135 Written HW 11-06 : Solutions

Problem 1

(a) $h(x) = f(g(x))$

$$h'(x) = f'(g(x)) \cdot g'(x)$$

$$h'(0) = f'(g(0)) \cdot g'(0) = f'(2) \cdot -3$$

$$= 3 \cdot -3 = \boxed{-9}$$

(b) $p(x) = [g(x)]^3$

$$p'(x) = 3 \cdot (g(x))^2 \cdot g'(x)$$

$$p'(2) = 3 \cdot (g(2))^2 \cdot g'(2) = 3 \cdot (-1)^2 \cdot 0 = \boxed{0}$$

(c) $K(x) = [g(f(x))]^{2/3}$

$$K'(x) = \frac{2}{3}(g(f(x)))^{-1/3} \cdot g'(f(x)) \cdot f'(x)$$

$$K'(1) = \frac{2}{3}(g(f(1)))^{-1/3} \cdot g'(f(1)) \cdot f'(1)$$

$$= \frac{2}{3}[g(-2)]^{-1/3} \cdot g'(-2) \cdot 5$$

$$= \frac{2}{3}(8)^{-1/3} \cdot (-3) \cdot 5 = \frac{2}{3}\left(\frac{1}{2}\right)(-15)$$

$$= \boxed{-5}$$

Problem 2: $x^2+4y^2=8$, tangent lines going through $(-4,0)$

Line tangent to $x^2+4y^2=8$ @ (a,b):

point: $(a,b)=(x_0,y_0)$

slope: $\left.\frac{dy}{dx}\right|_{x=a,y=b}=m$

$y-y_0=m(x-x_0)$

$\frac{d}{dx}(x^2+4y^2=8)$

$2x+8y\frac{dy}{dx}=0$

$\boxed{\frac{dy}{dx}=\frac{-x}{4y}}$ $\qquad \left.\frac{dy}{dx}\right|_{x=a,\,y=b}=\boxed{\frac{-a}{4b}}$

Tangent line: $y-b=\frac{-a}{4b}(x-a)$

Going through $(-4,0)$, so $x=-4$, $y=0$:

$0-b=\frac{-a}{4b}(-4-a)=\frac{a}{b}+\frac{a^2}{4b}$

$\left(0=\frac{a}{b}+\frac{a^2}{4b}+b\right)4b$

$0=4a+\boxed{a^2+4b^2}=4a+8$

Since (a,b) on $x^2+4y^2=8$, we have

$a^2+4b^2=8$

$\Rightarrow \boxed{a=-2}$

$a^2+4b^2=8$

$(-2)^2+4b^2=8$

$4+4b^2=8$

$b^2=1 \Rightarrow \boxed{b=\pm 1}$

$\underline{a=-2,\ b=1}$

$m = \frac{-a}{4b} = \frac{2}{4} = \frac{1}{2}$

$$\boxed{y-1 = \frac{1}{2}(x+2)}$$

$\underline{a=-2,\ b=-1}$

$m = \frac{-a}{4b} = \frac{2}{-4} = -1/2$

$$\boxed{y+1 = -\frac{1}{2}(x+2)}$$

Problem 3: Show $(f^{-1})'(x) = \dfrac{1}{f'(f^{-1}(x))}$

$$\boxed{y = f^{-1}(x)} \iff f(y) = x \quad \Big\} \frac{d}{dx} \text{ implicitly}$$

$$f'(y) \cdot \frac{dy}{dx} = 1$$

$$\frac{dy}{dx} = \frac{1}{f'(y)}$$

$$\frac{dy}{dx} = (f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))} \quad \checkmark$$