and magnetic field strength varies with time in a sinusoidal fashion, such that:

$$E = E'\sin(2\pi ft) \quad \text{[Eq 5]}$$

where:

E = instantaneous value of electric field

E' = peak electric field

f = frequency in Hz

t = time

$\frac{dE}{dt}$ is thus simply

$E'2\pi f\cos(2\pi ft)$

Since we are interested only in the peak magnitude of V_{oc}, we may further simplify *($\cos(2\pi ft)=1$, $E = E'$)* with the result that

$$\frac{dE}{dt} = E2\pi f$$

We have now arrived at the relationship between the incident electric field signal strength and the induced loop voltage:

$$V_{OC} = \frac{NAE2\pi f}{c}\cos\phi \quad \text{[Eq 6]}$$

Since f/c is simply the reciprocal of wavelength, λ, we can further simplify and recast Equation 6 into its familiar form:

$$V_{OC} = \frac{2\pi ENA}{\lambda}\cos\phi \quad \text{[Eq 7]}$$

where:

λ = wavelength in meters

A loop antenna is also, of course, an inductor. If the loop is resonated, the induced voltage will be increased. Recalling from basic circuit theory that Q is also the voltage magnification factor, Equation 7 can be modified to reflect the tuned case:

$$V_{OC} = \frac{2\pi ENAQ}{\lambda}\cos\phi \quad \text{[Eq 8]}$$

where:

Q = the loaded Q of the tuned circuit

Adding a ferrite rod core to the receiving loop increases V_{oc}. Theory provides that the ferrite core, having a large relative permeability, collects and concentrates the incident magnetic flux, thereby increasing dB/dt, and consequently V_{oc}. The literature shows V_{oc} increasing directly with the effective relative permeability, μ_{eff}, of the core:[8]

$$V_{OC} = \frac{2\pi ENAQ\mu_{eff}}{\lambda}\cos\phi \quad \text{[Eq 9]}$$

where:

μ_{eff} = effective permeability of the rod/coil combination

The cos ϕ factor is responsible for the familiar null as the loop is rotated in azimuth. When the loop plane is 90° to the magnetic field, cos(ϕ) = 0 and hence V_{oc} is also zero.

Equation 9 is deceptively simple — for a given frequency and incident field strength, the received signal level is directly proportional to the loop area, the number of turns, the Q, and the effective permeability of the core.

Air cored loops are well behaved and Equation 8 describes their behavior with remarkable accuracy, as μ_{eff} = 1.0 regardless of the loop geometry. However, as the remainder of this article demonstrates, the behavior of practical ferrite rod antennas is anything but simple! We will discover, for example, that μ_{eff} is not a constant, but rather depends upon:

- The type of material used in the core;
- The length of the winding in proportion to the length of the core;
- The ratio of the length of the core to its diameter;
- The position of the windings along the core;
- The ratio of wire diameter to wire spacing; and
- The frequency

The effect of these factors, and others, are presented in the remainder of this article.

Terminology

A number of terms are used through the text, and are defined below for convenience.

- L_0 — the inductance of a test coil without a ferrite rod inserted (with an air core).
- L — the inductance with a ferrite rod inserted.
- μ_{eff} — the ratio L/L_0, or the ratio of increase in inductance when a ferrite rod is inserted into a coil with a specific geometry.

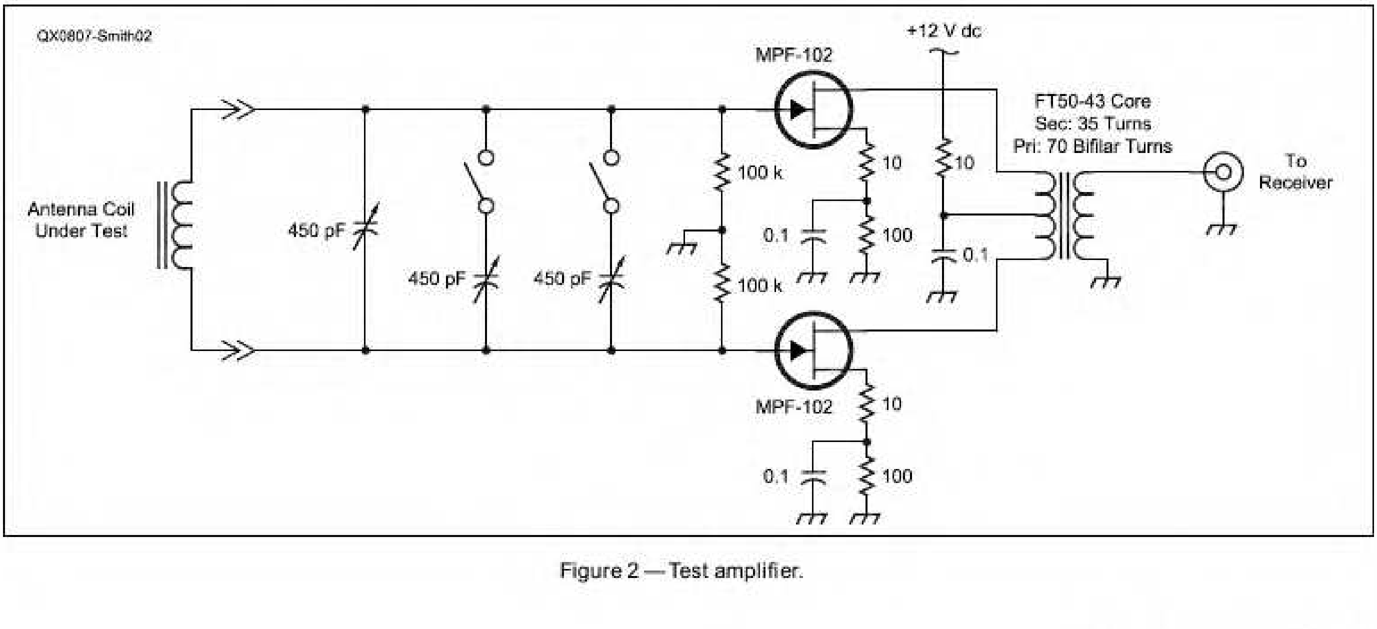

Figure 2 — Test amplifier.

• μ_0 — the permeability of free space; numerically $4\pi \times 10^{-7}$ henry/meter in SI units.

• μ_i — initial relative permeability of the *material* comprising the core, usually measured in a toroidal configuration.

• μ_{rod} — relative permeability of a ferrite rod of a specific geometry, determined using the theoretical demagnetization factor. Adjustments must be made to μ_{rod} to yield μ_{eff}.

Coil Construction

The data presented is based on more than 50 test coils I constructed and evaluated during June and July 2000. Each coil was wound over a paper core, with wire sizes from AWG no. 34 through AWG no. 16, as required for the purpose, and was given a unique number for identification. A sampling of the test coils is shown in Figure 1. Most coils were tested with at least two rod types, with varying positions, frequencies, and so on, so that several thousand individual measurements were taken.

The paper core consisted of approximately four wraps of standard copy paper, secured with household glue, wound over a steel mandrel. The mandrel was turned in a Myford Super-7 lathe to the appropriate size to permit the different ferrite cores to be snugly slid inside the core, 0.500-inch diameter for the Type 33 and Type 61 cores, and approximately 0.390-inch diameter for the surplus cores. Where multiple cores were used, such as the test of six surplus cores, arranged in a 3-long by 2-wide configuration, separate paper cores were used, which were then, in turn, wrapped with an outer paper core, upon which the windings were wrapped. After winding, the coil was coated with GC "Coil Dope" to secure the windings. The coil ends were additionally secured with several turns of 0.5-inch adhesive tape. Generous leads were allowed.

Standard paper is usually considered as an undesirable material for coil formers, as it is hygroscopic. However, for short-term test purposes, in a controlled environment, the cost and ease of working outweigh the long-term problems of the paper.

The coils were wound using a home-made wire feed installed on a Myford Super 7 lathe, equipped with a gearbox offering a range of pitch options from 8 through 56 TPI. Finer increments were available, but were not used during these tests.

Test Procedures

Inductance, Q and μ_{eff} — Inductance (both L_0 and L) and Q were measured using a Boonton Model 260A Q-meter. The rated accuracy in the frequency ranges used is ±5% for Q and ±3% for inductance. Checks using a selection of Boonton Model 103 inductance standards indicate that the Q-meter meets these performance specifications. L_0 measurements were in most cases taken at 7.9 MHz, while L measurements were at 790 kHz. These are "standard" test frequencies for Boonton/HP Q meters, at which the instrument's inductance scales may be directly read. Inductances outside the range permitted at these two test frequencies used a different frequency, based on the instrument's recommended test frequency for the inductance being measured. Since μ_{eff} is defined as L/L_0, it was calculated based upon these two measured inductances, not directly measured. Ideally, L and L_0 would both be measured at the same frequency, but practical limitations make this difficult at best.

Signal Level — To measure the receiving effectiveness of various configurations, a simple balanced input resonator/amplifier, based upon a published design by DeMaw was used.[9] See Figure 2. The amplifier output was fed into a Racal RA6790/GM receiver, as illustrated in Figure 3. The receiver was operated with AGC off and the IF-gain manually adjusted to be in the linear mode. Under these conditions, the 455 kHz IF output of the receiver is proportional to the received signal level. This level was read with a Hewlett Packard 3400A true RMS analog voltmeter, with an upper frequency limit of 10 MHz.

A local standard AM broadcast station, WMAL, 630 kHz, was used for medium wave signal comparisons. Some long wave comparisons were made using non-directional beacons, DC, at 332 kHz and IA at 346 kHz. Limited measurements were also made with WWVB, 60 kHz.

Each test antenna was connected to the amplifier and the resonating capacitor adjusted for peak signal. Additional sections of the tuning capacitor were selected, as necessary for resonance. The 3400A voltmeter reading was recorded, with care taken to read the meter during speech pauses or quiet carrier times for the non-directional beacons. WWVB was read at modulation peaks. At the outset of any test run, the antenna fixture was adjusted to peak the received signal with initial antenna tested. The orientation was not moved for subsequent tests of different antennas.

The signal readings are *relative* and thus indicate only the performance of one antenna against another. In most cases, comparison readings were taken at one sitting. However, some later measurements were made as new configurations were developed. In these cases readings were also taken with a reference antenna, thereby enabling later measurements to be correlated with earlier measurements.

Out-Of-Range Tests — A final series of tests were run on several coils to determine their performance as inductors at frequencies between 4 and 50 MHz using a HP 8754 network analyzer with the configuration shown at Figure 4. The inductor under test is placed in series with the swept output of the network analyzer using a homemade test fixture. A Tektronix TDS430 digital oscilloscope was used to capture the network analyzer output in a convenient digital form.

Unloaded or Loaded Q?

Unless explicitly stated to the contrary, references to the coil quality factor, Q, in this article are to the values measured using the 260A Q-meter. For practical purposes, the 260A measures "unloaded" Q.[10] When a coil is used as an antenna in the test circuit of Figure 2, it is shunted by the 100 kΩ gate resistors. Since the gate resistors are effectively in series, 200 kΩ is imposed across the tuned circuit, thereby lowering the Q, and the resultant voltage magnification of the signal level.

For a 200 μH antenna coil — a value typical of those measured--the effect of the gate resistor shunt can be significant. For example, at 1 MHz, an unloaded Q of 200 is reduced to 88.6 by the effect of the 200 kΩ shunt impedance.

The following approach may be used to calculate Q_{loaded}.

Q is defined, for the parallel model, as:

$$Q_{unloaded} = \frac{R_P}{2\pi fL} \quad \text{[Eq 10]}$$

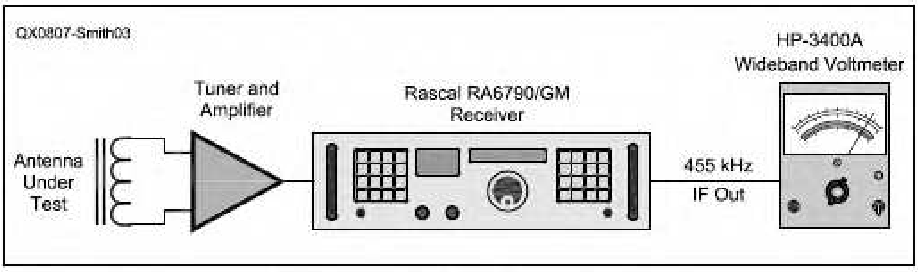

Figure 3 — Signal measurement setup.

Or, rearranging:

$$R_p = 2\pi fLQ_{unloaded}$$

where:

R_P = equivalent parallel resistance of the internal coil losses

f = frequency

L = inductance

Using the unloaded Q, calculate R_P

Determine the effective shunt resistance,

$$R'_p = \frac{R_p R_G}{R_p + R_G}$$

R_G = the gate resistance shunting the coil

The loaded Q is then:

$$Q_{loaded} = \frac{R'_p}{2\pi fL}$$

The most significant effect of using 100 kΩ gate resistors in the test amplifier is to diminish the benefit of using the higher Q, Type 61 core material. Thus, the relative benefits of the Type 61 material are understated in this article's signal level comparisons. However, in practice, a relatively low value of R_G may be desirable to intentionally reduce the loaded Q. At 1000 kHz, for example, a Q of 300 — a value easily achievable with Type 61 material — yields a 3 dB bandwidth of 3.3 kHz. This narrow bandwidth imposes tuning difficulties, may give rise to temperature stability concerns and will muffle the audio of AM broadcast radio signals. These concerns are ameliorated if the Q is reduced through parallel resistance. In this example, the 100 kΩ gate resistors reduce the loaded Q to 104, assuming L = 200 µH. The 3 dB bandwidth is correspondingly increased to about 10 kHz, and tuning is much easier. Of course, the price paid for the lower Q is the received signal voltage is reduced to about one-third the Q = 300 case. The gate resistors in Figure 2 could be increased to several megohms, if desired, to improve loaded Q.

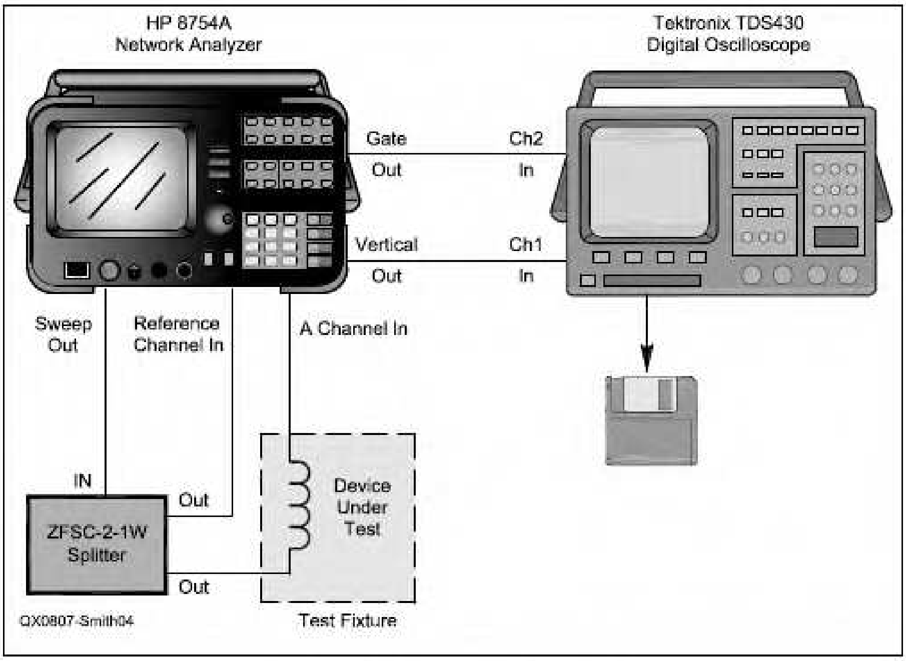

Figure 4 — Out-of-band sweep configuration.

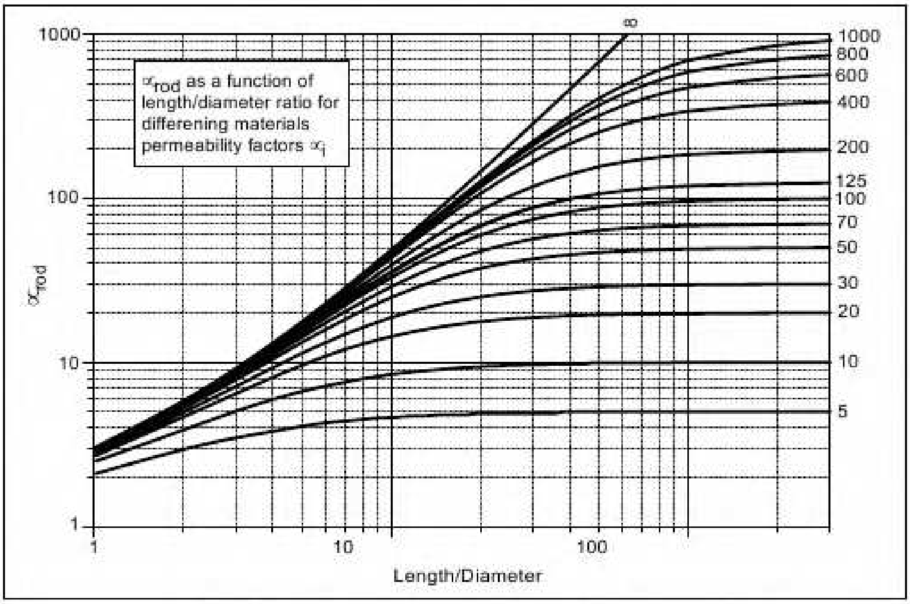

Figure 5 — Effect of rod length / rod diameter of rod permeability.

Length/Diameter Ratio Effect on Rod Permeability

If a test coil is wound over the full length of a ferrite core and the inductance of the test coil is measured with and without the rod in place, μ_{eff} will be found to be, for reasonable rod dimensions, much less than the permeability of material from which the rod is constructed, μ_i. (The permeability value based upon theoretical considerations for a coil wound over 100% of the rod length is commonly referred to as the "rod permeability," or μ_{rod}. However, the actual measured permeability μ_{eff} is almost always less than μ_{rod}, and if the coil occupies less than the full length of the rod, μ_{eff} will be even lower.)

The *theoretical* relationship between μ_{rod} and μ_i is:[11, 12]

$$\mu_{rod} = \frac{\mu_i}{1 + D(\mu_i - 1)} \quad \text{[Eq 11]}$$

where:

D = demagnetization factor

As Lenz's law provides, an induced current generates a magnetic field in the surroundings, including the core, in a direction opposing the inducing current's magnetic field. The demagnetizing factor, D, is a measure of effectiveness of the opposing field. Computing D for an arbitrary core shape is difficult or impossible. Fortunately, D can be determined for a simple cylindrical rod-shaped core. In this case, D is a function of the ratio of the rod length, l, to the rod diameter, d. For an infinitely long cylindrical rod, D=0 and $\mu_{rod} = \mu_i$, while for a rod with a length/diameter ratio = 1.00, D = 0.27, and μ_{rod} = 3.6 if μ_i = 125.

Based upon Equation 11 and published D values, I calculated the relationship between μ_{rod} and μ_i and Figures 5 and 6 show my results.[13] For l/d ratios below 10, both figures shows slightly lower μ_{rod} values than provided in other published curves.[14] Figure 5 presents the μ_{rod} versus l/d in the conventional fashion, while Figure 6 recasts the data to better illustrate the effect of l/d on μ_{rod} by parameterizing l/d.

Figure 7 plots D as a function of l/d to aid in making the calculations called for by Equation 11. Likewise, it is possible to work backwards and estimate μ_i from μ_{rod} measurements. Solving Equation 11 for μ_i yields:

$$\mu_i = \frac{\mu_{rod}(1-D)}{1-D\mu_{rod}} \quad \text{[Eq 12]}$$

In my experience, however, measured μ_{od} values will almost never match the theoretical μ_{rod} numbers for commonly available rod materials and dimensions, although making the adjustments described below reduces divergence between measured and theoretical data.

Effect of Winding Over Less than the Full Length of the Rod

If the winding occupies less than the full rod length, μ_{eff} decreases. *Q*, however, in my measurements, is maximized for short centered windings and decreases as the winding extends over a greater proportion of the rod length. This is contrary to *The ARRL Antenna Book* statement that maximum *Q* is found when the winding occupies the full coil length.[15] Figure 8 plots *Q* measured with air core, Type 33 core and Type 61 core for 28 coils versus core length to rod length ratio. The data is greatly scattered because the test coils have a variety of turns, wire diameters and pitch, but it shows an unmistakable trend to lower *Q* as the winding occupies a greater proportion of the core length. ($R = -0.42$ for Type 33 material and $R = -0.47$ for Type 61 material. No statistically valid similar trend can be seen for the air core *Q* data ($R = -0.06$) This data represents a limited range of coil geometry (0.5 inch diameter, 7.5 inch long rod) and hence may require caution in applying to the general case.

Since Equation 9 shows the received signal level in a tuned loop is proportional to product of μ_{eff} (increases with winding length) and *Q* (decreases with winding length), the length of winding providing maximum sensitivity requires further examination. We first turn to the relationship between V_{oc}, μ_{eff}, μ_{rod} and the ratio of winding length to rod length. We introduce an adjustment factor — the "Free End Factor" as it is known — relating μ_{eff} to μ_{rod} as a function of the length of the coil relative to the rod:

$$\mu_{eff} = \mu_{rod} \times f(l_c/l_r) \quad \text{[Eq 13]}$$

where:
l_c is the length of the coil
l_r is the length of the rod
$f(l_c/l_r)$ is a function providing an adjustment factor to relate the measured μ_{eff} to the theoretically predicted value, based on the ratio of winding length to total rod length.

I've found two versions of $f(l_c/l_r)$ in the literature, one presented in recent editions of *The ARRL Antenna Book* and the second in Johnson and Jasik's *Antenna Engineering Handbook*, 2nd Edition.

The ARRL Antenna Book version is:[16]

$$\mu' = \mu_{rod} \sqrt[3]{\frac{a}{b}} \quad \text{[Eq 14]}$$

where:
μ' is μ_{eff} in the terminology of this article
a is the length of the core (rod) (l_r)
b is the length of the winding (coil) (l_c)

At a minimum, Equation 14 seems to incorrectly exchange a and b, as it has μ'

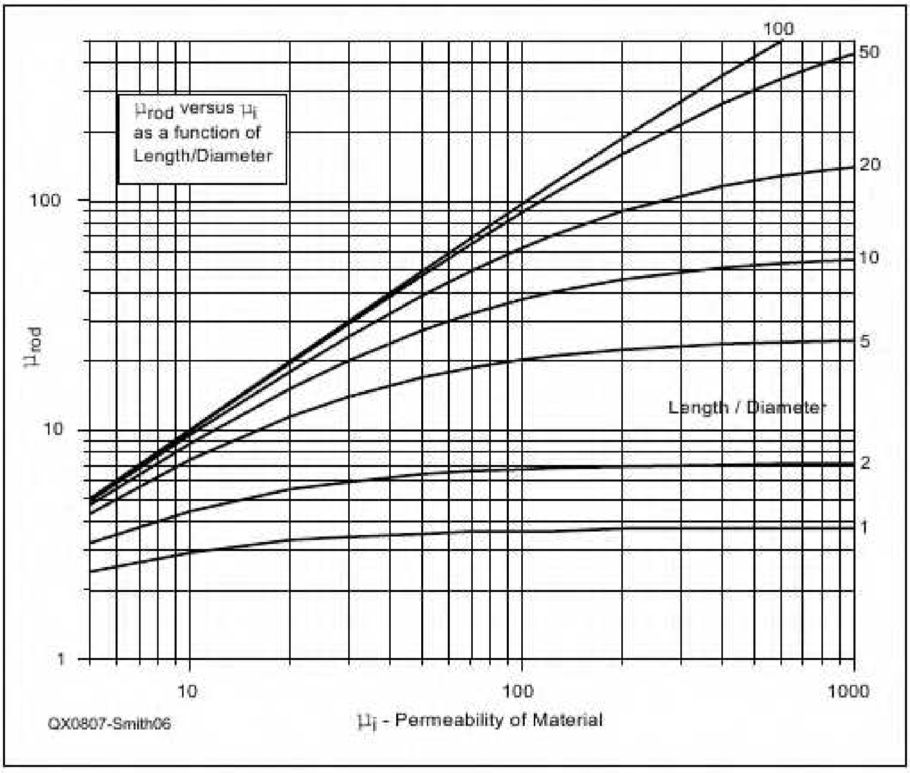

Figure 6 — μ_{rod} as a function of rod length/diameter.

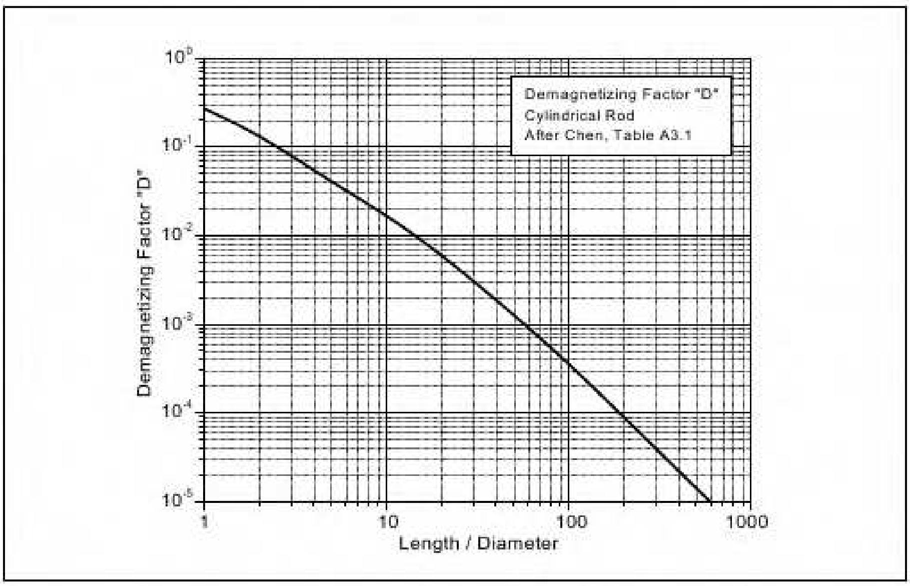

Figure 7 — Demagnetizing factor D versus l/d for cylindrical rod.

increasing without bounds as the winding length decreases as a proportion of core length. This is contrary to Johnson and Jasik, as well as other references and my measurements, in addition to being physically implausible. Since *The ARRL Antenna Book* provides the equation without source reference, it is not possible to determine if the error is a simple transcription error, inadvertently exchanging terms *a* and *b*, or if there are other problems with it. Assuming that, *arguendo*, parameters *a* and *b* were inadvertently reversed without other errors introduced, the corrected form of Equation 14 (with modified terminology consistent with this article) is:

$$\mu_{eff} = \mu_{rod} \sqrt[3]{\frac{l_c}{l_r}} \quad \text{[Eq 15]}$$

Johnson and Jasik provide only a graphical representation of the relationship between μ_{eff} and l_c/l_r, (which they call correction factor F_L) but I've extracted the data points and fitted a cubic equation to their data. Figure 9 shows excellent agreement between the curve fit and the underlying data (R = 0.99).

$$\mu_{eff} = \mu_{rod}(0.0699 + 1.547x - 1.109x^2 + 0.208x^3) \quad \text{[Eq 16]}$$

where x is the ratio l_c/l_r.

To determine the effect of varying the winding length as a proportion of the rod length, and to assess whether these various adjustment factors were accurate, I wound coils occupying 20%, 40%, 60%, 80% and 100% of the rod length over mandrels for cores of Type 61 and Type 33. The target inductance of the test coils for Type 33 cores was 200 μH. This was achieved within ±10%. The coils wound for the Type 33 rod were also used with the Type 61 core, accepting the lesser inductance.

As the coil length was increased to cover a greater percentage of the core length, the winding pitch (turns per inch) was necessarily reduced to maintain the target inductance value, reducing from 24 TPI for 20% coverage to 8 TPI for 100%. This decision to reduce the winding pitch to maintain (approximately) the same inductance complicates an already difficult analysis. As discussed below, μ_{eff} is also affected by the "winding fill ratio," in other words, as the ratio of copper to open space increases, so does μ_{eff}. Thus, the choice to maintain constant inductance with varying length potentially changes two variables at once.

Figure 10 compares the free end adjustment factors predicted by Equation 5-8 in *The ARRL Antenna Book*, as corrected above, Johnson and Jasik's F_L factor, and the free end factor I measured for both Type

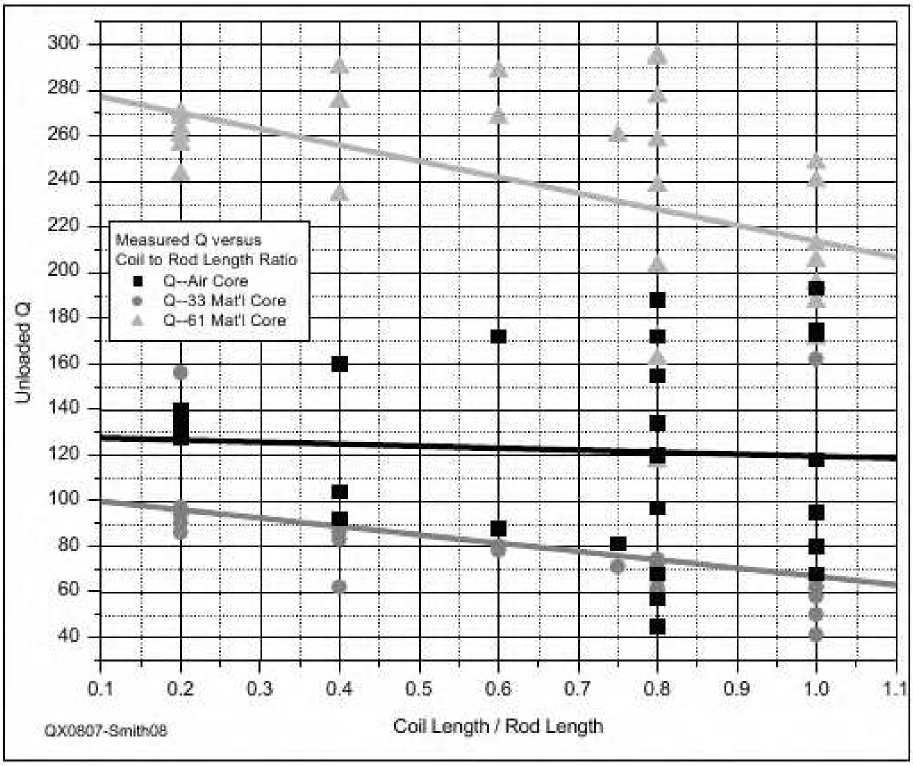

Figure 8 — Measured *Q* versus coil length to rod length ratio.

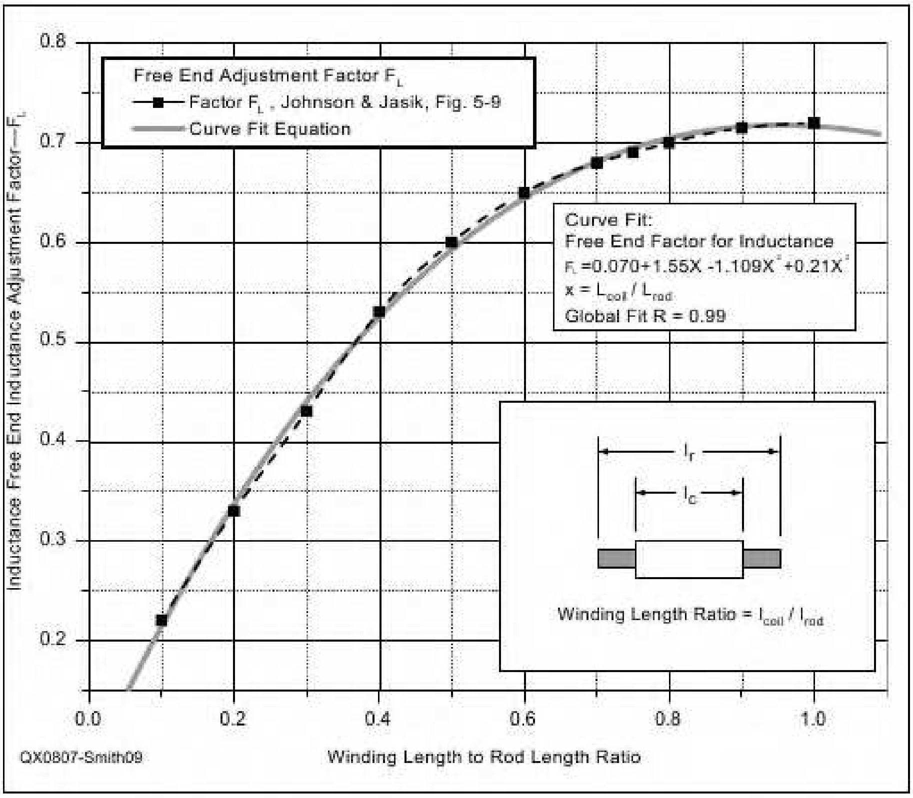

Figure 9 — Free end adjustment factor (μ_{eff}) (Johnson and Jasik).

33 and Type 61 material rods. The vertical bars in the measured data show the range of measured free end factor for different winding pitches and inductance values of my test coils. The figure shows significant divergence amongst the two prediction methods and my measured data. The reasons for the discrepancy remain unknown to me.

Further complicating an already complex matter, Johnson and Jasik note a different factor, F_V, relates the induced open circuit voltage to that computed with μ_{rod}:

$$V_{OC} = j\omega\mu_{rod} F_V NAB_z^i \quad \text{[Eq 17]}$$

where:

V_{oc} is open circuit voltage induced in the loop

$j\omega$ is the angular frequency, $2\pi f$ where f is frequency.

N is number of turns

A is the loop area

B_z^i is the component of the incident magnetic flux density normal to the plane of the loop.

Except for the factor Fv, Equation 17 is identical to Equation 7 when the simplifications introduced in this article are considered. F_V, like F_L, is provided in Johnson and Jasik only as a graphical factor, derived from experimental data. I've extracted data points for F_V and fitted a quadratic equation to it, as illustrated at Figure 11.

$$F_V = 0.998 + 0.0181x - 0.235x^2 \quad \text{[Eq 18]}$$

where x is the ratio l_c/l_r.

I have not, however, attempted to relate F_V to measured signal strength data.

Off-Center Windings

Where the coil winding occupies less than the entire length of the rod, it is possible to offset the winding so that it is no longer centered on the rod. To test this effect, I made a test coil of 20 turns close-wound, AWG no. 20 enamel magnet wire on a paper form. The length of the test coil was 0.62 inches and its measured inductance without a ferrite core was 3.2 µH at 7.9 MHz, with a Q of 154. Measurements of Q, inductance, μ_{eff} and received signal voltage were taken as the test coil was moved in 0.25-inch increments from the center position, with rods of both Type 33 and Type 61 material.

Figure 12 shows that as the winding is displaced from center, Q and inductance decrease. (I observed a slight Q peak at an offset ratio around 0.6 for Type 33 material.) Q is little changed with moderate displacements, remaining within a few percent of the maximum value up to 40% off-center for Type 61 and up to 60% off-center for Type

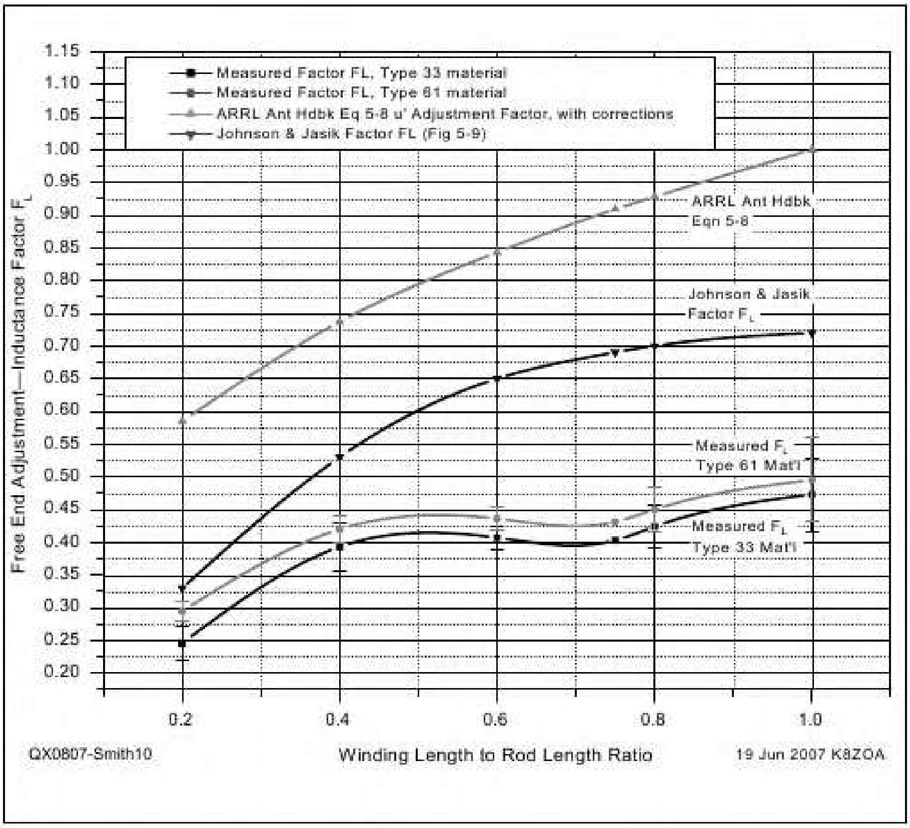

Figure 10 — Free end adjustment factors (μ_{eff}) compared with measured data.

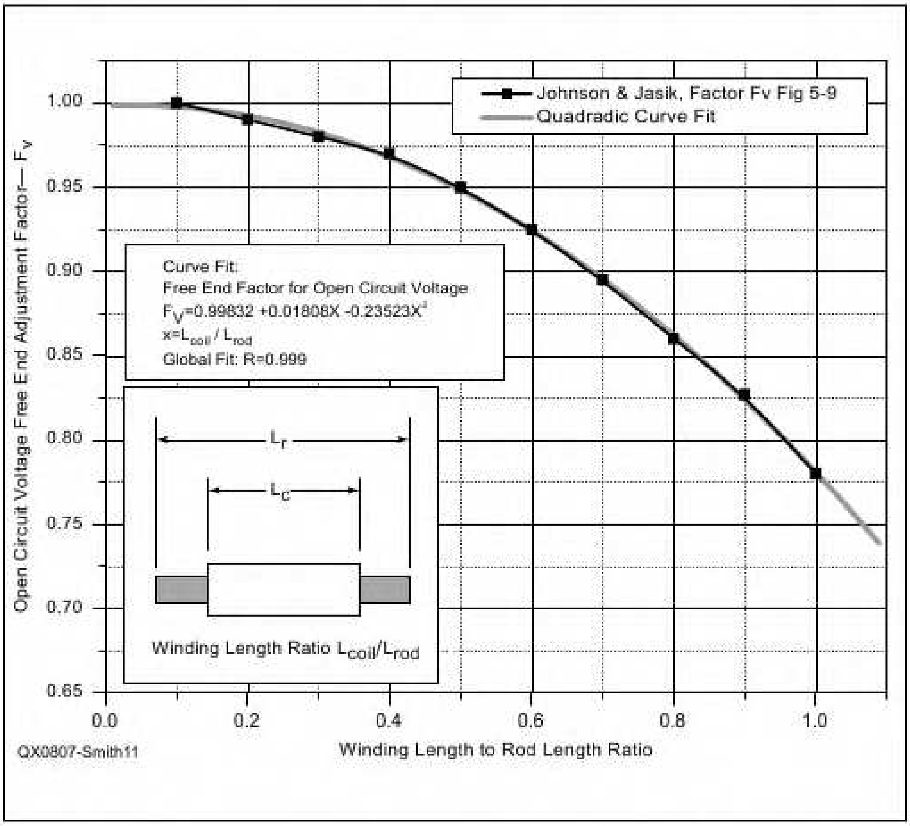

Figure 11 — Free end adjustment factor for open circuit voltage.

33 material. Inductance changes more with position, particularly with the higher μ Type 33 rod. Even here, however, 40% off-center placement reduces inductance by less than 10%. Both inductance and Q peak with the windings centered, consistent with statements in *The ARRL Antenna Book*. (One reason to offset the winding is to trim to a precise inductance value without materially affecting Q.)

Since μ_{eff} is defined as the observed inductance divided by the air-core inductance, the change in μ_{eff} with core offset the shape of inductance curve in Figure 12.

Equation 9 predicts that received signal voltage is proportional to $QN\mu_{eff}$. In this case, N is constant at 20 turns, while Q and μ_{eff} vary with the offset. Equation 9 therefore predicts the received signal voltage to be linearly proportional to the product $Q\mu_{eff}$. To test this, I collected relative signal strength data as the test coil was moved to different offsets and plotted the signal strength versus the product of the Q and μ_{eff} earlier measured for the same offset value. Figure 13 shows an excellent linear fit to the product of Q and μ_{eff} ($R = 0.997$) for the Type 61 rod, and almost as good fit for the Type 33 rod ($R = 0.985$). This analysis uses loaded Q values, and I have adjusted the measured Q to reflect the amplifier's input loading, although the amplifier results in less than a 10% reduction in Q for the test coil used. The signal readings in Figure 13 are relative, with the maximum observed signal level (Type 33 rod, coil centered) set to 1.00.

Figure 14 illustrates the variation in received voltage as the winding is offset from center. A twenty percent offset reduces the received signal only slightly.

Winding Fill Ratio

In addition to changes caused by core occupancy and off-center windings, μ_{eff} is a function of winding fill ratio. The winding fill ratio equals the conductor diameter divided by the winding pitch, as illustrated in Figure 15. The winding fill ratio is thus a measure of the "density" of the copper over the ferrite core. As the winding fill ratio approaches 1.00, the surface of the ferrite rod is nearly completely covered by the copper winding. (Of course, 1.00 is achievable only for one turn, as there must be some turn-to-turn gap.)

To study the effect of winding fill ratio, a series of 48-turn test windings were made, each 6.0 inches long, wound at 8 turns per inch on a paper core. To vary winding fill ratio, wire sizes from AWG no. 16 down to AWG no. 34 were used. Measurements were taken with both Type 33 and Type 61 cores.

Before discussing the results of varying the winding fill ratio on μ_{eff}, it is instructive to consider its effect upon L_0, the inductance without a ferrite core. Even though the number of turns and the coil length were held constant, L_0, the coil inductance without a core, still varied. Two countervailing physical effects are responsible for most of the variation:

- Since the coils must be wound with a fixed inside diameter of 0.50 inches — so as to accommodate the ferrite core — as the wire diameter increases, the mean diameter of the coil necessarily increases. As the inductance of a solenoid is proportional to the square of its diameter, this factor alone accounts for an

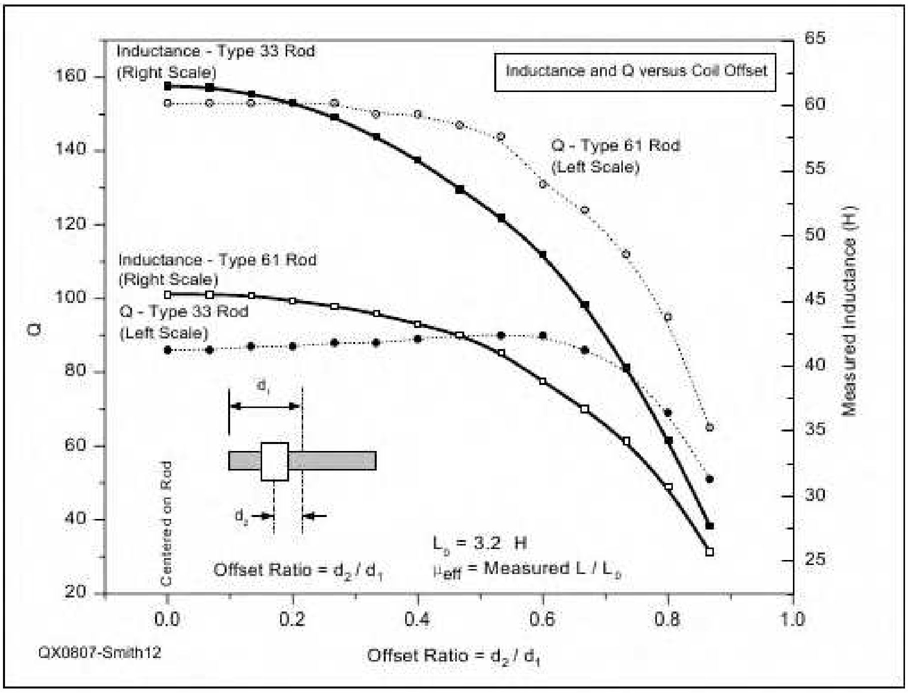

Figure 12 — Variation in L and Q with coil offset.

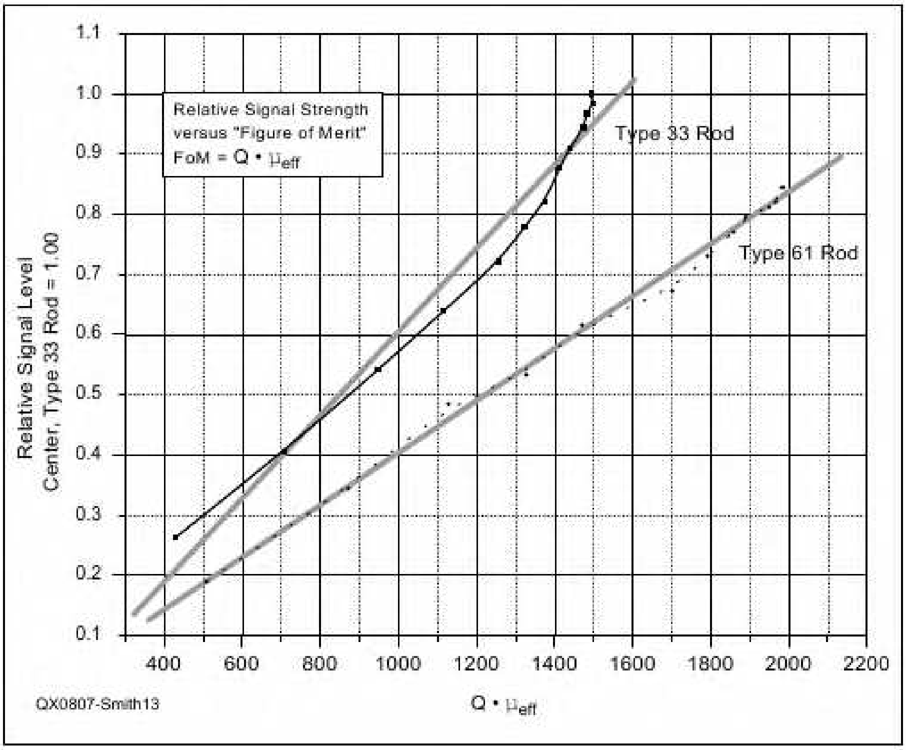

Figure 13 — Relative signal versus $Q\mu_{eff}$.

increase of 17% in the calculated inductance of the coil wound with 16 AWG wire over that of its 34 AWG counterpart.

• Commonly used inductance formulas — such as the classic Wheeler equation — are accurate only for current sheets, that is, where the current flows in an infinitesimally thin perfectly conducting tape with negligible gap between adjacent turns. A fixed pitch wound inductor looks less and less like a perfect current sheet as the wire diameter decreases and an appropriate correction must be applied if the coil is not close wound.[17] Because the space-wound correction factor is not easily located, Appendix 1 reproduces the key equations, including Wheeler's basic single winding solenoid inductance equation.

After calculating the current sheet inductance using Wheeler's equation, the wire spacing adjustment outweighed the increase in inductance with larger wire diameter, with the coil wound with 34 AWG having a calculated inductance some 15% above that of its 16 AWG counterpart.

Returning to the effect of the winding fill factor upon μ_{eff}, Figure 16 illustrates that as the ratio of copper to open space increases, so does μ_{eff}.

Choice of Core Type

Two rod-type core materials are readily available from suppliers:

• Type 33 material; μ_i = 600 to 800, optimum frequency 10 kHz to 1 MHz.[18]

• Type 61 material: μ_i = 125, optimum frequency 0.2 MHz to10 MHz.

These cores are commonly available in two sizes, both 0.5 inch diameter, with lengths of 4 inches and 7.5 inches. Some suppliers also stock other diameters and lengths.

When used as an antenna, our interest in ferrite rods is to maximize received signal strength at the frequency of interest, or, perhaps more correctly, to maximize the signal to noise ratio of the received signal. Of interest in the mass production environment is selecting a core that permits a reasonably sized tuning capacitor and meets the cost targets. In the one-off amateur environment, these latter considerations are of less importance and will not be further considered in this article.

Based upon the material specifications, it might be assumed that Type 33 material would be preferred for up to 1 MHz, including the new sub-200 kHz amateur bands available in some countries, while Type 61 material would be better for general broadcast band listening up through the 160 meter band.

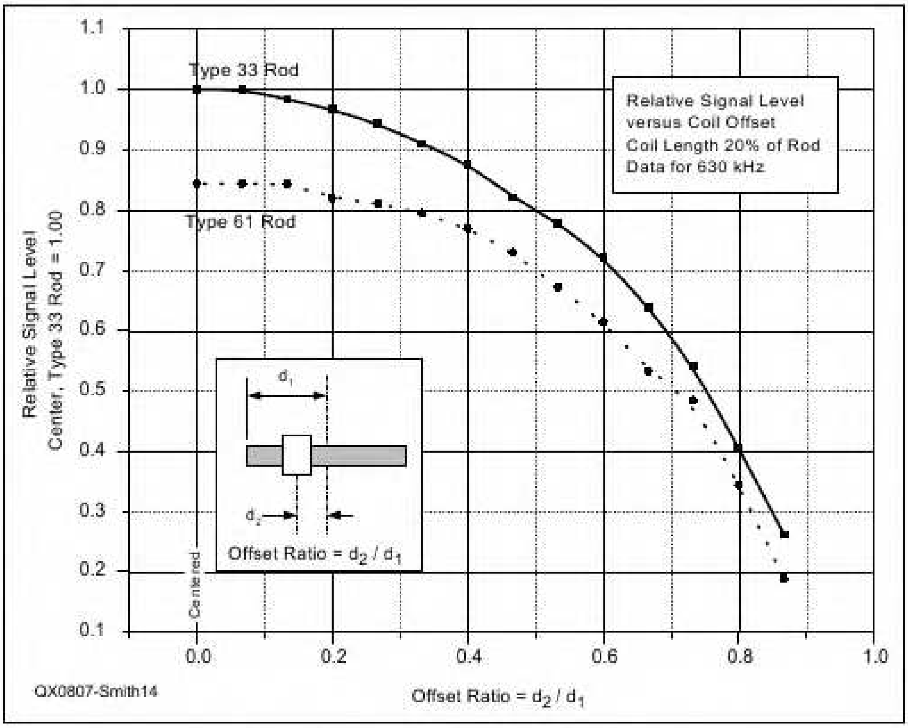

Figure 14 — Variation in received signal with coil offset.

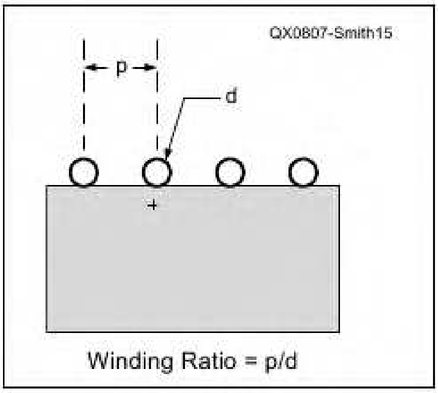

Figure 15 — Winding ratio.

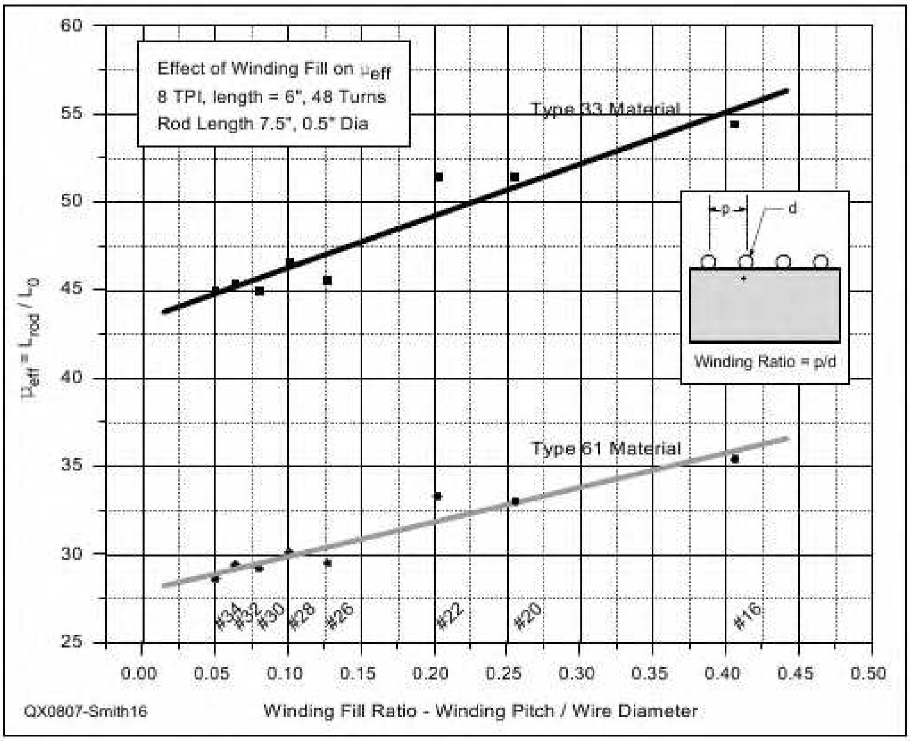

Figure 16 — μ_{eff} increases with increasing winding fill ratio.

This is not the case, however. My measurements demonstrate that Type 61 material is to be preferred for all frequencies between 60 kHz and 2 MHz.

Figure 17 illustrates the relative performance of three key parameters for Type 33 and Type 61 cores; received signal level, Q and μ_{eff}. The data is for a 6-inch long coil, 48 turns of AWG no. 22, wound at 8 turns/inch, and sets the performance of the Type 33 material version at 1.00. Ratios > 1 indicate Type 61 material is superior. Alternating measurements were made with a Type 33 rod and Type 61 rod cores, re-resonating the antenna after each core change. For almost all frequencies studied, from 60 kHz through 1600 kHz, the Type 61 rod provided more signal level than the same coil with a Type 33 core. Even at the lowest frequency measured, 60 kHz, well below the recommended minimum frequency for Type 61 material, the received signal level was essentially identical with Type 33 and Type 61 rods.

Also plotted in Figure 17 is the relative effect of the two rod materials upon measured Q and μ_{eff}. If variations in the received signal level were explained solely by differences in μ_{eff}, then we should have seen reverse performance, with Type 33 material core outperforming Type 61 material. The data shows a broadly parallel trend between Q and received signal level, although the sharp up-tick in received signal performance around 1300 kHz is not mirrored by a similar change in the ratio of Q_{rod61}/Q_{rod33}.

As may be apparent by now, when dealing with ferrite rod antennas changing any parameter has a knock-on effect upon others. However, the Type 61 rod retained its superiority for six winding configurations tested, albeit with varying margins.

Surplus Rods?

Purchasing new ferrite rods can be expensive, with the larger rods costing $20 or more. Can surplus AM broadcast band ferrite rods be used outside of the broadcast band? How do small surplus rods compare with large Type 61 cores?

I ran tests with two groups of surplus AM broadcast band ferrite cores:

• Type "X" rods, diameter cylindrical with two flats, measuring approximately 0.388 inches (9.85 mm) across the round and approximately 0.340 inches (8.64 mm) across the flats, 3.55 inches (90.5 mm) long. These rods are of crude finish, compared with the other rods available, having notable bumps, blemishes and imperfections. The quoted dimensions vary by 0.040 inches (1.0 mm) or more for the same rod and between samples. Six of these rods were recently (as of summer, 2000) procured from a mail order

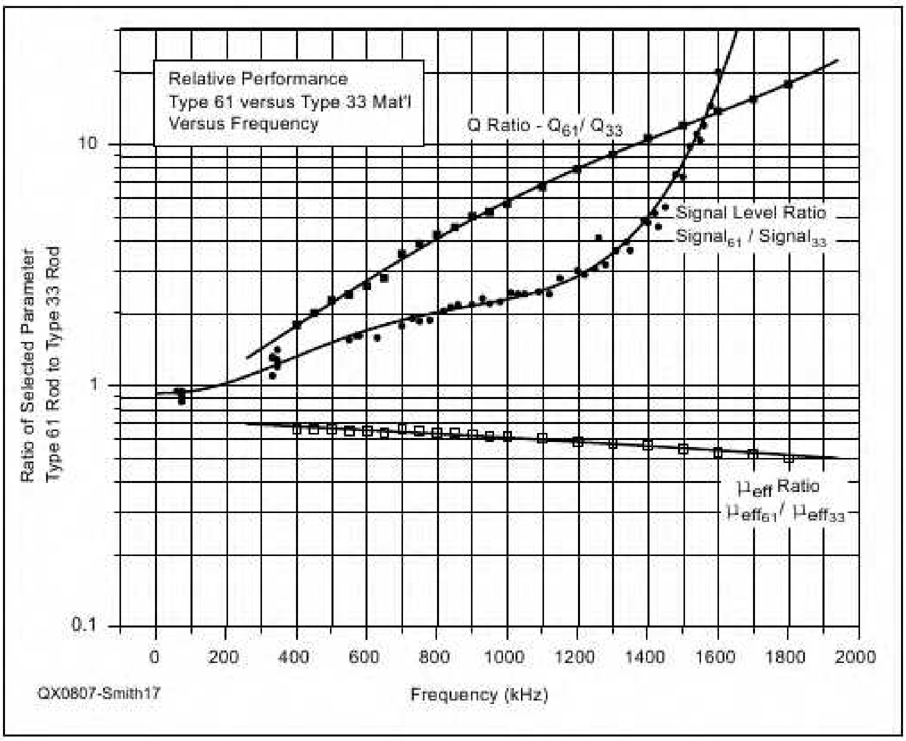

Figure 17 — Relative performance of Type 33 and Type 61 rods.

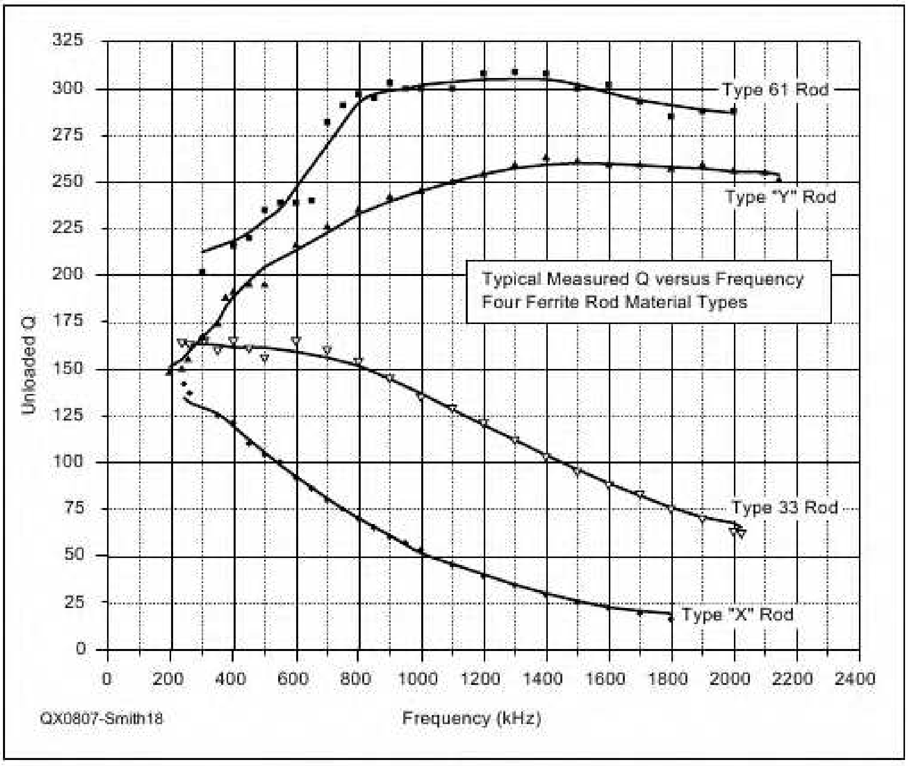

Figure 18 — Change in Q with frequency.

supplier for $2.25 each.[19]

• Type "Y" rods, same shape and diameter as the Type X rods, but 5.51 inches (140 mm) long. These rods have a superior finish and are dimensionally identical within 0.002 inches (0.05 mm). Three of these rods were acquired several years ago from a surplus house and have resided in my junk box since. These rods are obviously unlikely to be found by anyone else, but are included as a point of reference.

To compare the Q possible with Type X and Y cores, I wound a test coil for each, with approximately equal inductance (180 µH at 790 kHz with the rod inserted) and measured the Q between 200 kHz and 2000 kHz. For comparison, similar measurements were taken for test coils with Type 33 and Type 61 material. Figure 18 shows that the Type 61 and Type Y rods are similar, as are the Type 33 and Type X rods.

Although the Type X rod is a poor Q performer at most frequencies, its relative cheapness and availability leads to the question: "is it possible to use several relatively poor — but cheap — cores to match the performance of the expensive Type 61 rod?" The answer is yes, at least in the AM radio band. Figure 19 illustrates the relative signal level performance, measured at 630 kHz, for the test configurations described in Table 1. The relative comparison reference point is Test Configuration 21A, 48 turns at 8 TPI on a Type 61 core. Signal levels > 1.0 are better than the Type 61 core reference configuration.

Configurations 31, 32, 34 and 35, perform at least as well as the Type 61 rod reference. Reference coil 21A was the best performing Type 61 core configuration observed, so it is possible to duplicate the best Type 61 performance with less than one half the investment in ferrite. However these Type X rod configurations were not evaluated for signal reception outside of 630 kHz. The Q versus frequency data of Figure 18 strongly suggests, however, that Type X rods will deteriorate sharply at higher frequencies, and will likely be unsatisfactory above 1 MHz. However, Type X material may have merit below 500 kHz.[20]

Distributed Capacitance

Terman succinctly stated the cause of distributed capacitance in coils:

"In a coil there are small capacitances between adjacent turns, between turns that are not adjacent, between terminal leads, between turns and ground, etc. … The total effect that the numerous small capacitances have can be represented to a high degree of accuracy by assuming that they can be replaced by a single capacitor of appropriate size shunted across the coil terminals."[21]

Distributed capacitance sets a limit on the highest frequency at which a coil may be resonated; with no additional capacitance the inductance and C_{dist} parallel resonate at some frequency, referred to as the "self-resonant frequency," or f_{self}. In addition, self-capacitance causes a discrepancy between measured and true Q and inductance.[22]

In addition to increasing the inductance, adding a ferrite core to a coil increases the distributed capacitance. This is because some

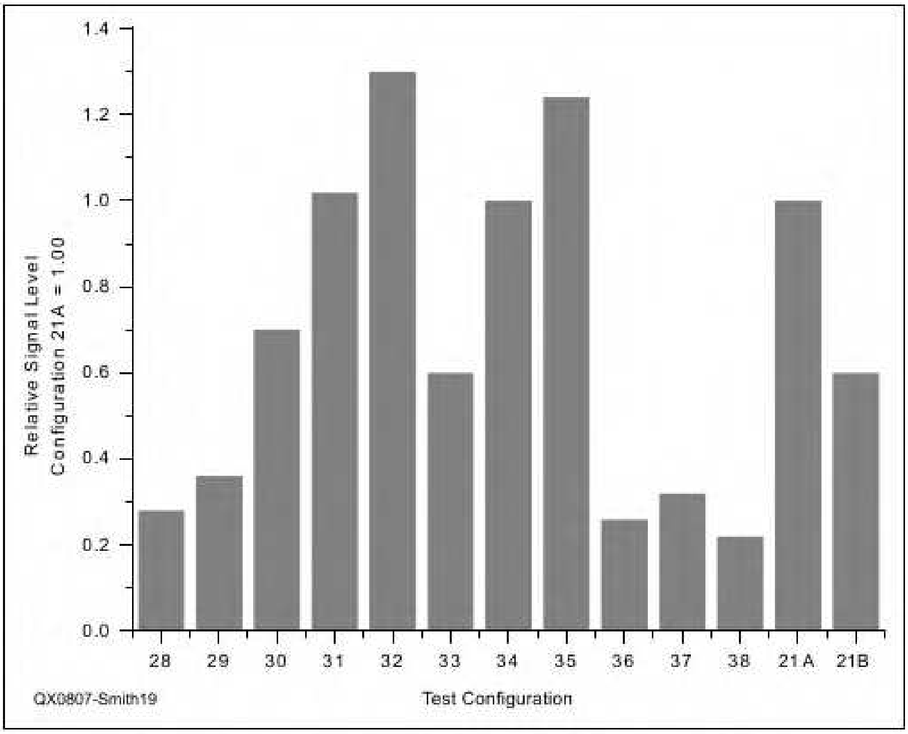

Figure 19 — Received signal level — Type X rod configuration.

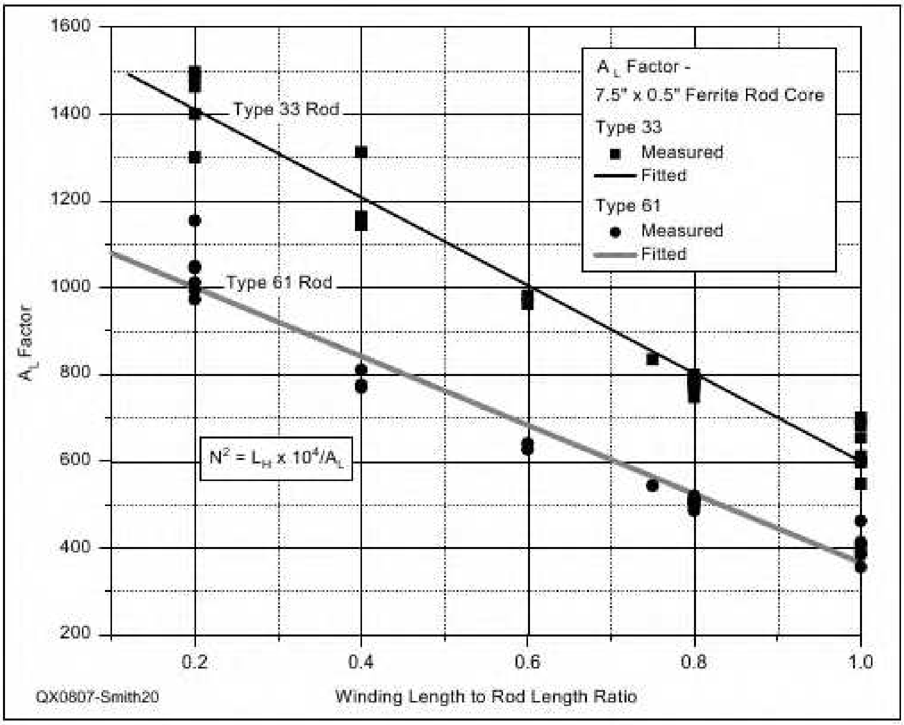

Figure 20 — A_L factor for Type 33 and Type 61 Material, 7.5 inch × 0.5 inch rods.

of the electric field lines that give rise to C_{dist} are contained within the ferrite. Typical ferrites have a relative permittivity (dielectric constant) of 10 to 11.[23] Thus, C_{dist} will increase. Since only part of the electric field lines are within the ferrite, the total increase in C_{dist} will be less than indicated by the relative permittivity of the ferrite rod.

As the ferrite core increases both inductance and distributed capacitance, F_{self} will be significantly lower than for the same coil without the rod inserted.

To evaluate the reduction in F_{self}, measurements were made on a typical antenna coil with and without the ferrite rod. The coil measured is the one identified as Configuration number 38 in Table 1. The radius of the coil is approximately 0.2 inches and the winding length is approximately 3.5 inches.

Without a core inserted, I measured C_{dist} as 1.9 pF, using the auxiliary work coil method described in the Model 260-A *Q* meter instruction manual. Since L_0 was measured as 18.3 µH, the f_{self} can be computed as 27 MHz.

With the Type X core inserted, C_{dist} increased to 8.5 pF. L increases to 1,100 µH (measured at 250 kHz), resulting in an approximate f_{self} of 1.6 MHz. (L is, in fact, a function of frequency since μ_{eff} varies with frequency. An exact f_{self} would require adjustment of L to match that at the self-resonant frequency. Alternatively, F_{self} can be measured directly.)

In this example, C_{dist} increased by a factor of 4.5 and f_{self} decreased by approximately 17:1.

It is also interesting to compare the measured (without ferrite core) C_{dist} to the computed value using the Medhurst formula.[24]

$$C_{dist} = 0.29l + 0.41R + 1.94\sqrt{\frac{R^3}{l}} \qquad \text{[Eq 19]}$$

where:

l = coil length in inches

R = coil radius in inches

Inserting the values for coil length and radius yields C_{dist} = 1.2 pF. The discrepancy of 0.7 pF between calculated and measured values can be attributed, at least in part, to the lead capacitance connecting the coil to the Q meter. Further, Medhurst's work assumes a specific geometry, where the coil is mounted vertically over a grounded chassis, a configuration significantly different than when the inductor is attached to a Q meter.

Simplified Inductance Calculation Approach

To avoid the complexity of the equations and adjustment factors presented in this paper, a simplified approach may prove adequate for Amateur Radio purposes. Ferrite and powdered iron toroidal are often designed using a simple equation, relating inductance, number of turns and the "A_L" factor. If we stick to the 7.5 inch × 0.5 inch Type 33 and Type 61 cores, it is possible to define a similar equation, presented below.

$$NumberTurns = 100\sqrt{\frac{DesiredL_{\mu H}}{A_L}} \qquad \text{[Eq 20]}$$

Instead of a single value for A_L for Type 33 and Type 61 cores, however, the particular A_L value must be selected from the curves shown at Figure 20. The data markers are A_L points based on my measured coils, and the straight line are curve fitted. The results will be within 10 to 15% in most cases, and the calculations are much easier than the traditional methods covered earlier in this article.

Sample Calculation:

Desired Inductance: 200 µH

Desired coil length: 6 inches (0.8 winding length to rod length ratio)

Core Material: Type 61.

From Figure 20 we read AL = 520. Hence the number of turns required is:

$$NumberTurns = 100\sqrt{\frac{200}{520}}$$

$$= 100\sqrt{0.385} = 62$$

62 turns, wound over 6 inches, or approximately 10 TPI, will yield 200 µH when centered over the 7.5 inch long Type 61 core.

Conclusion

Our rambling excursion through the world of ferrite cores and antennas might lead to the expedient strategy of bypassing analysis completely and trying, more or less randomly, combinations of core materials and windings until an acceptable result follows. And, the complexity and contradictory nature of the limited information available may reinforce those desires. However, it's often said "a day in the library is worth a month in the lab," meaning that experiments guided by past learning are far more productive than random efforts.

If designing a ferrite rod antenna, my data shows a good starting point to be the large 7.5 inch × 0.5 inch Type 61 material cores. For maximum performance, the windings should cover at least 80% of the core length, and reasonably large diameter wire should be used, such as AWG no. 18 to 16, depending on the number of turns required for the frequency band of interest. To determine the antenna inductance, one may start with the old rule of thumb of 1 to 2 pF of resonating capacitance per meter wavelength. From that capacitance, the inductance, and hence the number of turns can be determined from Equation 20 and Figure 20.

Table 1
Type X Core Configuration

ID	*Configuration*
28	1 Type X rod, 42 turns at 12 TPI
29	1 Type X rod, 67 turns at 20 TPI
30	2 Type X rods long, 70 turns at 10 TPI
31	3 Type X rods long, 85 turns at 8 TPI
32	4 Type X rods long, 112 turns at 8 TPI
33	1 Type X rod long, 5 rods in approx. cylindrical format, 32 turns at 10 TPI
34	2 Type X rods long, 3 rods in triangle (6 total rods) format, 70 turns at 10 TPI
35	3 Type X rods long, 2 rods side-by-side (6 total rods) format, 100 turns at 10 TPI
36	1 Type X rod, 35 turns at 10 TPI
37	1 Type X rod, 112 turns at 32 TPI
38	1 Type X rod, 168 turns at 48 TPI
Reference Configurations	
21A	48 turns at 8 TPI, Type 61 core
21B	48 turns at 8 TPI, Type 33 core

Appendix 1

Adjustment for Space-Wound Inductors

The following is based upon an equation given in the *Radiotron Designer's Handbook*.[24] The method described yields results "suitable for slide rule computation."

Step 1 — Calculate the inductance using one of the standard formulas, such as Wheeler's equation.

$$L_0 = \frac{a^2N^2}{9a+10l}$$

where:
L_0 = inductance in μH
a = coil radius to center of wire, in inches
N = number of turns
l = length of coil, in inches

Step 2 — The correction factor applied to the result in Step 1 is:

$L = L_0 - 0.0319\, a\, N\,(A + B)$
a, N as above
A, B as below

$$A \approx 2.3\log_{10}(1.7 \cdot S)$$

$$B \approx 0.336\left[1 - \frac{2.5}{N} + \frac{3.8}{N^2}\right]$$

$$S = \frac{D}{P}$$

where:
D = wire diameter
P = winding pitch
(D and P should be in the same units, such as inches)

Example:

48 turns, AWG no. 30 copper wire, wound at 8 turns/inch. The coil is wound on a paper core, consisting of 4 layers wound over a 0.500 inch diameter mandrel. The paper is standard typing paper, approximately 4 mils (0.004 inches) thick per sheet. AWG no. 30 copper wire has a diameter of 10 mils (0.010 inch).

Step 1 — Determine the parameters to use in the Wheeler formula and calculate L_0:

$$a = \frac{0.500 + 2\times 4\times 0.004}{2} + \frac{0.010}{2}$$

$$= 0.271 \text{ inches radius to center of wire}$$

$$l = \frac{48\,turns}{8\,turns/inch} = 6 \text{ inches}$$

$$N = 48 \text{ turns}$$

$$L_0 = \frac{0.271^2 \times 48^2}{9\times 0.271 + 10\times 6} = 2.71\,\mu\text{H}$$

Step 2 — Calculate the spaced winding correction

$$S = \frac{0.010}{0.125} = 0.080$$

$$A = 2.3\times\log_{10}(1.7\times 0.080) = -1.99$$

$$B = 0.336\times\left[1 - \frac{2.5}{48} + \frac{3.8}{48^2}\right] = 0.319$$

$$L = 2.71 - 0.0319\times 0.271\times 48\times (-1.99 + 0.319) = 2.71 - (-0.693) = 3.40\,\mu\text{H}$$

In this example, the spaced winding correction increases the inductance by more than 25%.

Measurement Comparison

The inductor in this example was constructed and measured with a Boonton 260A Q meter at 7.9 MHz. The distributed capacitance, C_{dist}, of the coil was measured at 2.5 pF. The measured inductance, after correction for C_{dist}, the inductance of the lead wires connecting the coil with the Q meter and the residual inductance of the Q meter, was 3.55 μH. Based upon calibration with 2.5 μH and 5.0 μH Boonton 103A standard inductors, the Q meter is known to have an error of +2.6% at 7.9 MHz. Adjusting for this instrument error, the final measured L is 3.46 μH, representing a difference between the calculated and measured values of 1.8%.

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Notes

1 Fair-Rite Soft Ferrites Catalog, 14th ed. (Fair-Rite Products Corp., Wallkill, NY, 2000), p 2: "During the 1930s research on 'soft' ferrites continued, primarily in Japan and the Netherlands. However, it was not until 1945 that J. L. Snoek of the Phillips Research Laboratories in the Netherlands succeeded in producing a 'soft' ferrite for **commercial applications**."

2 For example, "The Optima 160/80-Meter Receive Antenna," R.Q. Marris, in *The ARRL Antenna Compendium*, Vol 6, R.D. Straw, ed., (ARRL, Newington, CT, 1999); *The ARRL Antenna Book*, R.D. Straw, ed. (ARRL, Newington, CT, 1997), Chapter 14; *The ARRL Handbook for Radio Amateurs 2000*, R.D. Straw, ed. (ARRL, Newington, CT, 1999), Chapter 23; *Loop Antenna Handbook*, J.J. Carr (Universal Radio Research, Reynoldsburg, OH, 1999), Chapter 10; *The Low and Medium Frequency Radio Scrapbook*, 10th ed., K. Cornell (self-published, 1996).

3 M.F. "Doug" DeMaw, *Ferromagnetic-Core Design and Application Handbook*, Section 2.2, (MFJ Publishing Co, Inc, Starkville, MS, 1996); R.D. Straw, ed., *The ARRL Antenna Book*, 18th Ed, (ARRL, Newington, CT, 1997), pp 5-6 through 5-8.

4 J.D. Krause, *Antennas*, 2nd ed., (McGraw Hill, 1988), Section 6-12; W.L. Weeks, *Antenna Engineering*, Section 8.6.1, (McGraw-Hill Book Co., New York, NY, 1968; R.C. Johnson and H. Jasik, eds,. *Antenna Engineering Handbook*, 2nd ed., (McGraw-Hill, New York, 1984) pp 5-5 through 5-9.

5 The derivation of Equation 1 through Equation 9 is based upon several sources. Of particular interest is *Sensitivity of Multi Turn Receiving Loops*, by W.E. Payne, N4YWK, unpublished, but available at **www.lwca.org/library/articles/ywk/looptheo.htm**. See also the references cited at Note 4.

6 One weber per square meter is also known as one Tesla. Because stating a "density" in terms of a value per unit area offers a more intuitive understanding, the discussion uses the older weber/m² terminology. It is simple to see that a uniform flux density of 5 webers/m² through a loop area of 2 square meters yields a total flux of 10 webers.

7 Of more familiarity in radio engineering is the magnetic field strength, H, of an electromagnetic field, in amperes/meter, given by

$$H = \frac{E}{\eta_0}$$

where
η_0 is the intrinsic impedance of free space, 120π ohms. H is used, for example, to set safe exposure limits of low frequency signals where the biological effects of magnetic induction, not electric field coupling, is of concern. Of course, $B = \mu_0 H$ in free space and

$$\eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}}$$

where ε_0 is the relative permittivity of space, 8.85×10^{-12} F/m. Substituting, we find that

$$B = E\sqrt{\mu_0\varepsilon_0}$$

But we know

$$c = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$$

Therefore,

$$B = \frac{E}{c} \text{ or } E = Bc.$$

8 R. Dean Straw, N6BV, *The ARRL Antenna Book*, Equation 10, page 5-8. Note, however, that Johnson and Jasik, (op. cit Note 4) add an additional empirical correction factor F_v to the open circuit voltage, where F_v is an experimentally derived function of the ratio of the overall winding length to the core length. See Johnson and Jasik (op cit Note 4) Section 5-2, Figure 5-9 and Equation 5-15. F_L runs from 1.0 (Lc/Lr = 0 to 0.8 (Lc/Lr = 1, where Lc/Lr is the ratio of the winding (coil) length Lc to the ferrite

core length Lr.). *The ARRL Antenna Book* has a similar correction factor, but of different magnitude, due to the "free end effect." Note, however, that *The ARRL Antenna Book* Equation 5-8 appears to erroneously invert Lc and Lr.

[9] M.F. "Doug" DeMaw, *Ferromagnetic-Core Design and Application Handbook*, Section 2.2, (MFJ Publishing Co., Inc., Starkville, MS, 1996).

[10] In general, adjustments in *Q* and L for distributed capacitance were not made, nor were adjustments made for the Boonton 260A Q Meter finite input impedance and other imperfections.

[11] W.L. Weeks, *Antenna Engineering*, Section 8.6.1, (McGraw-Hill Book Co., New York, NY, 1968).

[12] I've emphasized the term "theoretical" since, as we see later, further adjustments are necessary to obtain even an estimate of the true relationship between the measured rod permeability and the material permeability.

[13] C. Chen, *Magnetism and Metallurgy of Soft Magnetic Materials*, Page 536, Table A3.1, (North-Holland Publishing Co., Amsterdam, 1977), reprinted by Dover Publications, Inc., New York, 1986, provides a table of "D" values versus rod geometry.

[14] M.F. "Doug" DeMaw, *Ferromagnetic-Core Design and Application Handbook*, Figure 2.2, (MFJ Publishing Co., Inc., Starkville, MS, 1996); R.C. Johnson and H. Jasik, eds., *Antenna Engineering Handbook*, 2nd ed., Figure 5-8, (McGraw-Hill, New York, 1984); *Fair-Rite Soft Ferrites Catalog*, 14th ed., page 131 (Fair-Rite Products Corp., Wallkill, NY, 2000).

[15] R. Dean Straw, N6BV, *The ARRL Antenna Book*, 18th Ed., op. cit., p 5-7 says "The *Q* of a short coil on a long rod is greatest at the center. On the other hand, if you require a higher *Q* than this, it is recommended that you spread the coil turns along the whole length of the core, even though this will result in a lower value of inductance. (The inductance can be increased to the original value by adding turns.)" J.J. Carr (*Joe Carr's Loop Antenna Handbook*, 1st Ed., 1999, Universal Radio Research, Reynoldsburg, OH) makes a similar statement: "The *Q* is worst when the coil is concentrated in a small portion of the rod length. To improve *Q*, at the expense of overall possible inductance, then wind the coil over the entire length of the rod." No measured data or citation is advanced to support this statement.

[16] R. Dean Straw, N6BV, *The ARRL Antenna Book*, 18th Ed., op. cit., p 5-7, Eq. 8.

[17] F. Langford-Smith, ed., *Radiotron Designer's Handbook*, 4th ed., (Wireless Press, Sydney, Aust., 1953), reprinted by RCA Electronics Components, Harrison, N.J., 1968. A comprehensive discussion of the effect of spaced windings is given at Section 10.1(ii).

[18] Type 33 Material is described in catalogs and the literature as "permeability 800." See, for example DeMaw, Section 2.1. However, Fair-Rite, the manufacturer of Type 33 material, states, in Catalog no. 14, that it has an initial permeability of 600. Fair-Rite's graph of initial permeability versus frequency, however, shows an initial permeability of 700 below about 750 kHz, increasing to 800 at 1 to 2 MHz. Fair-Rite specifies permeability within ±20%.

[19] Ocean State Electronics, part no. LA-540. Ocean State Electronics, 6 Industrial Drive, P.O. Box 1458, Westerly, RI 02891, Order telephone number 1-800-866-6626, Web site: **www.oselectronics.com**.

[20] Although the Type "X" rod is described in Ocean State Electronics' catalog as "tunes the broadcast band, 540 kHz – 1600 kHz," the *Q* performance leads to the suspicion that, in fact, the material is optimized for the long wave broadcast band, 200-400 kHz.

[21] F.E. Terman, *Electronic and Radio Engineering*, 4th Edition, (McGraw Hill, New York, 1955), Section 2.7.

[22] Hewlett Packard provides the following correction factors in *Operating and Service Manual: Q Meter 4342A* (1983) at Pages 3-13 through 3-16:

$$Q_t = Q_i \frac{C + C_d}{C}$$

Where:
Q_t is the true *Q*
Q_i is the indicated (instrument reading) *Q*
C is the instrument capacitance to achieve resonance
C_d is the inductor's distributed capacitance

$$L_t = L_i \frac{C_d}{C + C_d}$$

where:
L_t is the true inductance
L_i is the indicated (instrument reading *Q*)
C and C_d are as above

Similar correction formulas are provided in the Instruction Manual for the Q Meter Type 260A, (undated) pages 19-21.

[23] H.P. Westman, ed., *Reference Data for Radio Engineers*, 5th Edition, (Howard W. Sams, Indianapolis, 1970), Chapter 4, Table 25.

[24] A different formulation of Medhurst's formula is presented in *Radiotron Designer's Handbook, id*, Chapter 11, Section 2(i), in which a table of coil length versus diameter must be used. I don't know the provenance of the formula presented — widely used by the Tesla coil builder fraternity — but it closely matches values computed using the tabular approach presented in the *Radiotron Designer's Handbook*. Of course, Medhurst's formula is based upon a vertically oriented coil, installed on a metal chassis and is not necessarily accurate for other configurations. However, it provides reasonable results for the examples that I investigated.

QEX