Dynamical Models for Tracking with the Variable Rate Particle Filter

Pete Bunch and Simon Godsill

Cambridge University Engineering Department
Signal Processing & Communications Lab

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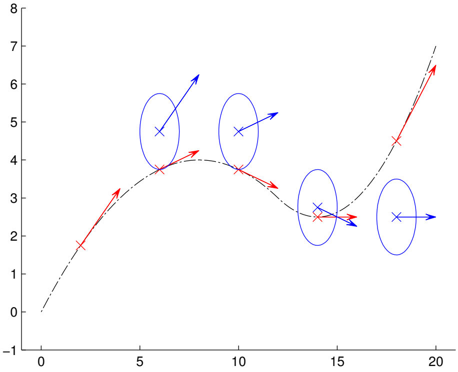
What's in store...

- What is a variable rate model?
- What is a variable rate particle filter?
- What dynamical models can we use for tracking with a VRPF?
- What's wrong with them?
- How can make them better?
- Does it work in 3 dimensions?

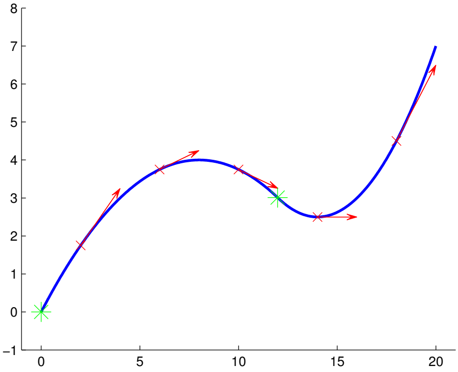
What is a variable rate model?

Conventional Hidden Markov Models for Tracking



- Discrete-time
- Stochastic Markovian dynamics
- Simple
- Hard to handle manoeuvres

Variable Rate Models for Tracking



- Continuous-time
- Conditionally-deterministic dynamics
- Always nonlinear
- Handles manoeuvres

What is a variable rate particle filter?

The Variable Rate Particle Filter

The Variable Rate Particle Filter

Bootstrap particle filter

- Proposal: Add a new changepoint or just extend trajectory
- Weight update: $w_n \propto w_{n-1} p(y_n | x(t_n))$

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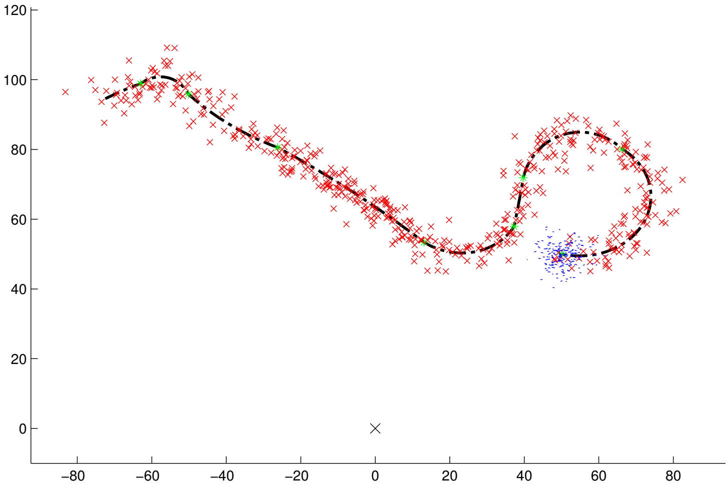
Improvements

- Resample-move
- SMC sampler
- RJ-MCMC


120
100
80
60
40
20
0
−80
−60
−40
−20
0
20
40
60
80

What dynamical models can we use for tracking with a VRPF?

VR Tracking Models I: Cartesian Coordinates

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \ddot{x}_1(t) \\ \ddot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} a_{1,k} \\ a_{2,k} \end{bmatrix}$$

$$\tau_k \leq t < \tau_{k+1}$$

VR Tracking Models I: Cartesian Coordinates

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \ddot{x}_1(t) \\ \ddot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} a_{1,k} \\ a_{2,k} \end{bmatrix}$$

$$\tau_k \leq t < \tau_{k+1}$$

$$\begin{bmatrix} x_1(t) \\ x_2(t) \\ \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 1 & 0 & \Delta t & 0 \\ 0 & 1 & 0 & \Delta t \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1(\tau_k) \\ x_2(\tau_k) \\ \dot{x}_1(\tau_k) \\ \dot{x}_2(\tau_k) \end{bmatrix} + \begin{bmatrix} \frac{1}{2}\Delta t^2 & 0 \\ 0 & \frac{1}{2}\Delta t^2 \\ \Delta t & 0 \\ 0 & \Delta t \end{bmatrix} \begin{bmatrix} a_{1,k} \\ a_{2,k} \end{bmatrix}$$

$$\Delta t = t - \tau_k$$

VR Tracking Models II: Intrinsic Coordinates

$$\begin{aligned} \dot{s}(t) &= a_{T,k} \\ s(t)\dot{\psi}(t) &= a_{N,k} \\ \dot{x}_1(t) &= s(t)\cos(\psi(t)) \\ \dot{x}_2(t) &= s(t)\sin(\psi(t)) \end{aligned}$$

$$\tau_k \leq t < \tau_{k+1}$$

$$\dot{s}(t) = \dot{s}(\tau_k) + a_{T,k}\Delta t$$

$$\psi(t) = \psi(\tau_k) + \frac{a_{N,k}}{a_{T,k}} \log\left(\frac{\dot{s}(t)}{\dot{s}(\tau_k)}\right)$$

$$x(t) = x(\tau_k) + \frac{\dot{s}(t)^2}{4a_{T,k}^2 + a_{N,k}^2}\left[a_{N,k}\sin(\psi(t)) + 2a_{T,k}\cos(\psi(t))\right]$$
$$- \frac{\dot{s}(\tau_k)^2}{4a_{T,k}^2 + a_{N,k}^2}\left[a_{N,k}\sin(\psi(\tau_k)) + 2a_{T,k}\cos(\psi(\tau_k))\right]$$

$$y(t) = y(\tau_k) + \frac{\dot{s}(t)^2}{4a_{T,k}^2 + a_{N,k}^2}\left[-a_{N,k}\cos(\psi(t)) + 2a_{T,k}\sin(\psi(t))\right]$$
$$- \frac{\dot{s}(\tau_k)^2}{4a_{T,k}^2 + a_{N,k}^2}\left[-a_{N,k}\cos(\psi(\tau_k)) + 2a_{T,k}\sin(\psi(\tau_k))\right]$$

$$\Delta t = t - \tau_k$$

What's wrong with them?

A Problem: Model Deficiency

- 4 state variables governed by 2 motion parameters
- From a given state and time, not all future states are achievable

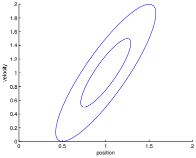
Problems:

- Poor robustness to model errors
- Forward-backward smoothing does not work

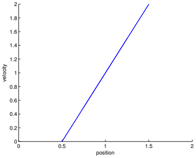
A Familiar Example

$$x_n = A x_{n-1} + w_n$$
$$w_n \sim \mathcal{N}(.|0, Q)$$

$$Q_1 = \sigma^2 \begin{bmatrix} \frac{1}{3}\Delta t^3 & \frac{1}{2}\Delta t^2 \\ \frac{1}{2}\Delta t^2 & \Delta t \end{bmatrix} \qquad Q_2 = \sigma^2 \begin{bmatrix} \frac{1}{4}\Delta t^3 & \frac{1}{2}\Delta t^2 \\ \frac{1}{2}\Delta t^2 & \Delta t \end{bmatrix}$$



Q_1



Q_2

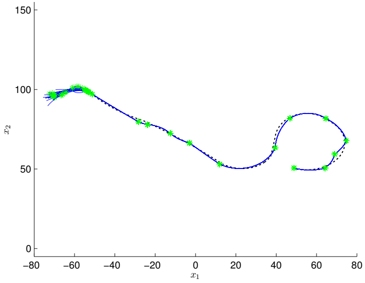
How can make them better?

The Solution: Add More Parameters

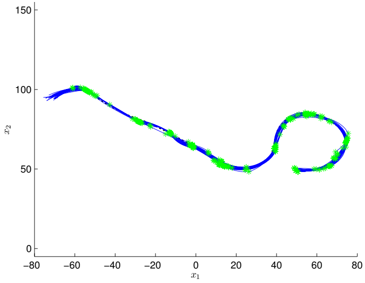
$$
\begin{aligned}
\dot{s}(t) &= a_{T,k} \\
s(t)\dot{\psi}(t) &= a_{N,k} \\
\dot{x}_1(t) &= s(t)\cos(\psi(t)) + d_{1,k} \\
\dot{x}_2(t) &= s(t)\sin(\psi(t)) + d_{2,k}
\end{aligned}
$$

$$\tau_k \leq t < \tau_{k+1}$$

Variable Rate Smoothing



Filter



Smoother

Does it work in 3 dimensions?

3D Intrinsic Coordinate Tracking

$$
\begin{aligned}
\dot{s}(t) &= a_{T,k} \\
\dot{\mathbf{e}}_T(t) &= \frac{a_{N,k}}{s(t)}\mathbf{e}_N(t) \\
\dot{\mathbf{e}}_N(t) &= -\frac{a_{N,k}}{s(t)}\mathbf{e}_T(t) \\
\dot{\mathbf{e}}_B(t) &= \mathbf{0}.
\end{aligned}
$$

3D Intrinsic Coordinate Tracking

$$\dot{s}(t) = \dot{s}(\tau_k) + a_{T,k}\Delta t$$

$$\begin{bmatrix} \mathbf{e}_T(t)^T \\ \mathbf{e}_N(t)^T \\ \mathbf{e}_B(t)^T \end{bmatrix} = \begin{bmatrix} \cos(\Delta\psi(t)) & \sin(\Delta\psi(t)) & 0 \\ -\sin(\Delta\psi(t)) & \cos(\Delta\psi(t)) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \mathbf{e}_T(\tau_k)^T \\ \mathbf{e}_N(\tau_k)^T \\ \mathbf{e}_B(\tau_k)^T \end{bmatrix}$$

$$\Delta\psi(t) = \frac{a_{N,k}}{a_{T,k}} \log\left(\frac{\dot{s}(t)}{\dot{s}(\tau_k)}\right)$$

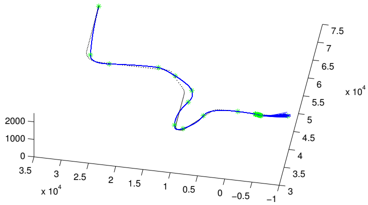
$$\mathbf{v}(t) = \dot{s}(t)\mathbf{e}_T(t)$$

$$\mathbf{r}(t) = \mathbf{r}(\tau_k) + \frac{1}{a_{N,k}^2 + 4a_{T,k}^2} \times \begin{bmatrix} \mathbf{e}_T(\tau_k) & \mathbf{e}_N(\tau_k) \end{bmatrix} \begin{bmatrix} \zeta_1(t) & \zeta_2(t) \\ \zeta_2(t) & -\zeta_1(t) \end{bmatrix} \begin{bmatrix} 2a_{T,k} \\ a_{N,k} \end{bmatrix}$$

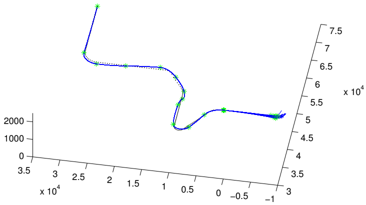
$$\zeta_1(t) = \cos(\Delta\psi(t))\dot{s}(t)^2 - \dot{s}(\tau_k)^2$$

$$\zeta_1(t) = \sin(\Delta\psi(t))\dot{s}(t)^2$$

3D Intrinsic Coordinate Tracking



Cartesian



Intrinsic

Summary

- Variable rate framework elegantly handles manoeuvres, changepoints and model-switching.
- Variable rate particle filter may be used for inference in this framework.
- Basic VR tracking models are uncontrollable - thwarts smoothing algorithms.
- New augmented model improves estimation and may be used for smoothing.
- Intrinsic coordinate model has been extended to work in 3D - piecewise planar assumption.