First Year Report:
Fixed Lag Particle Filtering for Target Tracking

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Contents

1

Introduction

Everyone loves to track things. Yay!

2

Literature Review

2.1 Particle Filtering

2.1.1 *Bayes of our lives - what is filtering?*

Many tasks in signal processing, science in general, and indeed life, require us to make some estimate of an unknown quantity from indirect, incomplete, or inaccurate observations. By constructing a model to explain how these observations depend on the underlying state, we can infer something about that state. We will express this observation model in terms of a likelihood function:

$$P(Y|X) \tag{2.1}$$

where X is the state and Y the observations. This is not the whole story - in many cases we are not estimating our unknown state "from scratch". Previous experience, prejudice, and prior knowledge can also contribute to our estimates. The likelihood and prior terms can be combined through our friend, Bayes rule (**?**), to calculate the posterior probability of the state, i.e. the probability of the state given the observations:

$$P(X|Y) = \frac{P(Y|X)P(X)}{P(Y)} \tag{2.2}$$

We may not be able to pinpoint an exact value with certainty, but the probability of different candidates may now be compared. Mathematically, we can assign a probability distribution to the state space of X.

Often the quantity in which we are interested, X, is changing over time, and we would like to estimate its value at each point in time given only the observations received so far. In this case we will generally assume that the state is Markovian, i.e. that $X_t|X_{t-1}$ is independent of $X_{1:t-2}$. This is the traditional filtering problem. By constructing a model for the evolution of the unknown state, we can now derive our prior information from our estimate at the previous time step. In discrete time, we now write:

$$P(X_t|Y_{1:t}) = \frac{\int P(Y_t|X_t)P(X_t|X_{t-1})P(X_{t-1}|Y_{1:t-1})dX_{t-1}}{P(Y_t|Y_{1:t-1})} \tag{2.3}$$

where the subscript indicates the time and ranges are notated by : in the MATLAB style.

Instead of marginalising the previous state, we may sometimes want to consider the joint state distribution over all time instances. This may similarly be expanded as:

$$P(X_{1:t}|Y_{1:t}) = \frac{P(Y_t|X_t)P(X_t|X_{t-1})P(X_{1:t-1}|Y_{1:t-1})}{P(Y_t|Y_{1:t-1})} \tag{2.4}$$

Note that in this expression we are estimating the distribution over previous time instances given the latest observations.

So far, we have expressed the problem in terms of distributions, but sometimes we will need to consider recursive random variable equations. In the most general form:

$$X_t = f_t(X_{t-1}, V_t) \tag{2.5}$$

$$Y_t = g_t(X_t, W_t) \tag{2.6}$$

where f_t and g_t are known deterministic functions and V_t and W_t and random variables. Again, we assume X_t is Markovian.

2.1.2 *Keep Kalman carry on - the Kalman filter and its extensions*

In a few simple cases the filtering set-up permits the derivation of closed form posterior distributions at each time instant. Most notably, the Kalman filter (KF) (**?**) is an analytic filter for models with continuous state and observation variables, in which both transition and observation models are linear transformations with Gaussian innovations.

$$x_t = Ax_t + v_t \tag{2.7}$$

$$y_t = Cx_t + w_t \tag{2.8}$$

where v_t and w_t are now Gaussian random variables with zero mean and covariance matrices Q and R. We use lower case variables here to emphasise that these are "nice", continuous vectors. (As we shall see, our state variable will later be sets or lists).

Kalman's solution for the linear-Gaussian case is given by:

$$P(x_t|y_{1:t}) = \mathcal{N}(x_t|\mu_t, \Sigma_t) \tag{2.9}$$

$$P(x_t|y_{1:t-1}) = \mathcal{N}(x_t|\hat{\mu}_t, \hat{\Sigma}_t) \tag{2.10}$$

where μ_t, Σ_t, etc. are given by the following recursions.

Time Updates:

$$\hat{\mu}_t = A\mu_{t-1} \tag{2.11}$$

$$\hat{\Sigma}_t = A\Sigma_{t-1}A^T + Q \tag{2.12}$$

Measurement Updates:

$$z_t = y_t - C\hat{\mu}_t \tag{2.13}$$

$$S_t = C\hat{\Sigma}_t C^T + R \tag{2.14}$$

$$K_t = \hat{\Sigma}_t C^T S_t^{-1} \tag{2.15}$$

$$\mu_t = \hat{\mu}_t + K_t z_t \tag{2.16}$$

$$\Sigma_t = (I - K_t C)\hat{\Sigma}_t \tag{2.17}$$

The KF is delightful because it not only provides us with a closed-form analytic solution, but the complexity of that solution does not increase as we receive additional measurements. This is a consequence of the fact that the Gaussian distribution is its own conjugate prior. Unfortunately, no other such convenient cases have been discovered (**?**). Analytic solutions to non-linear, non-Gaussian filtering problems generally require unacceptable conditions, such as zero process noise $Q = 0$ (**?**).

Given the loveliness of the KF, the instinct when faced by an intractable non-linear filtering problem is to linearise it. This produces the Extended Kalman Filter (EKF), and is achieved by replacing the A and C matrices in equations 2.11 through 2.17 above with Jacobians:

$$A_t = \left. \frac{\partial f}{\partial x_t} \right|_{\mu_{t-1}} \tag{2.18}$$

$$C_t = \left. \frac{\partial g}{\partial x_t} \right|_{\hat{\mu}_t} \tag{2.19}$$

ADD SOMETHING ABOUT SMOOTHING.

2.1.3 *Tough as old bootstraps - the traditional approach*

In general we will not be so lucky as to have a problem with linear-Gaussian dynamics. In this case, a particle filter (PF) may be the best alternative. With a PF, we approximate a probability distribution with a set of (weighted) samples drawn from that distribution.

$$P(X) \approx \frac{1}{N} \sum_m W^{(m)} \delta_X(x^{(m)}) \tag{2.20}$$

where $\delta(x)$ represents a unit probability point mass at a point x, and $\sum_m W^{(m)} = 1$. Consider the posterior joint state distribution of equation 2.4.

2.1.4 *MCMC in da house - series vs. parallel*

S-MCMC (Godsill) approach

2.2 Approaches to Tracking

A brief overview JPDAF MHT Brief mention of PFs

3

Fixed Lag Estimation for Tracking

4

Fixed Lag Particle Filters

5

Plan Of Future Research

radar, insects, crowds, cells, jump-diffusions, cursor tracking...

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