# Math 135 Written HW 10-16: Solutions

Exercise 1:

$$\lim_{x\to 0} \frac{\sin(8x)}{\sin(9x)} = \lim_{x\to 0} \frac{\sin(8x)}{x} \cdot \frac{x}{\sin(9x)}$$

$$= \lim_{x\to 0} \frac{\sin(8x)}{x} \cdot \lim_{x\to 0} \frac{1}{\sin(9x)/x}$$

$t = 8x$ $\qquad$ $\Delta = 9x$

$$= \lim_{t\to 0} \frac{\sin(t)}{t/8} \cdot \frac{1}{\lim_{\Delta\to 0} \sin(\Delta)/\Delta/9}$$

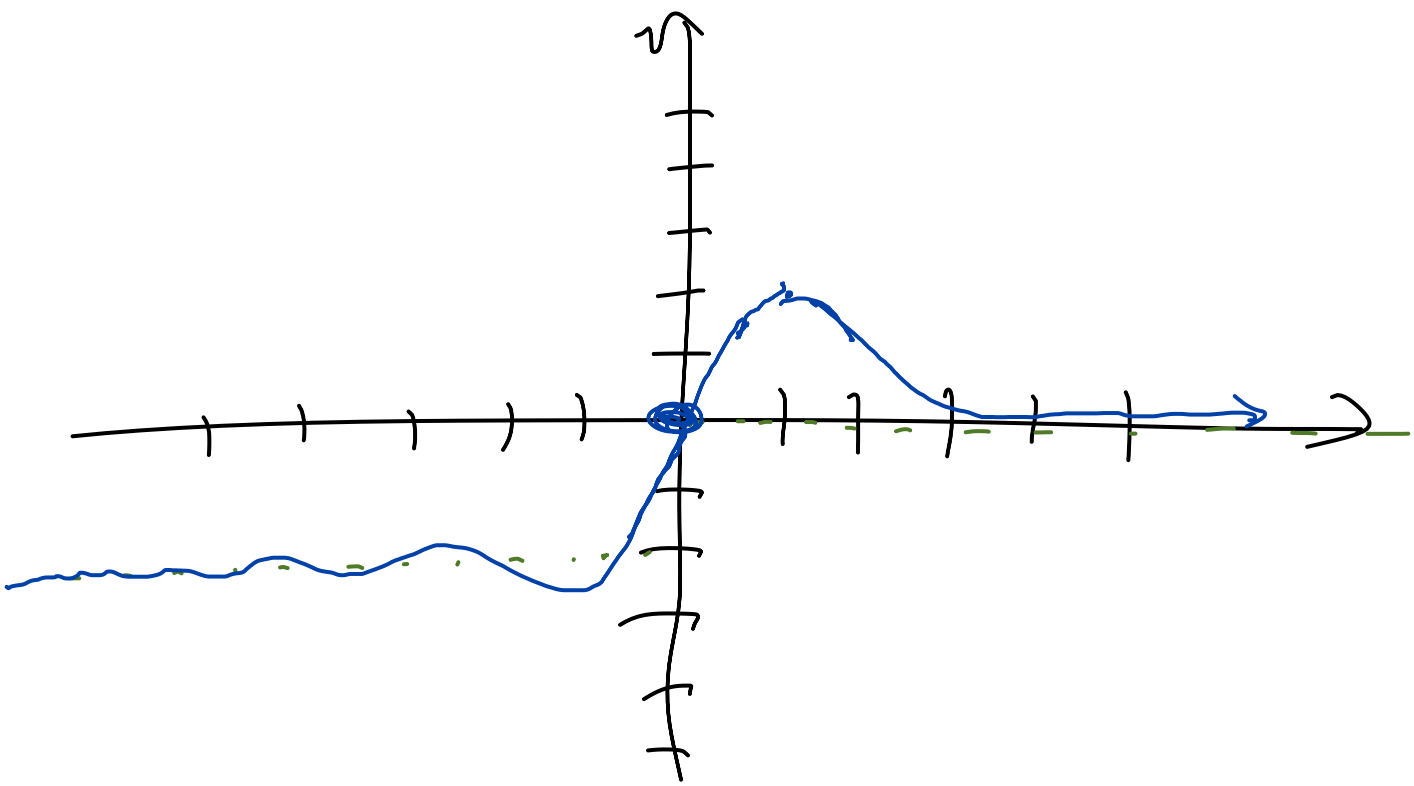
$$= 8 \cdot \lim_{t\to 0} \frac{\sin(t)}{t} \cdot \frac{1}{9 \cdot \lim_{\Delta\to 0} \frac{\sin(\Delta)}{\Delta}}$$

$$= (8 \cdot 1) \cdot \frac{1}{9 \cdot 1} = \boxed{8/9}$$

Exercise 2: Sketch differentiable function $f$

s.t.

- $f(0)=0$
- $f'(0)=3$
- $f'(1)=0$
- $f'(2)=-1$
- $\lim_{x\to\infty} = 0$
- $\lim_{x\to-\infty} = -2$



Exercise 3: $y=x^2$, tangent lines passing through $(-1,-3)$

- Tangent Line: $y = f'(a)(x-a) + f(a)$

- $f'(x) = 2x$

$f'(a) = 2a$, $f(a) = a^2$

$\Rightarrow$ Tangent Line: $y = 2a(x-a) + a^2$

$$\boxed{y = 2ax - a^2}$$

Find values for $a$ s.t. $(-1,-3)$ on this line

$-3 = 2a(-1) - a^2$

$a^2 + 2a - 3 = 0$

$(a+3)(a-1) = 0$

$\boxed{a = -3}$ $\boxed{a = 1}$

Lines: $\boxed{\begin{aligned} y &= -6x - 9 \\ y &= 2x - 1 \end{aligned}}$