Math 135 Written HW 11-13: Solutions

Problem 1: $y = f(x)\cdot g(x)$

$$\ln(y) = \ln\left(f(x)\cdot g(x)\right) = \ln\left(f(x)\right) + \ln\left(g(x)\right)$$

$\frac{d}{dx}$

$$\frac{1}{y}\cdot\frac{dy}{dx} = \frac{1}{f(x)}\cdot f'(x) + \frac{1}{g(x)}\cdot g'(x)$$

$$\frac{dy}{dx} = \left(\frac{f'(x)}{f(x)} + \frac{g'(x)}{g(x)}\right)\cdot y = \left(\frac{f'(x)}{f(x)} + \frac{g'(x)}{g(x)}\right)\cdot\left(f(x)\cdot g(x)\right)$$

$$= \frac{f'(x)\cdot f(x)\cdot g(x)}{f(x)} + \frac{g'(x)\cdot f(x)\cdot g(x)}{g(x)}$$

$$\boxed{\frac{dy}{dx} = f'(x)\cdot g(x) + g'(x)\cdot f(x)}$$

This is the product rule!

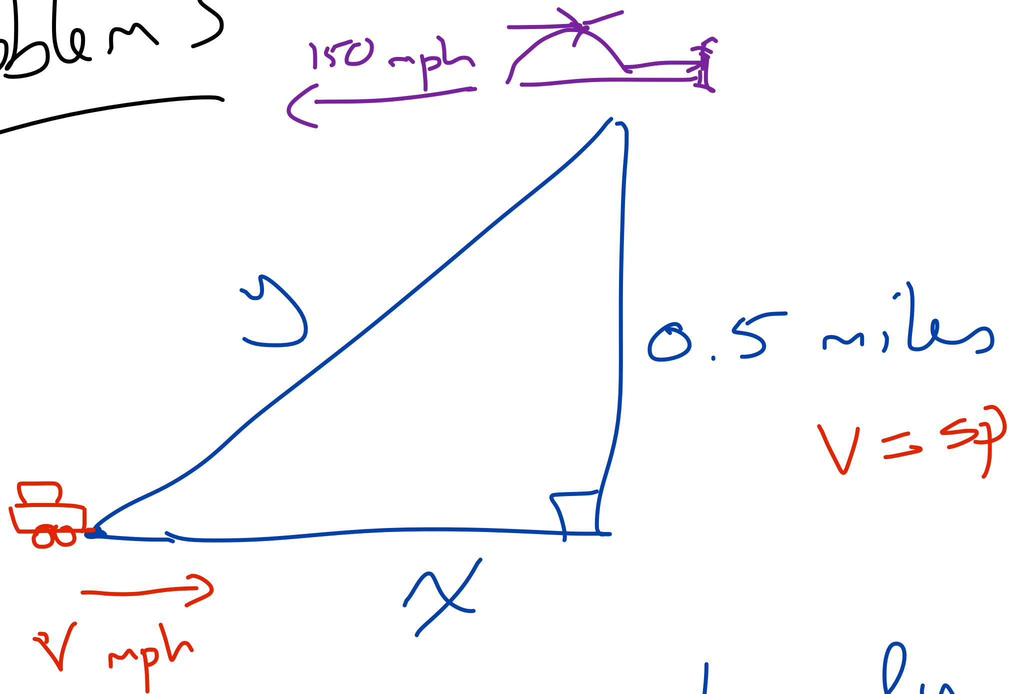
Problem 2: $h(t)$ = change in height from height at midnight

(a) $h(7) = 2.75$ means that by 7am, the tide has risen 2.75 meters.

(b) $h'(7) = 0.21$ means that at 7am, the tide is rising by 0.21 meters/hour.

(c) $h^{-1}(-1.5) = 13.2$ means that $-1.5 = h(13.2)$, so that the tide will have decreased by 1.5 meters by 1:12pm.

Problem 3



v = speed of car

<u>Known</u>: • When $y = 1$ mile, $\frac{dy}{dt} = -190$ mph:

$$\left.\frac{dy}{dt}\right|_{y=1} = -190$$

• The horizontal distance is decreasing by the speed of the helicopter plus the speed of the car:

$$\frac{dx}{dt} = -(150 + v) \text{ mph}$$

<u>Find</u>: v.

Equation:

$$x^2 + (0.5)^2 = y^2$$

$\frac{d}{dt}$

$$2x\frac{dx}{dt} = 2y\frac{dy}{dt}$$

• When $y=1$,

$$x^2 + 0.25 = 1$$

$$x^2 = 3/4$$

$$x = \sqrt{3}/2$$

• When $y=1$,

$$2\left(\frac{\sqrt{3}}{2}\right)\cdot\left(-(150+v)\right) = 2(1)(-190)$$

$$-150\sqrt{3} - \sqrt{3}\cdot v = -380$$

$$\boxed{v = \frac{150\sqrt{3}-380}{-\sqrt{3}} \text{ mph} \qquad v \approx 69.4 \text{ mph}}$$