Math 135 Written HW 10-23: Solutions

Problem 1: Show $y = \sin(Ax)$ is a solution to

$$y'' = -A^2 y$$

If $y = \sin(Ax)$, then
$$y' = \frac{d}{dx}(\sin(Ax))$$
$$= \cos(Ax) \cdot \frac{d}{dx}(Ax)$$
$$= A \cdot \cos(Ax)$$

and
$$y'' = \frac{d}{dx}(A\cos(Ax))$$
$$= A\frac{d}{dx}(\cos(Ax))$$
$$= -A\sin(Ax)\frac{d}{dx}(Ax)$$
$$= -A^2 \sin(Ax)$$
$$= -A^2 \cdot y$$

So the equation

$$y'' = -A^2 y$$

holds.

- Another solution is: $y = \cos(Ax)$:
  $y' = -A\sin(Ax)$, $y'' = -A^2\cos(Ax) = -A^2 y$.
- Another solution is:
  $$y = B\sin(Ax) + C\cos(Ax).$$

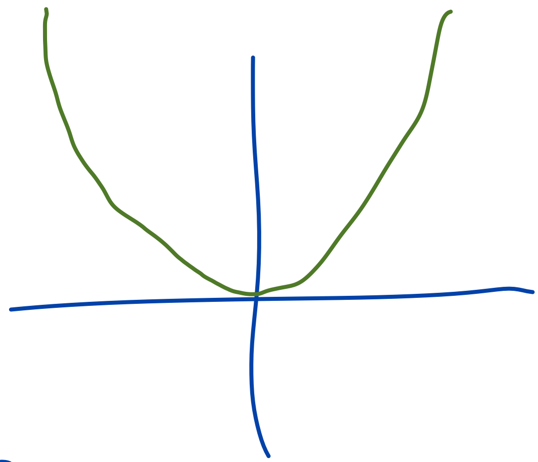
Problem 2

(a) For <u>some</u> functions f, if $f''(x) > 0$ then $f'(x) < 0$.

Consider $f(x) = x^2$

$f'(x) = 2x$

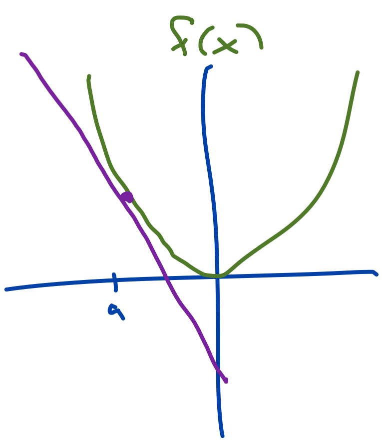
$f''(x) = 2$

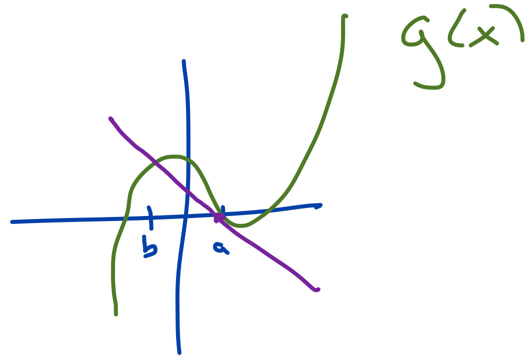


- For $(a,b) = (-\infty, 0)$,
  we have $f''(x) > 0$ & $f'(x) < 0$
- <u>But</u> for $(a,b) = (0, \infty)$, we have $f''(x) > 0$ & $f'(x) > 0$.

(b) For <u>some</u> functions f, the tangent line to $f(x)$ @ $x = a$ will intersect the graph of $f(x)$ at exactly one point.

Consider the two functions $f(x)$ & $g(x)$ below:





Each has a tangent line drawn in purple. On $f(x)$, this line only intersects the graph of $f(x)$ at $(a, f(a))$. However, on $g(x)$, this line intersects the graph of $g(x)$ at $(a, g(a))$ <u>and</u> at some point $(b, g(b))$.

Problem 3

"The town's expenses are increasing, but at a decreasing rate."

Let $f(t)$ denote the town's expenses at time t.

Then $f'(t)$ denotes the change in the town's expenses.

- "The town's expenses are increasing" means that $f'(t) > 0$.
- "but at a decreasing rate" means that $f'(t)$ is decreasing, so the derivative of $f'(t)$ is negative.

That is, $\boxed{f''(t) < 0}$