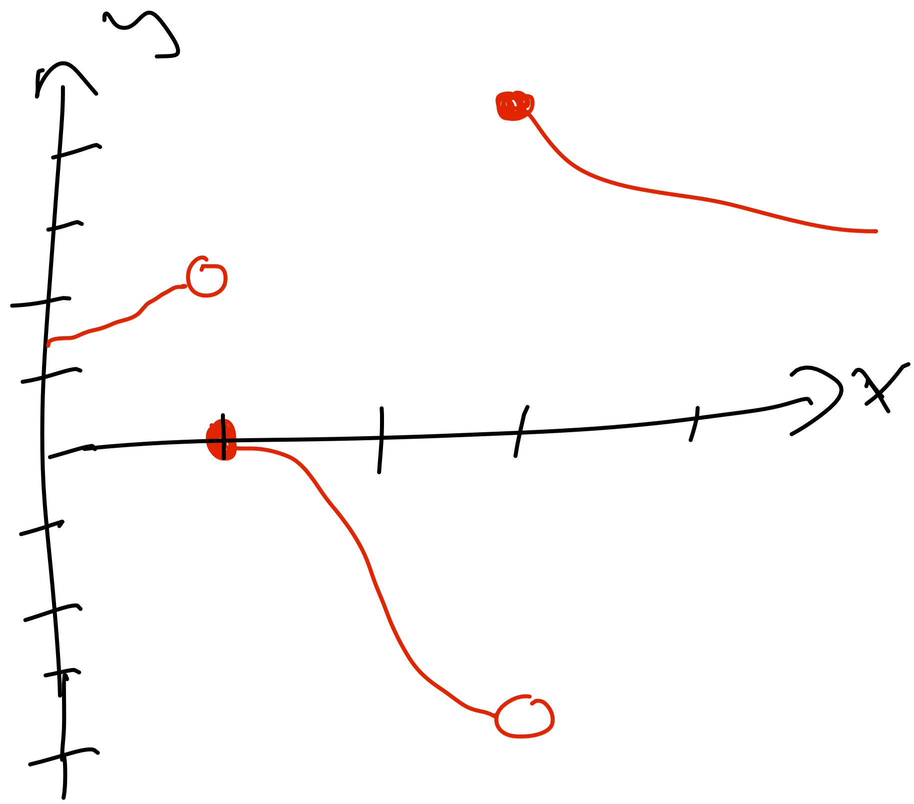
Math 135 Written HW 09-25 — Solutions!

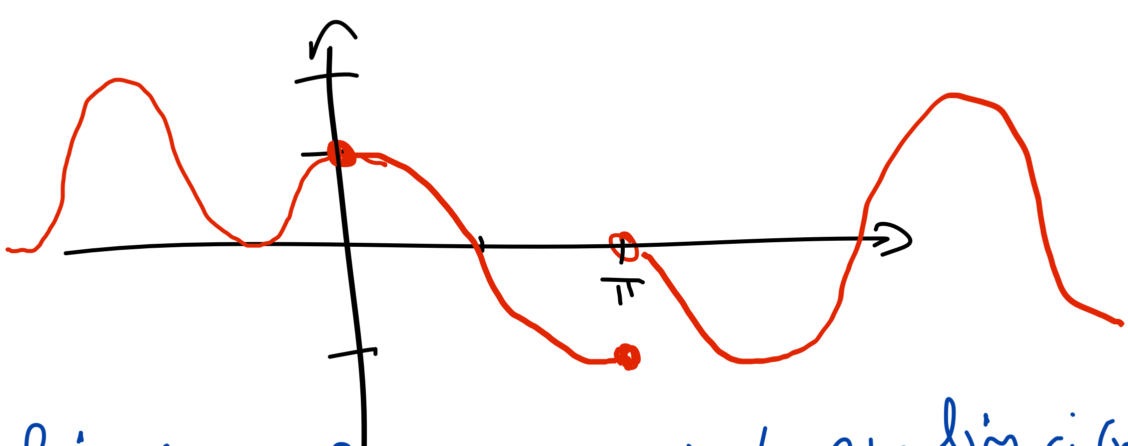
Exercise 1: Need $f(x)$ so that:

- $\lim_{x \to 1} f(x) = 2$
- $\lim_{x \to 3^-} f(x) = -4$
- $\lim_{x \to 3^+} f(x) = 4$
- $f(1) = 0$
- $f(3) = 4$



Exercise 2

$$f(x) = \begin{cases} 1+\sin(x) & x<0 \\ \cos(x) & 0 \le x \le \pi \\ \sin(x) & x>\pi \end{cases}$$



$$\lim_{x \to \pi^-} f(x) = \lim_{x \to \pi^-} \cos(x) = -1 \neq 0 = \lim_{x \to \pi^+} \sin(x) = \lim_{x \to \pi^+} f(x)$$

Exercise 3

(a) $\dfrac{x^4-8x^2+16}{x^2-4}$ vs x^2-4

LHS: DNE at $x=2$ & $x=-2$

Otherwise, these two functions are the same.

(b) Since the actual function value at $x=2$ does not influence the limit as $x \to 2$, we can conclude

$$\lim_{x\to 2} \frac{x^4-8x^2+16}{x^2-4} = \lim_{x\to 2} x^2-4.$$

$$x^4-8x^2+16=(x^2-4)(x^2-4)$$