Math 135, Written HW 10-10: Solutions

Exercise 1:

$$\lim_{t\to 0}\left(\frac{1}{t}-\frac{1}{t^2-1}\right) = \lim_{t\to 0}\left(\frac{1}{t}\cdot\frac{(t-1)}{(t-1)} - \frac{1}{t(t-1)}\right)$$

$$= \lim_{h\to 0}\left(\frac{t-1-1}{t(t-1)}\right) = \lim_{h\to 0}\left(\frac{t-2}{t(t-1)}\right)$$

Evaluating we get $\frac{-2}{0\cdot -1} = \frac{-2}{0}$, so the limit is either $+\infty$, $-\infty$, or DNE.

Left hand limit: $\frac{-2}{(0^-)(-1)} = -\infty$

Right hand limit: $\frac{-2}{(0^+)(-1)} = +\infty$

So limit $\boxed{\text{DNE}}$.

Exercise 2: let $h(x) = \frac{|x-2|}{x}$.

Since $|x-2| = \begin{cases} x-2 & \text{if } x-2 \geq 0, \text{ or } x \geq 2 \\ -(x-2) = -x+2 & \text{if } x-2 \leq 0, \text{ or } x \leq 2 \end{cases}$

we have

- $\lim_{x\to\infty} \frac{|x-2|}{x} = \lim_{x\to\infty} \frac{(x-2)}{x} = \lim_{x\to\infty} \frac{x/x - 2/x}{x/x}$

$= \lim_{x\to\infty} \frac{1-2/x}{1} = \boxed{1}$ horizontal asymptote at $y=1$

- $\lim_{x\to-\infty} \frac{|x-2|}{x} = \lim_{x\to-\infty} \frac{-x+2}{x} = \lim_{x\to-\infty} \frac{-1+2/x}{1}$

$= \boxed{-1}$ horizontal asymptote at $y=-1$

- Vertical asymptote @ $x=0$:

$\lim_{x\to0} \frac{|x-2|}{x} = \lim_{x\to0} \frac{-x+2}{x} = \frac{2}{0}$ is $\pm\infty$

$\lim_{x\to0^-} \frac{|x-2|}{x} = \frac{2}{0^-} = -\infty$,

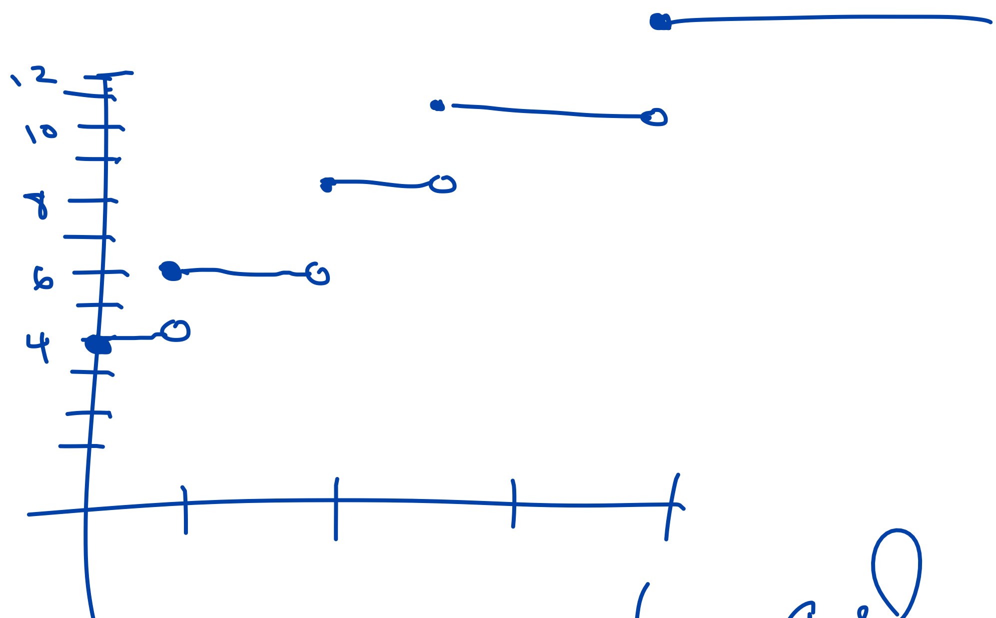
$\lim_{x\to0^+} \frac{|x-2|}{x} = \frac{2}{0^+} = +\infty$

Exercise 3

- \$4 for 1st hour (or part thereof)
  \$2 for each additional hour (or part)
- max: \$12
- let's make a table:

time parked	0.5	1	1.1	2	2.9	3	4+
cost	4	4+2=6	6	6+2=8	6+2=8	8+2=10	12

(a)



(b) left-continuous everywhere, and right-continuous everywhere except $x = 1, 2, 3, \& 4$.

Significance: the difference btwn parking for 59 minutes and 1 hour & 2 seconds is a big jump.