Filtering and Smoothing Algorithms for Nonlinear Variable Rate Models with Applications in Tracking

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***Abstract*—The abstract goes here.**

***Index Terms*—**

I. INTRODUCTION

CONVENTIONAL sequential state-space inference methods use discrete-time hidden Markov models to estimate an unkown quantity evolving over time. These models may be developed by discretisation of a continuous-time model, or by formulation directly in discrete time. In either case, it is common practice to use a grid of time points synchronous with the observations. Such "fixed rate" models may give poor representations of processes which exhibit structured behaviour over longer time scales. In such cases, a "variable rate" model may prove advantageous - the model is discretised onto a set of set of random times, which characterise the transitions in system behaviour.

Using a variable rate model, it is necessary to estimate both the set of changepoints and the (nonlinearly) evolving state. This is not analytically tractable so numerical approximations must be employed. The particle filter (introduced by [1]) and smoother (see [2], [3]) are such schemes which approximate the posterior state distribution using a set of samples, or "particles", drawn sequentially from it. A thorough introduction to particle filtering and smoothing methods can be found in [4], [5].

Target tracking algorithms are commonly based upon fixed rate models (see, e.g. [6] for a thorough survey), in which the target kinematics (position, velocity, etc.) are estimated at a set of fixed times at which observations (e.g. radar measurements) are made. In [7]–[9], variable rate models were introduced for tracking, in which the state trajectory is divided up by a set of changepoints between which the motion follows a deterministic path governed by kinematic parameters (accelerations, etc.) which are fixed for that segment. The changepoint times and segment parameters may be estimated using a numerical approximation called the variable rate particle filter (VRPF).

In this paper we review the VRPF and introduce improvements using the resample-move method of [10]. We then describe a new variable rate particle smoother (VRPS) algorithm, which uses a novel Markov Chain Monte Carlo (MCMC) formulation to circumvent difficulties of the conventional methods. In section **??**, we adapt the intrinsic-coordinate models of [8], [9] for use with the new smoothing algorithm. Simulations are presented in section **??**.

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II. VARIABLE RATE MODELS

We consider a general model from time 0 to T, between which observations, $\{y_1 \dots y_N\}$, are made at times $\{t_1 \dots t_N\}$. During this period, an unknown number, K, of changepoints occur at times $\{\tau_1 \dots \tau_K\}$, each with associated motion parameters, $\{w_1 \dots w_K\}$. Discrete sets containing multiple values over time will be written as, e.g. $y_{1:n} = \{y_1 \cdots y_n\}$.

Changepoints are treated as random variables, dependent on the previous sequence of times.

$$\tau_k \sim p(\tau_k | \tau_{1:k-1}) \tag{1}$$

In continuous time, the dynamics of the latent state, $x(t)$, may be written as a differential equation.

$$dx(t) = g(x(t), w(t), \tau_{1:k})dt \tag{2}$$

The dynamics are thus dependent on the sequence of changepoints which have occured so far, $\tau_{1:k}$, and a continuously varying random innovation, $w(t)$. Such a system may be discretised by assuming that this random innovation is constant between changepoints. This yields an equivalent discrete time system represented by a difference equation, in which the state at time τ_k is written as x_k.

$$x_k = f(x_{k-1}, w_{k-1}, \tau_{1:k}) \tag{3}$$

In order to calculate measurement likelihoods, it is also necessary to estimate the latent state at the time of the observations, t_n. Once τ_{k-1}, x_{k-1} and w_{k-1} have been estimated, this may be calculated deterministically using the same state equation. The state at time t_n at which the nth observation is made is denoted $\hat{x}_n$.

$$\hat{x}_n = f(x_{k-1}, w_{k-1}, \tau_{1:k-1}, t_n) \tag{4}$$

To keep the following derivations concise, we introduce an additional variable for each changepoint-parameter pair $\theta_k = \{\tau_k, x_k, w_k\}$.

It will be desirable to denote the set of changepoints and parameters (of unspecified size) which occur within some range of time. This will be written as $\theta_{[s,t]} = \{\theta_k \forall k : s < \theta_k < t\}$. Note that such a variable not only conveys where changepoints occur, but also where they do not within the specfied range.

- Notation for variable rate models
- Basic p-d model

III. The Variable Rate Particle Filter

- Basic filter
- Improved performance using RM and UKF

IV. The Variable Rate Particles Smoother

- Failure of ordinary backward sampling method
- MCMC method

V. Variable Rate Models for Tracking

- 2D Intrinsic coordinate model
- Full-rank 2D model

VI. Simulations

- Improved RMSE performance
- Improved particle diversity

VII. Conclusion

The conclusion goes here.

Appendix A

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Appendix B

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