Variable Demand Equilibrium Problems

A Comparison of Automated Methods

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[3]Numeral Software Limited

Mathematics in Transport, 2005

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Outline

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General Context.

- Arose within the Department for Transport project DIADEM.

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  - The Universities of York, Leeds and Napier.

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We have just one route joining each OD pair.

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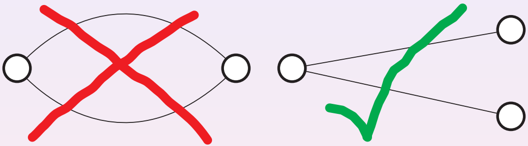

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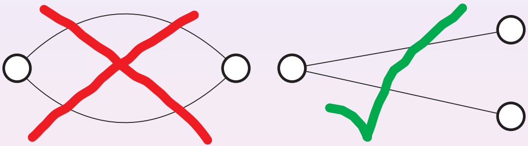

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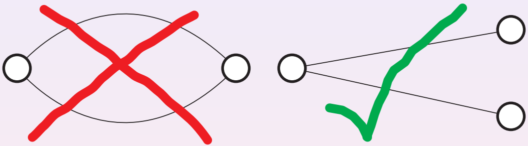

Main variables are as follows:

$\mathbf{X}_i$ = "the flow along the route between OD-pair i";

$\mathbf{Y}_i$ = "the cost of travel along the route between OD-pair i".

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Cost and Demand Functions.

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Definition (Cost and demand functions)

- Cost function, $\mathbf{C} : [0^N, +\infty^N) \rightarrow [0^K, +\infty^K)$;
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- Cost function $\mathbf{C}(\mathbf{X})$ gives us the cost of traversing each route with flows $\mathbf{X}$;
- Demand function $\mathbf{D}(\mathbf{Y})$ gives us the demand on each route with costs $\mathbf{Y}$.

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$$\begin{pmatrix} \mathbf{D}(\mathbf{Y}) - \mathbf{X} \\ \mathbf{Y} - \mathbf{C}(\mathbf{X}) \end{pmatrix} = 0,$$

or equivalently

$$\mathbf{D}(\mathbf{C}(\mathbf{X})) - \mathbf{X} = 0.$$

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- But how do we find the solutions to variable demand equilibrium?

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A Basic Algorithm.

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- Move a distance λ.

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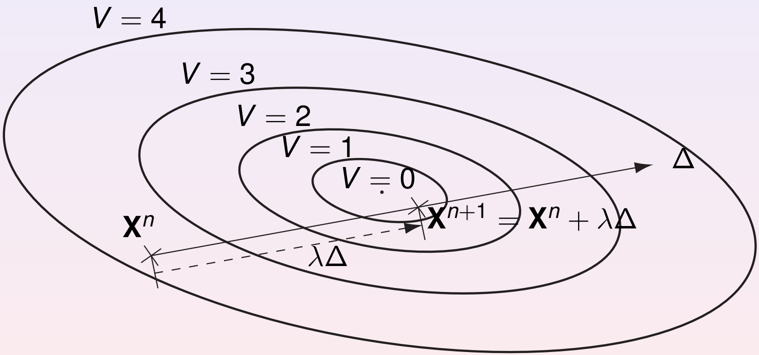
A Basic Algorithm.



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Our First Algorithm – Algorithm 1.

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Definition

Our search direction is

$$\Delta^{A1}(\mathbf{X}) = \mathbf{D}(\mathbf{C}(\mathbf{X})) - \mathbf{X}.$$

Our objective function is

$$V^{A1}(\mathbf{X}) = \frac{1}{2}\Delta^{A1}(\mathbf{X}) \cdot \Delta^{A1}(\mathbf{X}).$$

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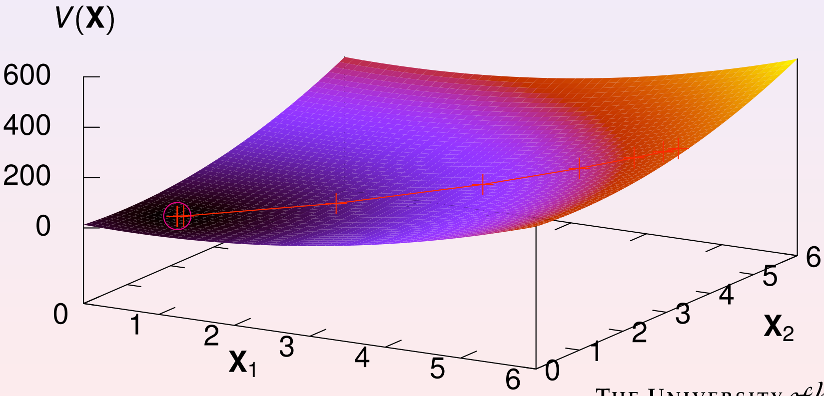
Choose step-lengths via Armijo-like method.

Algorithm 1 In Practise.



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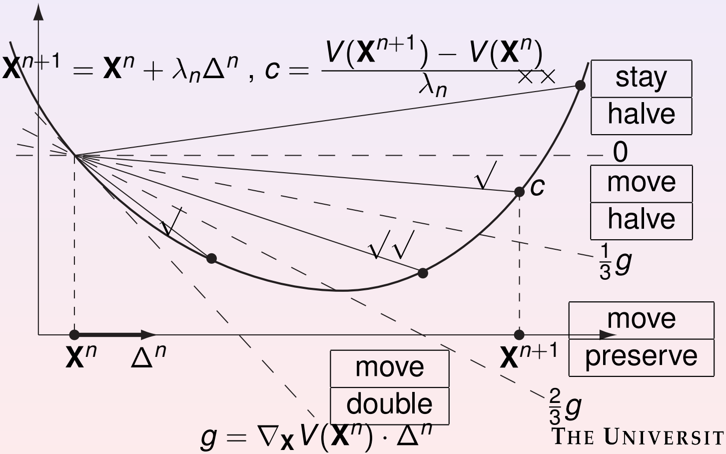
Armijo-Like Step-Lengths.



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$$\mathbf{C}(\mathbf{X}) = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} \mathbf{X}, \quad \mathbf{D}(\mathbf{Y}) = \begin{pmatrix} 4 \\ 94 \end{pmatrix} - \begin{pmatrix} 1 & 0 \\ 0 & 31 \end{pmatrix} \mathbf{Y}$$

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- Along the line $\mathbf{X} = (3 - 2t, t)^\top$, $t \in [0, 1.5]$ the search direction is uphill.

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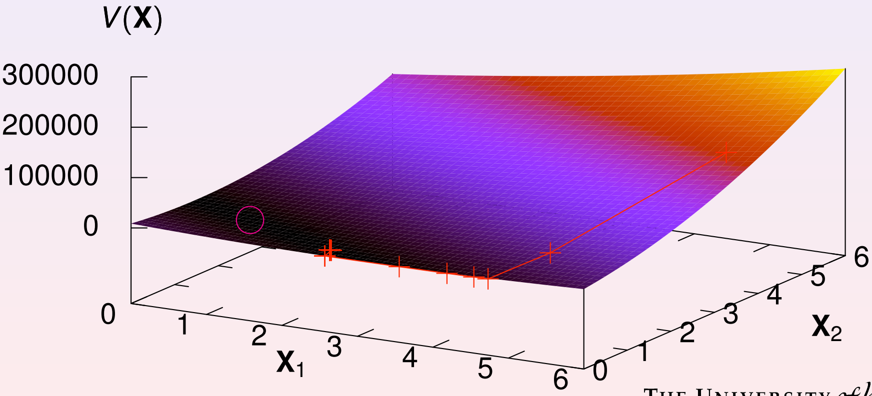
Algorithm 1 Failing In Practise.



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- New objective function:

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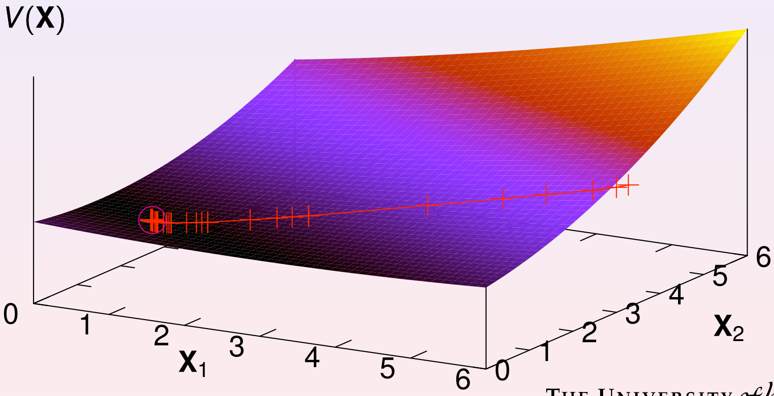
'Most Obvious' Direction Fixing The Flaw.



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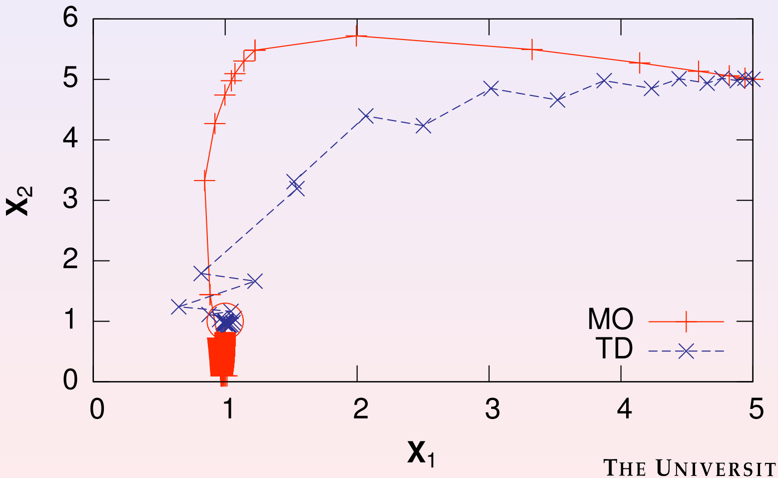
    - ‘Alternating Search’ algorithm.

‘Alternating Search’ Fixing The Further Flaw.



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$$\Delta^{\mathrm{ND}}(\mathbf{X}) = P(\mathbf{X})\Delta^{\mathrm{A1}}(\mathbf{X}).$$

- Can we find a good definition for $P(\mathbf{X})$?

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Can We Make Our Directional Derivative Negative?

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- Our directional derivative is

$$\nabla_{\mathbf{X}} V^{\mathrm{A1}} \cdot \Delta^{\mathrm{ND}} = \left(P(\mathbf{X})\Delta^{\mathrm{A1}}\right)^{\top} \mathbf{C}'(\mathbf{X})^{\top} \left(\mathbf{D}'(\mathbf{C}(\mathbf{X}))^{\top}\Delta^{\mathrm{A1}}\right) - \left(\Delta^{\mathrm{A1}}\right)^{\top} P(\mathbf{X})^{\top} \left(\Delta^{\mathrm{A1}}\right).$$

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  - $P(\mathbf{X}) \equiv -\mathbf{D}'(\mathbf{C}(\mathbf{X}))^{\top}$.

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First Choice — $P(\mathbf{X}) \equiv \mathbf{C}'(\mathbf{X})$.

Using this our directional derivative is now equal

$$\underbrace{\left(\Delta^{A1}\right)^{\top}\left[\mathbf{C}'(\mathbf{X})^{\top}\mathbf{C}'(\mathbf{X})^{\top}\mathbf{D}'(\mathbf{C}(\mathbf{X}))^{\top}\right]\left(\Delta^{A1}\right)}_{\text{unsure of sign}}$$

$$\underbrace{-\left(\Delta^{A1}\right)^{\top}\mathbf{C}'(\mathbf{X})\left(\Delta^{A1}\right)}_{<0 \text{ as } \mathbf{C}'(\mathbf{X}) \text{ strictly positive definite}}.$$

Second Choice — $P(\mathbf{X}) \equiv -\mathbf{D}'(\mathbf{C}(\mathbf{X}))$.

Using this our directional derivative is now equal

$$\underbrace{-\left(\mathbf{D}'(\mathbf{C}(\mathbf{X}))\Delta^{A1}\right)^{\top} \mathbf{C}'(\mathbf{X})^{\top}\left(\mathbf{D}'(\mathbf{C}(\mathbf{X}))^{\top}\Delta^{A1}\right)}_{\text{unsure of sign because } \mathbf{D}'(\mathbf{C}(\mathbf{X})) \text{ doesn't match } \mathbf{D}'(\mathbf{C}(\mathbf{X}))^{\top}}$$

$$+\underbrace{\left(\Delta^{A1}\right)^{\top}\mathbf{D}'(\mathbf{C}(\mathbf{X}))\left(\Delta^{A1}\right)}_{<0 \text{ as strictly negative definite}}.$$

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Third Choice — $P(\mathbf{X}) \equiv -\mathbf{D}'(\mathbf{C}(\mathbf{X}))^\top$.

Using this our directional derivative is now equal

$$\underbrace{-\left(\mathbf{D}'(\mathbf{C}(\mathbf{X}))^\top \Delta^{A1}\right)^\top \mathbf{C}'(\mathbf{X})^\top \left(\mathbf{D}'(\mathbf{C}(\mathbf{X}))^\top \Delta^{A1}\right)}_{<0 \text{ as } \mathbf{C}'(\mathbf{X}) \text{ strictly positive definite}}$$

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$$\underbrace{+\left(\Delta^{A1}\right)^\top \mathbf{D}'(\mathbf{C}(\mathbf{X}))\left(\Delta^{A1}\right)}_{<0 \text{ as strictly negative definite}}.$$

So we have a definition of $P(\mathbf{X})$ that looks good under standard constraints.

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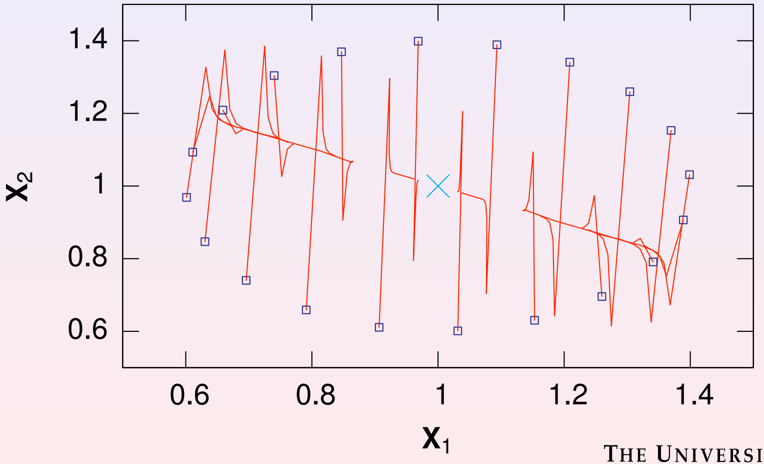
Outline

Demonstration of The Failure of Algorithm 1.



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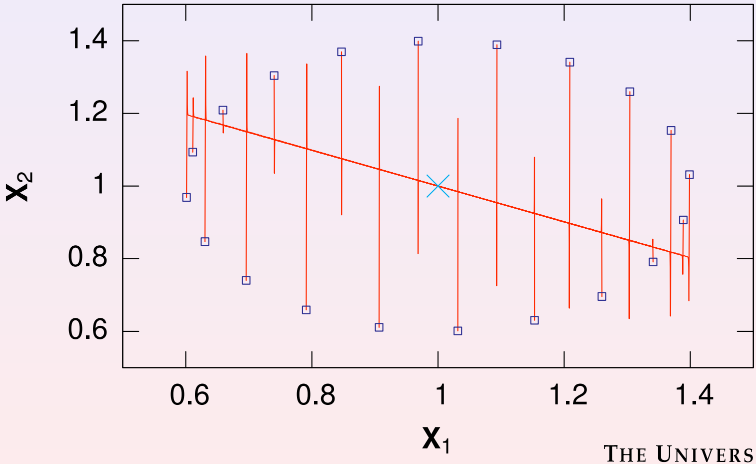
New Search Direction Succeeding Where A1 Fails.



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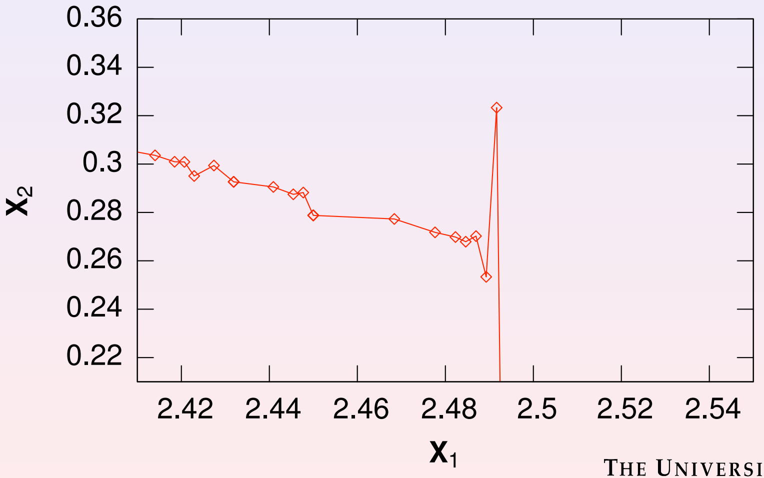
Why The New Direction Is Slow.



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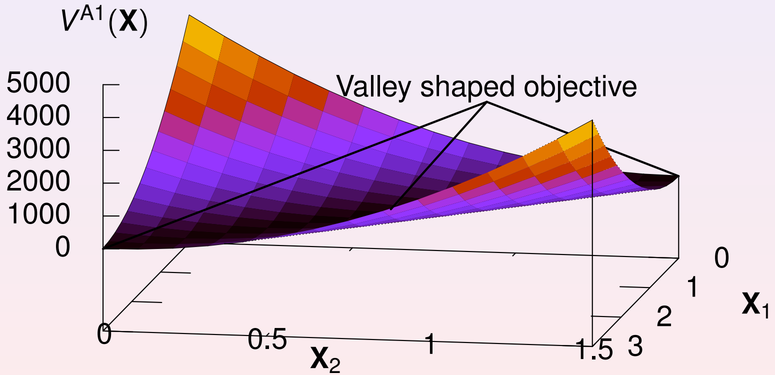
Maybe The Cost Function Is The Cause.



Two Example Problems.

Definition (First Problem)

$$\mathbf{C}(\mathbf{X}) = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} \mathbf{X}, \quad \mathbf{D}(\mathbf{Y}) = \begin{pmatrix} 4 \\ 3k+1 \end{pmatrix} - \begin{pmatrix} 1 & 0 \\ 0 & k \end{pmatrix} \mathbf{Y}$$

Definition (Second Problem)

$$\mathbf{C}(\mathbf{X}) = \begin{pmatrix} k \\ 3k \end{pmatrix} + \begin{pmatrix} 2 & 1-k \\ 1-k & 2(1-k) \end{pmatrix} \mathbf{X},$$

$$\mathbf{D}(\mathbf{Y}) = \begin{pmatrix} 4 \\ 1+96(1-k) \end{pmatrix} - \begin{pmatrix} 1 & 0 \\ 0 & 32(1-k) \end{pmatrix} \mathbf{Y}$$

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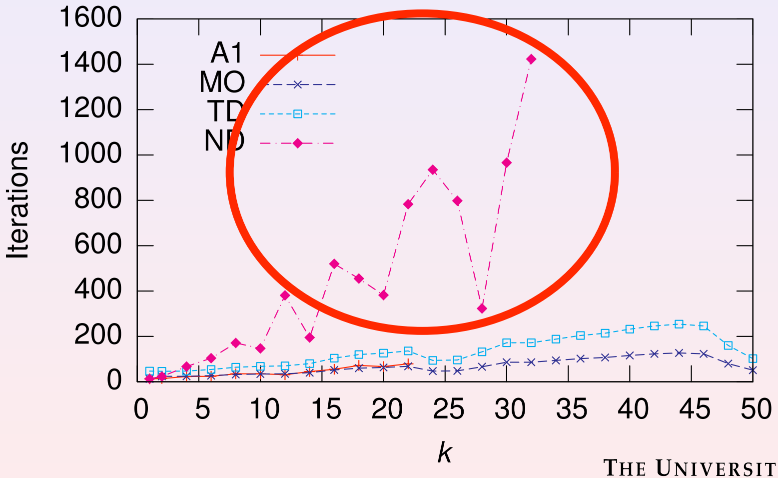
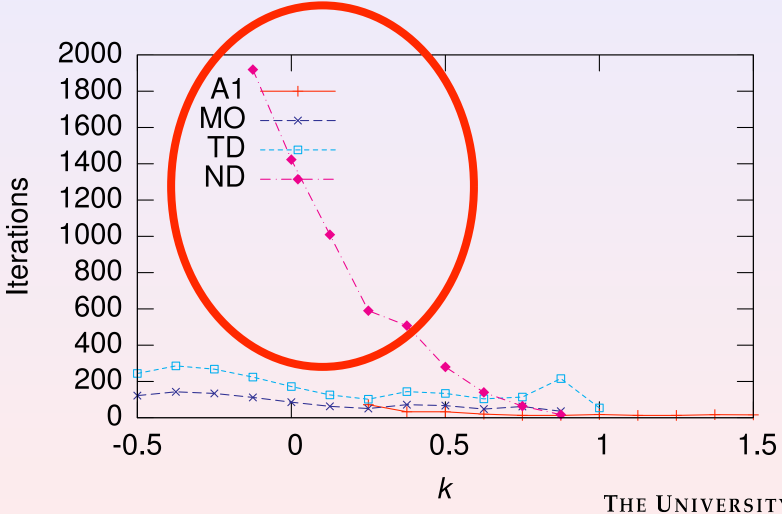
Armijo-Like Step-Lengths And Problem 1.



Armijo-Like Step-Lengths And Problem 2.



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- Outlook
  - Find a way to improve the performance of the new direction.

For Further Reading I

M. J. Smith
Two-direction methods of calculating variable demand equilibria.
DIADEM research note, 2004.

J. Springham
On solving variable demand equilibrium problems.
Thesis submitted for the degree of MSc at the University of York, 2004.

A. Woods
A Comparison Of Automated Methods For Solving Variable Demand Equilibrium Problems.
Thesis submitted for the degree of MSc at the University of York, 2005.