Math 135, Written HW 09-11: Solutions

Exercise 1:

(a) For <u>some</u> real numbers x, $(x+2)^4 = x^4 + 16$:

- When $x=0$, $(0+2)^4 = 16 = 0^4 + 16$ ✓
- When $x \neq 0$, say $x=1$, $(1+2)^4 = 81$

  $1^4 + 16 = 17$

  but $17 \neq 81$ ✗

(b) For <u>all</u> real numbers x, $\sqrt{x^4 + 8x^2 + 16} = x^2 + 4$

- $x^4 + 8x^2 + 16 = (x^2+4)^2$, so

  $\sqrt{x^4+8x^2+16} = \sqrt{(x^2+4)^2} = |x^2+4| = x^2+4$ ✓

(c) For <u>no</u> real numbers x, if

$(x+2)(x+3) = 2$ then $x+2=2$ & $x-3=2$.

- If $x+2=2$ & $x-3=2$, then

  $(x+2)(x+3) = (2)(2) = 4 \neq 2$

- Also, if $x+2=2$ & $x-3=2$ then

  $x=0$ & $x=5$,

  which can't happen.

(d) For all functions f & g,
if both f & g are even, then so is $f+g$

- f & g are even $\Rightarrow f(-x) = f(x)$ &
$g(-x) = g(x)$,

So $(f+g)(-x) = f(-x) + g(-x)$
$= f(x) + g(x)$
$= (f+g)(x)$

So $f+g$ is even.

(e) For some real numbers k, x, y,
if $x<y$ then $kx<ky$.

- If $k>0$, $x<y$ implies $kx<ky$.
However, if $k<0$, $x<y$ implies $kx>ky$
& if $k=0$, $kx=ky$.

Exercise 2:

This will be a piecewise function, with different rules for:

$0 \leq x \leq 400$,

$400 \leq x \leq 900$, and

$x \geq 900$.

- When $0 \leq x \leq 400$, we just pay

$$0.10x + 6$$

- When $400 \leq x \leq 900$, we pay:
  - \$6
  - 0.10 for each of the first 400 kWh
  - 0.11 for each remaining kWh

so we pay

$$6 + 0.10(400) + 0.11(x - 400)$$

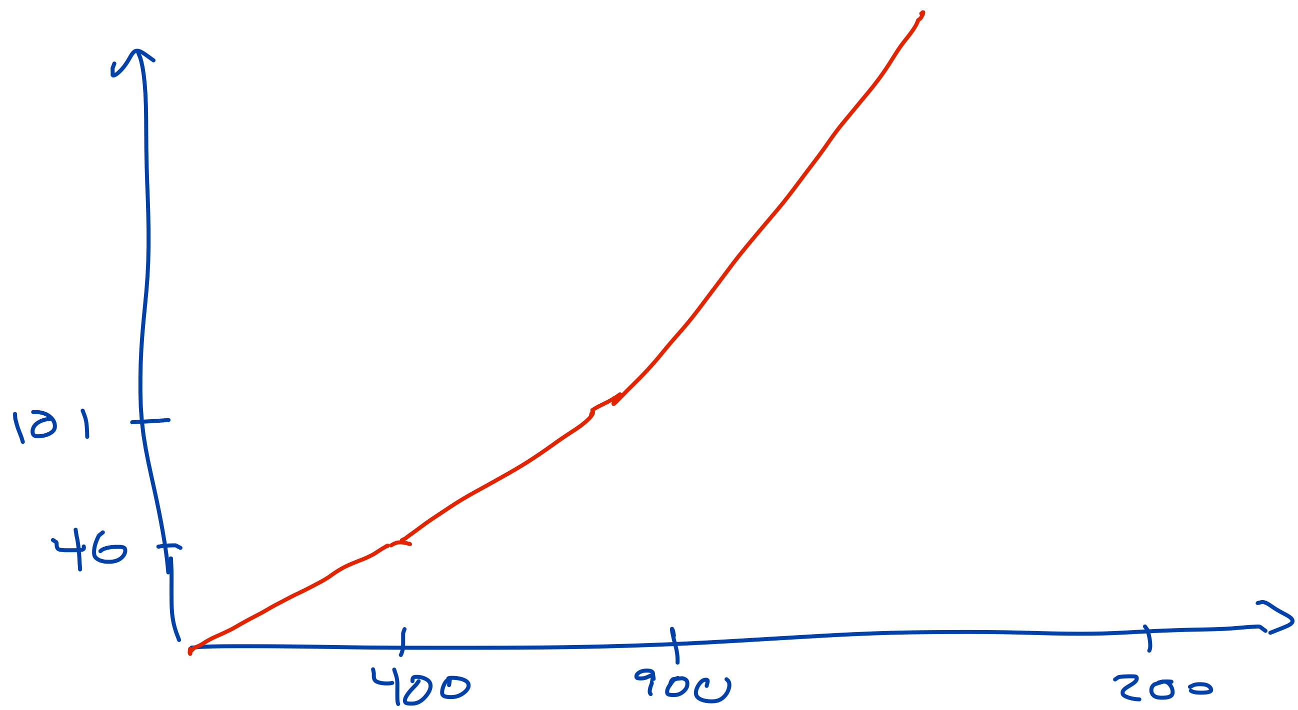
- When $x \geq 900$, we pay:
  - \$6
  - 0.10 for each of the first 400 kWh
  - 0.11 for each of the next 500 kWh
  - 0.15 for each remaining kWh

$$6 + 0.10(400) + 0.11(500) + 0.15(x - 900)$$

So, in total, we have

$$E(x) = \begin{cases} 6 + 0.10x & 0 \le x \le 400 \\ 46 + 0.11(x-400) & 400 \le x \le 900 \\ 101 + 0.15(x-900) & x \ge 900 \end{cases}$$

(b)



(c) We have 3 different slopes (increasing each time) corresponding to the 3 different rates. However, the line segments all line up, as we only pay the new rate on new kWh.