Math 135 Written HW 11-20: Solutions

Problem 1 $s(t) = 80t - 16t^2$

(a) The ball is at ground level when $s(t) = 0$:

$$80t - 16t^2 = 0$$

$$8t(10 - 2t) = 0$$

$8t = 0$ $\quad$ $10 - 2t = 0$

$\boxed{t = 0}$ $\quad$ $2t = 10$

$\boxed{t = 5}$

Since $s(1) = 80 - 16 > 0$, the ball is at or above ground level on the closed interval $\boxed{[0, 5]}$.

(b) CPs:

$$s'(t) = 80 - 32t \stackrel{!}{=} 0$$

$$32t = 80$$

$$t = \frac{80}{32} = \boxed{\frac{5}{2}}$$

EVT:

$$s(5/2) = 80(5/2) - 16(5/2)^2 = 200 - 20 = \boxed{180}$$

Absolute max on $[0, 5]$.

$$s(0) = 0$$

$$s(5) = 0$$

(c) $$s(t) = 80t - 16t^2 \stackrel{!}{=} 96$$

$$16t^2 - 80t + 96 = 0$$

$$t^2 - 5t + 6 = 0$$

$$(t - 3)(t - 2) = 0$$

$t = 2$ $\quad$ $t = 3$

$$\boxed{\begin{aligned} s'(2) &= 80 - 32(2) = 16 \\ s'(3) &= 80 - 32(3) = -16 \end{aligned}}$$

Problem 2: $g(x) = \dfrac{x^2}{(x-2)^2}$

(a) $$g'(x) = \frac{(2x)(x-2)^2 - (x^2)\cdot 2(x-2)}{(x-2)^4}$$

$$= \frac{2x(x^2-4x+4) - (x^2)(2x-4)}{(x-2)^4}$$

$$= \frac{2x^3 - 8x^2 + 8x - 2x^3 + 4x^2}{(x-2)^4}$$

$$= \frac{-4x^2+8x}{(x-2)^4} = \frac{-4x(x-2)}{(x-2)^4} = \frac{-4x}{(x-2)^3}$$

$g'(x) = \dfrac{-4x}{(x-2)^3} = 0$

$-4x = 0$

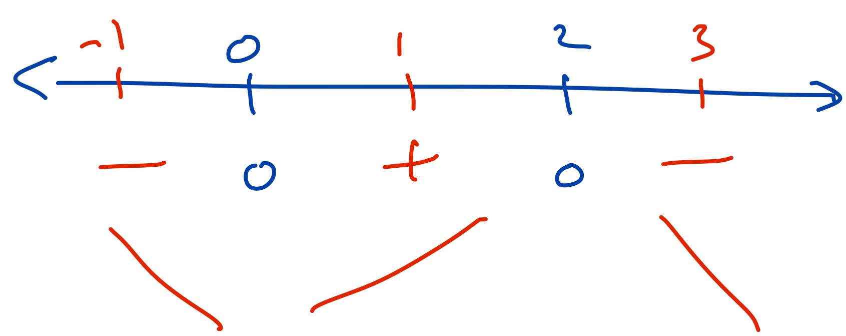
$\boxed{x=0}$

$g'(x)$ DNE

$(x-2)^3 = 0$

$\boxed{x=2}$

(b)



$f'(-1) = \dfrac{-4(-1)}{(-3)^3} = \dfrac{+}{-} < 0$

$f'(1) = \dfrac{-4}{(-1)^3} = \dfrac{-}{-} > 0$

$f'(3) = \dfrac{-12}{1^3} = \dfrac{-}{+} < 0$

(c) By FDT,

$\boxed{x=0 \text{ is a local min}}$

- $f(2)$ DNE ($x=2$ is a vertical asymptote),

So $x=2$ is neither.