Math 135 : Project 3 Solutions

§A: $f(x) = e^x \cdot \sin(x)$ [40 pts]

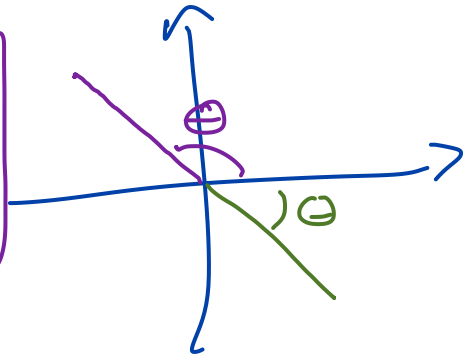
① CPs on $[-2\pi, 2\pi]$ (5 pts)

$$f'(x) = e^x \cdot \sin(x) + e^x \cdot \cos x$$
$$= e^x(\sin x + \cos x) \overset{!}{=} 0$$

$e^x \neq 0$ never

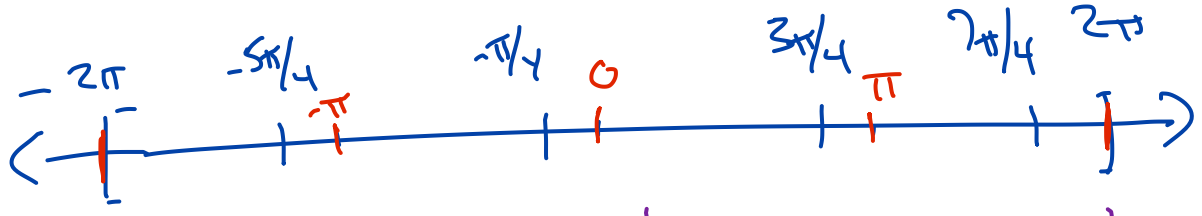
$$\sin x + \cos x = 0$$
$$\sin x = -\cos x$$

$$\theta = \frac{-5\pi}{4}, \frac{-\pi}{4}, \frac{3\pi}{4}, \frac{7\pi}{4}$$



4 critical points

② Sign chart for $f'(x)$ (5 pts)



$f'(x)$: + 0 − 0 + 0 − 0 +

$f(-2\pi) \sim f(0) \sim f(2\pi) = (+)\cdot(\sin(0) + \cos(0)) = (+)(1) > 0$

$f(-\pi) \sim f(\pi) = (+)\cdot(\sin(\pi) + \cos(\pi)) = (+)(0-1) < 0$

(3) Sign chart for $f''(x)$ (10 pts)

$$f''(x) = e^x(\sin x + \cos x) + e^x \cdot (\cos x - \sin x)$$

$$= e^x(\sin x + \cos x + \cos x - \sin x)$$

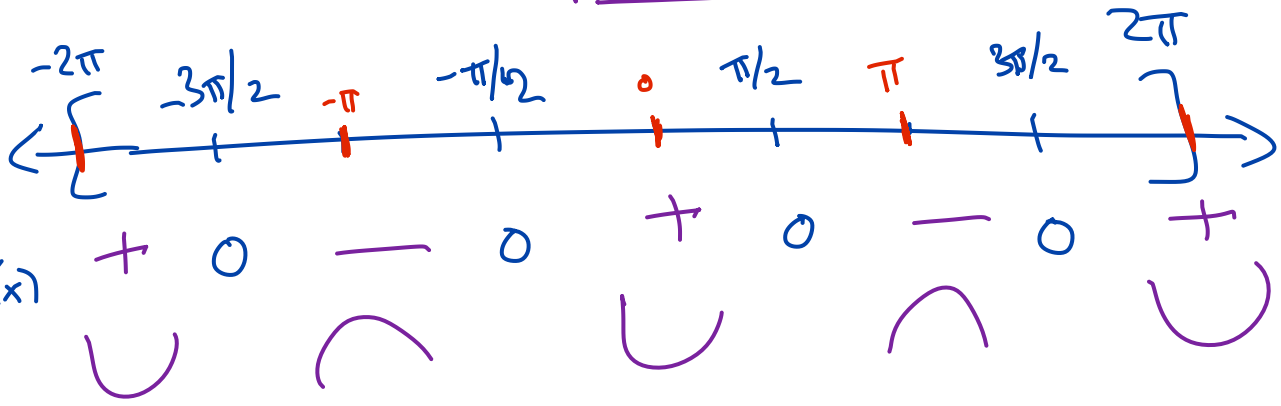
$$= e^x \cdot 2\cos x \overset{!}{=} 0$$

$e^x \neq 0$ never

$2\cos x = 0$

$\cos x = 0$

$$x = -3\pi/2, -\pi/2, \pi/2, 3\pi/2$$



$f''(-2\pi) \sim f''(0) \sim f''(2\pi) = (+)\cdot 2\cos(0) = (+)\cdot 2 > 0$

$f''(-\pi) \sim f''(\pi) = (+)\cdot 2\cos(\pi) = (+)\cdot -2 < 0$

$x = -3\pi/2, -\pi/2, \pi/2, 3\pi/2$ all inflection points!

4 inflection points

④ Classify CPs (10 pts)

CPs: $-5\pi/4,\ -\pi/4,\ 3\pi/4,\ 7\pi/4$

First Derivative Test:
- $x = -5\pi/4,\ 3\pi/4$ local maxes
- $x = -\pi/4,\ 7\pi/4$ local mins

Second Derivative Test:
- $f''(-5\pi/4),\ f''(3\pi/4) < 0 \Rightarrow$ local maxes
- $f''(-\pi/4,\ 7\pi/4) > 0 \Rightarrow$ local mins

⑤ EVT on $[-2\pi, 2\pi]$ (10 pts)

$f(-2\pi) = e^{-2\pi}\sin(-2\pi) = e^{-2\pi}\cdot 0 = 0$

$f(-5\pi/4) = e^{-5\pi/4}\sin(-5\pi/4) = e^{-5\pi/4}\cdot\frac{\sqrt{2}}{2} \approx 0.0139$

$f(-\pi/4) = e^{-\pi/4}\sin(-\pi/4) = e^{-\pi/4}\cdot\frac{-\sqrt{2}}{2} \approx -0.3224$

$f(3\pi/4) = e^{3\pi/4}\sin(3\pi/4) = e^{3\pi/4}\cdot\frac{\sqrt{2}}{2} \approx$ 7.4605 Abs max value

$f(7\pi/4) = e^{7\pi/4}\sin(7\pi/4) = e^{7\pi/4}\cdot\frac{-\sqrt{2}}{2} \approx$ -172.6409 Abs min value

$f(2\pi) = e^{2\pi}\sin(2\pi) = e^{2\pi}\cdot 0 = 0$

§B: Approximations {45 pts}

⑥ $L(x) = A + Bx$, $L(a) = f(a)$, $L'(a) = f'(a)$

- $L'(x) = B$

  $L'(0) = B \doteq f'(0) = e^0(\sin(0) + \cos(0)) = 1$

  $\boxed{B = 1}$

- $L(0) = A \doteq f(0) = e^0 \cdot \sin 0 = 0$

  $\boxed{A = 0}$

$\boxed{L(x) = x}$

⑦ CPs

$L'(x) = 1 \neq 0$ $\boxed{\text{None}}$

⑧ EVT

$L(-2\pi) = \boxed{-2\pi}$ Abs min value

$L(2\pi) = \boxed{2\pi}$ Abs max value

(9) $Q(x) = A + Bx + Cx^2$, $Q(a)=f(a)$, $Q'(a)=f'(a)$, $Q''(a)=f''(a)$

$Q'(x) = B + 2Cx$

$Q''(x) = 2C$

- $Q''(0) = 2C \overset{!}{=} f''(0) = e^0 \cdot 2\cos(0) = 2$

$\boxed{C = 1}$

- $Q'(0) = B \overset{!}{=} f'(0) = 1$

$\boxed{B = 1}$

- $Q(0) = A \overset{!}{=} f(0) = 0$

$\boxed{A = 0}$

$\boxed{Q(x) = x + x^2}$

(10) CPs

$Q'(x) = 1 + 2x \overset{!}{=} 0$

$\boxed{x = -1/2}$ One CP

(11) EVT

$Q(-2\pi) = -2\pi + 4\pi^2 \approx 33.1952$

$Q(-1/2) = -1/2 + 1/4 = -1/4 = \boxed{-0.25}$ Abs min value

$Q(2\pi) = 2\pi + 4\pi^2 \approx \boxed{45.7616}$ Abs max value

(12) $P(x) = A + Bx + Cx^2 + Dx^3$

$P'(x) = B + 2Cx + 3Dx^2$

$P''(x) = 2C + 3 \cdot 2Dx$

$P'''(x) = 6D$

$f''(x) = e^x \cdot 2\cos x$

$f'''(x) = e^x \cdot 2\cos x - e^x 2\sin x$

$= e^x \cdot 2(\cos x - \sin x)$

- $P'''(0) = 6D \overset{!}{=} f'''(0) = e^0 \cdot 2(\cos(0) - \sin(0)) = 1 \cdot 2 \cdot 1 = 2$

$\boxed{D = 1/3}$

- $P''(0) = 2C + 0 \overset{!}{=} f''(0) = 2$

$\boxed{C = 1}$

- $P'(0) = B + 0 + 0 \overset{!}{=} f'(0) = 1$

$\boxed{B = 1}$

- $P(0) = A + 0 + 0 + 0 \overset{!}{=} f(0) = 0$

$\boxed{A = 0}$

$\boxed{P(x) = x + x^2 + \frac{1}{3}x^3}$

(13) CPs

$P'(x) = 1 + 2x + x^2 \overset{!}{=} 0$

$(x+1)(x+1) = 0$

$\boxed{x = -1}$ one CP

(14) EUT

$P(-2\pi) = -2\pi + 4\pi^2 - \frac{8\pi^3}{3} \approx \boxed{-49.4882}$ Abs min valu

$P(-1) = -1 + 1 - \frac{1}{3} = -1/3$

$P(2\pi) = 2\pi + 4\pi^2 + \frac{8\pi^3}{3} \approx \boxed{128.4450}$ Abs max valu

(15) See other fib

§C: General Approximations [15 pts]

(16) $L(x) = A + B(x-a)$, $L(a) = f(a)$ & $L'(a) = f'(a)$

$L'(x) = B$

- $L'(a) = B \overset{!}{=} f'(a)$ $\boxed{B = f'(a)}$
- $L(a) = A + 0 \overset{!}{=} f(a)$ $\boxed{A = f(a)}$

$\boxed{L(x) = f(a) + f'(a)(x-a)}$

(17) $Q(x) = A + B(x-a) + C(x-a)^2$,

$Q(a) = f(a)$, $Q'(a) = f'(a)$, $Q''(a) = f''(a)$

$Q'(x) = B + 2C(x-a)$

$Q''(x) = 2C$

- $Q''(a) = 2c \overset{!}{=} f''(a)$ $\quad \boxed{C = f''(a)/2}$

- $Q'(a) = B + 0 \overset{!}{=} f'(a)$ $\quad \boxed{B = f'(a)}$

- $Q(a) = A + 0 + 0 \overset{!}{=} f(a)$ $\quad \boxed{A = f(a)}$

$$\boxed{Q(x) = f(a) + f'(a)(x-a) + f''(a)(x-a)^2}$$

(18) $P(x) = A + B(x-a) + C(x-a)^2 + D(x-a)^3$

$P(a) = f(a)$, $P'(a) = f'(a)$, $P''(a) = f''(a)$, $P'''(a) = f'''(a)$.

$P'(x) = B + 2C(x-a) + 3D(x-a)^2$

$P''(x) = 2C + 6D(x-a)$

$P'''(x) = 6D$

- $P'''(a) = 6D \overset{!}{=} f'''(a)$ $\quad D = f'''(a)/6$

- $P''(a) = 2C + 0 \overset{!}{=} f''(a)$ $\quad C = f''(a)/2$

- $P'(a) = B + 0 + 0 \overset{!}{=} f'(a)$ $\quad B = f'(a)$

- $P(a) = A + 0 + 0 + 0 \overset{!}{=} f(a)$ $\quad A = f(a)$

$$\boxed{P(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2}(x-a)^2 + \frac{f'''(a)}{6}(x-a)^3}$$