Sketch of Answers for Micro II Set 3 Exercises

Q1: d.

Q2: c.

Q3: d.

Detailed solution:

Firm 1's problem is

$$\begin{aligned}\max \pi_1 &= p(q_1, q_2)q_1 - cq_1 \\ &= \frac{1000000}{(q_1+q_2)}q_1 - 10q_1 \\ FOC \Rightarrow &\ 1000000\left(\frac{1}{q_1+q_2} - \frac{q_1}{(q_1+q_2)^2}\right) - 10 = 0 \\ \Rightarrow &\ 1000000\frac{q_2}{(q_1+q_2)^2} - 10 = 0 \\ \Rightarrow &\ \frac{q_2}{(q_1+q_2)^2} = \frac{1}{100000}.\end{aligned}$$

Similarly, for firm 2 we have $\frac{q_1}{(q_1+q_2)^2} = \frac{1}{100000}$. And in equilibrium we know $q_1 = q_2$. Therefore, $\frac{1}{4q} = \frac{1}{100000}$. Hence, $q_1 = q_2 = 25000$, and $p = \frac{1000000}{50000} = 20$.

Q4: d.

Detailed solution:

We know that the elasticity is

$$\epsilon = \frac{p(y)}{y}\frac{dy}{dp(y)} = \frac{\frac{p(y)}{y}}{\frac{dp(y)}{dy}}.$$

Here a representative airline's problem is

$$\begin{aligned}\max &\quad \pi_i = p(Q)q_i - cq_i \\ FOC_{q_i} &= \frac{dp(Q)}{dQ}\frac{dQ}{dq_i}q_i + p(Q) - c = 0.\end{aligned}$$

Since $Q = q_1 + q_2 + q_3 + q_4$, $\frac{dQ}{dq_i} = 1$. Thus, the above equation can be simplified as

$$\frac{dp(Q)}{dQ}q_i + p(Q) = c.$$

In equilibrium, all these four airlines should provide the same level of output (because they have the same cost functions). Therefore, $q_i = \frac{Q}{4}$. We get

$$\begin{aligned}
&\frac{1}{4}\frac{dp(Q)}{dQ}Q + p(Q) = c \\
\text{Divided by } p(Q):\quad &\frac{1}{4}\frac{dp(Q)}{dQ}\frac{Q}{p(Q)} + 1 = \frac{c}{p(Q)} \\
&\frac{1}{4}\frac{1}{(-1.5)} + 1 = \frac{c}{p(Q)} \\
\text{So,}\quad &\frac{p(Q)}{c} = \frac{6}{5}.
\end{aligned}$$

You get the same result by using the pricing formula for the Cournot oligopoly with many firms, which is $p(Q)\left[1 + \frac{1}{\epsilon/s_i}\right] = MC(q_i)$, with s_i being the market share of firm i, see also section 'Many Firms in Cournot Equilibrium' in the Varian textbook (in the 9th edition this is section 28.8):

$$\begin{aligned}
p(Q)\left[1 - \frac{1}{1.5/0.25}\right] &= c \\
p(Q)\left[1 - \frac{1}{6}\right] &= c \\
p(Q)\frac{5}{6} &= c \\
\frac{p(Q)}{c} &= \frac{6}{5}
\end{aligned}$$

Q5: d.

Detailed solution:

The coordination problem is

$$\begin{aligned}
\max \pi_{1+2} &= pY - 8y_1 - y_2^2 \\
&= (72 - y_1 - y_2)(y_1 + y_2) - 8y_1 - y_2^2 \\
FOC_{y_1} \Rightarrow &\ 72 - 2y_1 - 2y_2 - 8 = 0 \\
\Rightarrow &\ y_1 + y_2 = 32 \\
FOC_{y_2} \Rightarrow &\ 72 - 2y_1 - 2y_2 - 2y_2 = 0 \\
\Rightarrow &\ y_1 + 2y_2 = 36.
\end{aligned}$$

So, $y_1 = 28$ and $y_2 = 4$.

Q6: c.

Detailed solution:

The follower's problem is

$$\begin{aligned} \max \quad & \pi_2 = (36 - q_1 - q_2)q_2 \\ FOC_{q_2} \quad & = 36 - q_1 - 2q_2 = 0 \\ \Rightarrow \quad & q_2 = 18 - \frac{q_1}{2}. \end{aligned}$$

The leader's problem is then

$$\begin{aligned} \max \pi_1 \quad & = (36 - q_1 - q_2)q_1 - 9q_1 \\ & = \left(36 - q_1 - (18 - \frac{q_1}{2})\right) q_1 - 9q_1 \\ & = \left(18 - \frac{q_1}{2}\right) q_1 - 9q_1 \\ FOC_{q_1} \quad & = 18 - q_1 - 9 = 0 \\ \Rightarrow \quad & q_1 = 9. \end{aligned}$$

Solution to Question 7:

(i) With 3 firms profits of ith are

$$\pi_i = px_i - x_i^2 = (3 - x_1 - x_2 - x_3)x_i - x_i^2$$

FOC's are

$$\begin{aligned} 3 - x_2 - x_3 - 4x_1 &= 0, \\ 3 - x_1 - x_3 - 4x_2 &= 0, \\ 3 - x_1 - x_2 - 4x_3 &= 0. \end{aligned}$$

Thus, the reactions curves are

$$\begin{aligned} x_1 &= \frac{3}{4} - \frac{1}{4}x_2 - \frac{1}{4}x_3 \ , \\ x_2 &= \frac{3}{4} - \frac{1}{4}x_1 - \frac{1}{4}x_3 \ , \\ x_3 &= \frac{3}{4} - \frac{1}{4}x_1 - \frac{1}{4}x_2 \ . \end{aligned}$$

(ii) Solution must be symmetric:
$3 - 6x_i = 0 \ \Rightarrow \ x_i^* = \frac{1}{2}$, consequently $\pi_i = (3 - \frac{3}{2})\frac{1}{2} - \left(\frac{1}{2}\right)^2 = \frac{1}{2}$.

(iii) With only two firms,

$$\pi_i = (3 - x_1 - x_2)x_i - x_i^2$$

FOC's are

$$\begin{aligned} 3 - x_2 - 4x_1 &= 0, \\ 3 - x_1 - 4x_2 &= 0. \end{aligned}$$

Thus, the reactions curves are

$$\begin{aligned} x_1 &= \frac{3}{4} - \frac{1}{4}x_2 \ , \\ x_2 &= \frac{3}{4} - \frac{1}{4}x_1 \ . \end{aligned}$$

Again, symmetry yields
$3 - 5x_i = 0 \ \Rightarrow \ x_i^* = \frac{3}{5}$, and $\pi_i = (3 - \frac{6}{5})\frac{3}{5} - \left(\frac{3}{5}\right)^2 = \frac{18}{25}$.

(iv) For firm 1:

$$\max_{x_1} \pi_1 = (3 - x_1 - x_2 - x_3)x_1 - x_1^2.$$

The FOC is

$$3 - 4x_1 - (x_2 + x_3) = 0 \ \Rightarrow \ x_1 = \frac{3}{4} - \frac{x_2 + x_3}{4}. \tag{1}$$

However, for firms 2 and 3:

$$\max_{x_2, x_3} \pi_{2,3} = (3 - x_1 - x_2 - x_3)(x_2 + x_3) - x_2^2 - x_3^2.$$

FOC's for firms 2 and 3 are

$$\begin{aligned} 3 - x_1 - 2(x_2 + x_3) - 2x_2 &= 0, \\ 3 - x_1 - 2(x_2 + x_3) - 2x_3 &= 0. \end{aligned}$$

This leads to

$$x_2 = \frac{1}{2} - \frac{1}{6}x_1 = x_3.$$

Substituting into Equation (3.1) yields

$$\begin{aligned} x_1^* = \frac{6}{11} \ , \ x_2^* = x_3^* = \frac{9}{22} \ , \\ \pi_1 = \frac{72}{121} \ , \ \pi_2 = \pi_3 = \frac{243}{484} \ . \end{aligned}$$

Note that $\pi_2 + \pi_3 = \frac{243}{242} > 1$.

END