# Mass Properties Tool Manual

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## I. Nomenclature

| | | |
|---|---|---|
| $A$ | = | Aspect Ratio |
| $b_L$ | = | Distance Between Point Fuel Loads |
| $B_w$ | = | Wingspan [feet] |
| CG | = | Center of Gravity |
| EWF | = | Empty Weight Fraction |
| GA | = | General Aviation |
| $K_b$ | = | Point Load Span Factor |
| $K_d$ | = | Distributed Load Weight Factor |
| $K_p$ | = | Point Load Weight Factor |
| L | = | Fuselage Length [feet] |
| $L/D$ | = | Fuselage Length to Structural Depth Ratio |
| $L/D_b$ | = | Boom Length to Structural Depth Ratio |
| $L_m$ | = | Main Landing Gear Length [feet] |
| $L_n$ | = | Nose Landing Gear Length [feet] |
| $L_t$ | = | Wing Quarter Chord to Tail Quarter Chord Distance [feet] |
| $N_{en}$ | = | Number of Engines |
| $N_L$ | = | Landing Load Factor |
| $N_t$ | = | Number of Tanks |
| $N_z$ | = | Ultimate Load Factor |
| $S_b$ | = | Boom Wetted Area |
| $S_f$ | = | Fuselage Wetted Area [square feet] |
| $S_{HT}$ | = | Horizontal Tail Area [square feet] |
| $S_{VT}$ | = | Vertical Tail Area [square feet] |
| $S_w$ | = | Wing Planform Area [square feet] |
| $t_c$ | = | Wing Root thickness/chord |
| $V_H$ | = | Design Cruise Speed [knots] |
| $V_i/V_t$ | = | Fraction of Fuel Tanks that are Integral |
| $V_t$ | = | Fuel Tank Volume [gallons] |
| $W_d$ | = | Weight of Distributed Fuel |
| $W_{dg}$ | = | Design Gross Weight [pounds] |
| $W_E$ | = | Empty Weight [pounds] |
| $W_{en}$ | = | Uninstalled Weight of one Engine [pounds] |
| $W_{fw}$ | = | Fuel Weight in Wings [pounds] |
| $W_L$ | = | Design Max Landing Weight [pounds] |
| $W_p$ | = | Total Weight of Point Fuel Loads |
| $W_{press}$ | = | Added Weight due to Pressurization [pounds] |
| $W_{uav}$ | = | Uninstalled Avionics Weight [pounds] |
| $\Delta P$ | = | Max Cabin Pressure Differential (0 for unpressurized) [psi] |
| $\lambda$ | = | Taper Ratio |
| $\Lambda_{1/4}$ | = | Quarter Chord Wing Sweep Angle [degrees] |
| $\Lambda_{1/4HT}$ | = | Horizontal Tail Quarter Chord Sweep Angle [degrees] |
| $\Lambda_{1/4VT}$ | = | Vertical Tail Quarter Chord Sweep Angle [degrees] |
| $\lambda_{HT}$ | = | Horizontal Tail Taper Ratio |
| $\lambda_{VT}$ | = | Vertical Tail Taper Ratio |

# II. Background

With the goal of designing an aircraft that will circumnavigate the world non-refueled and non-stop, all three design teams finished Fall quarter with three important design elements that we will highlight. These are an EWF estimate, Aircraft Gross Weight ($W_{dg}$), and three-view drawing. As it stands, all these design elements come from selecting parameters such as EWF and $\frac{L}{D}$ and assuming that we will be able to meet them. The mass properties team is responsible to building a tool that will allow the design groups to improve these into choices backed by engineering analysis.

This tool allows users to employ a more sophisticated methodology to estimate aircraft group weights, $W_{dg}$, and aircraft CG x-location. Knowing group and total weights allows designers to assess their EWF estimates and adjust them according to results. Knowing the CG location allows designers to adjust aircraft elements such as wing, fuselage, stabilizer, or landing gear locations until CG is in a desirable location. This tool provides reasonable results for aircraft with low EWF and multiple fuselages which are poorly represented by the methods available in the literature.

# III. Methodology

This Mass Properties Tool employs a statistical weight estimation method developed by Daniel P. Raymer. An evaluation of this method on a variety of existing aircraft was conducted. Aircraft were selected to include a wide variety of characteristics, including single engine general aviation aircraft such as the Piper Archer III, Beechcraft J35 Bonanza, and Cirrus SR22, light twins such as the Cessna 310C and Diamond DA-42, and sailplanes such as the ASW 22BL as well as the two aircraft to have flown around the world without refueling, Voyager and GlobalFlyer. The results of this showed that the method was accurate to within about 10 percent on the general aviation aircraft and gliders. However, it showed large overestimates for the Voyager and GlobalFlyer. While Raymer's method provides good results for typical general aviation aircraft and still produces good results for higher aspect ratios than it was intended for, it does not account for aircraft with low EWF and the unique design features of Voyager and GlobalFlyer.

Comparing Raymer's method's results against real weight group data showed that the discrepancies were primarily in the wing and fuel system groups. The wing group errors were determined to be caused by a lack of accounting for fuel distribution over the span of high aspect ratio wings on aircraft with large fuel fractions. This was corrected by developing a first principles based multiplier to apply to wing weights that accounted for fuel weight located on booms or in-wing fuel tanks. This required defining three new inputs: $K_b$, $K_p$, and $K_d$, representing the position of a point load like a boom tank, the weight in the boom tanks, and the fuel weight distributed in the wing.

$$K_b = \frac{b_L}{B_w} \tag{1}$$

$$K_p = \frac{W_p}{W_{dg}} \tag{2}$$

$$K_d = \frac{W_d}{W_{dg}} \tag{3}$$

The correction factor was developed by starting with a first principles approximation and adjusting it to fit cases for which information was available as well as display the theoretically expected behavior in the limits of its parameters.

Analysis showed that there was a correlation between EWF and error in the fuel system weight estimate. Accordingly, a correction factor of the ratio of EWF to a reference average EWF of several single engine general aviation aircraft was applied. While the empty weight fraction based correction factor gave good results, a solution that did not require EWF as a variable would produce a more useful design tool. Fuel weight fraction also provided a good correction, again using an average from a selection of general aviation aircraft as a reference. This produced a corrected fuel system weight estimate without requiring an additional variable. Below are the resulting equations, correction factors included, that are used in this mass properties tool.

**Wing:**

$$W_w = corr_{fuel} * .036 * S_w^{.758} * W_{fw}^{.0035} * [\frac{A}{cos(\Lambda_{1/4})^2}]^{.6} * q^{.006} * \lambda^{.04} * [\frac{100 * t_c}{cos(\Lambda_{1/4})}]^{-.3} * [N_z * W_{dg}]^{.49} \tag{4}$$

where:

$$corr_{fuel} = (1 - 1.2 * (K_b^{.42} - .7 * K_p^{.3} * K_b) * K_p) * (1 - .08 * K_d) \tag{5}$$

**Vertical Tail:**

$$W_{VT} = N_{VT} * .073 * (1 + .2 * (\frac{H_T}{H_V})) * (N_z * W_{dg})^{.376} * q^{.122} * S_{VT}^{.873} * [\frac{100 * t_c}{cos(\Lambda_{1/4VT})}]^{-.49} * [\frac{A}{cos(\Lambda_{1/4VT})^2}]^{.357} * \lambda_{VT}^{.039} \quad (6)$$

**Horizontal Tail:**

$$W_{HT} = .016 * (N_z * W_{dg})^{.414} * q^{.168} * S_{ht}^{.896} * [\frac{100 * t_c}{cos(\Lambda)}]^{-.12} * [\frac{A}{cos(\Lambda_{HT})^2}]^{.043} * \lambda_h^{-.02} \quad (7)$$

**Landing Gear:**

$$W_{main} = 0.095 * (N_L * W_L)^{0.768} * L_M{}^{0.409} \quad (8)$$

$$W_{nose} = 0.125 * (N_L * W_L)^{0.566} * L_N{}^{0.845} \quad (9)$$

**Fuselage:**

$$W_{fuse} = 0.052 * S_f{}^{1.086} * (N_z * W_{dg})^{0.177} * L_t^{-0.051} * (L/D)^{-0.072} * q^{0.241} + W_{press} \quad (10)$$

where:
if $\Delta_P = 0$

$$W_{press} = 0 \quad (11)$$

if $\Delta_P > 0$

$$W_{press} = 11.9 + (V_{pr} * \Delta_P)^{0.271} \quad (12)$$

**Boom:**

$$W_{boom} = 0.052 * S_f{}^{1.086} * (N_z * W_{dg})^{0.177} * L_t^{-0.051} * (L/D_b)^{-0.072} * q^{0.241} + W_{press} \quad (13)$$

**Furnishings:**

$$W_{furnishing} = .0582 * W_{dg} - 65 \quad (14)$$

**Fuel System:**

$$W_{fuelsys} = K * 2.49 * V_t^{.726} * [\frac{1}{1 + V_i/V_t}]^{.363} * N_t^{.242} * N_{en}^{.157} \quad (15)$$

where:

$$K = \frac{1 - 0.9 * [\frac{W_F}{W_{DG}}]^{0.8}}{0.1992} \quad (16)$$

**Flight Control System:**

$$W_{flightcon} = .053 * L^{1.536} * B_w^{.371} * (N_z * W_{dg} * .0004)^{.8} \quad (17)$$

**Engine:**

$$W_{engine} = 2.575 * W_{en}^{.922} * N_{en} \quad (18)$$

**Avionics:**

$$W_{avionics} = 2.117 * W_{uav}^{.933} \tag{19}$$

**Electrical System:**

$$W_{elec} = 12.57 * (W_{fuelsys} + W_{avionics})^{.51} \tag{20}$$

## IV. Limitations and Assumptions

The method presented here is based on a method that was designed for typical general aviation aircraft, simple conventional layouts with relatively high EWF and low aspect ratios. Validation data indicates that it still provides good results for aircraft with the characteristics expected for around the world record setting designs. It is, however, limited to a few configurations. There can be between one and three fuselages, but its applicability to a flying wing design has not been assessed.

The wing weight correction factor is based on the assumption that bending load is the most significant contributor to wing structural weight. If the torsional rigidity necessary to prevent flutter is instead driving, the correction may not be valid. The distributed load correction assumes that fuel tankage extends the entire span. In the authors' judgment this was a reasonable assumption for this type of aircraft in which all available wing volume was likely to be used for fuel, but when this is not the case the wing weight prediction will be too low. Lift distribution and taper ratio are also expected to have small effects that are not accounted for.

For typically equipped aircraft, accuracy of about 10 percent is expected for structural weight and 15 percent for empty weight. CG is expected to be accurate to within about 3 percent of aircraft length. This provides sufficient accuracy for the conceptual design stage.

## V. Validation

To validate our method, we tested it on the same aircraft used for the evaluation of Raymer's method. The results are shown in table 1.

| Airplane | Actual $W_E$ [$pounds$] | Estimated $W_E$ [$pounds$] |
|---|---|---|
| Voyager | 2250 | 2950 |
| GlobalFlyer | 3699 | 3651 |
| ASW 22BL | 1025 | 936 |
| Cessna 310C | 3,032 | 2879 |
| DA-42 | 2646 | 3083 |
| Beech J-35 | 1821 | 1874 |
| SR 22 | 2269 | 1789 |
| Piper Archer 3 | 1,639 | 1331 |

**Table 1 Actual and Estimated Empty Weights**

In all of the general aviation cases the results were still acceptably accurate, within 10 percent. In the case of the Cessna 310, which has tip tanks, the wing weight correction brought the weight estimate noticeably closer to reality. For Voyager, the error in total weight was 31 percent, while the error in wing and fuselage weight was 6 percent. GlobalFlyer's weight was estimated to within one percent. It appears that most of the error in Voyager's weight estimate comes from fixed equipment. Center of gravity estimation was checked on the Beechcraft Bonanza, the Voyager, and the GlobalFlyer. For the Bonanza the estimated empty CG fell on the aft CG limit, while it fell slightly ahead of the mean wing quarter chord for the GlobalFlyer and about 6 inches ahead of the mean wing quarter chord for the Voyager. These CG results are accurate enough to validate a configuration at the conceptual stage as long as the CG positions of each weight group are correctly determined.

# VI. User Guide

To use this tool on a specific airplane configuration, first the configuration vector should be defined. The configuration vector takes the form **config = [a, b, c, d]** where each position should be filled as follows:

a - 0 for conventional tail or canard; 1 for t-tail
b - number of booms/fuselages, can be 1, 2, or 3
c - number of vertical tails
d - number of horizontal tails

Next the user should complete the 'inputs' vector. This vector should be 1x37 and shall include each of the following inputs in the following order and indicated units:

| | | |
|---|---|---|
| $S_w$ | = | Wing Planform Area [square feet] |
| $W_{dg}$ | = | Design Gross Weight [pounds] |
| $N_z$ | = | Ultimate Load Factor |
| $A$ | = | Aspect Ratio |
| $\Lambda_{1/4}$ | = | Quarter Chord Wing Sweep Angle [degrees] |
| $V_H$ | = | Design Cruise Speed [knots] |
| $\lambda$ | = | Taper Ratio |
| $t_c$ | = | Wing Root thickness/chord |
| $W_{fw}$ | = | Fuel Weight in Wings [pounds] |
| $K_b$ | = | Point Load Span Factor |
| $K_p$ | = | Point Load Weight Factor |
| $K_d$ | = | Distributed Load Weight Factor |
| $S_{VT}$ | = | Vertical Tail Area [square feet] |
| $\Lambda_{1/4VT}$ | = | Vertical Tail Quarter Chord Sweep Angle [degrees] |
| $\lambda_{VT}$ | = | Vertical Tail Taper Ratio |
| $S_{HT}$ | = | Horizontal Tail Area [square feet] |
| $\Lambda_{1/4HT}$ | = | Horizontal Tail Quarter Chord Sweep Angle [degrees] |
| $\lambda_{HT}$ | = | Horizontal Tail Taper Ratio |
| $N_L$ | = | Landing Load Factor |
| $W_L$ | = | Design Max Landing Weight [pounds] |
| $L_m$ | = | Main Landing Gear Length [feet] |
| $L_n$ | = | Nose Landing Gear Length [feet] |
| $S_f$ | = | Fuselage Wetted Area [square feet] |
| $L_t$ | = | Wing Quarter Chord to Tail Quarter Chord Distance [feet] |
| $L/D$ | = | Fuselage Length to Structural Depth Ratio |
| $V_t$ | = | Fuel Tank Volume [gallons] |
| $V_i/V_t$ | = | Fraction of Fuel Tanks that are Integral |
| $N_t$ | = | Number of Tanks |
| $N_{en}$ | = | Number of Engines |
| $B_w$ | = | Wingspan [feet] |
| L | = | Fuselage Length [feet] |
| $W_{en}$ | = | Uninstalled Weight of one Engine [pounds] |
| $W_{uav}$ | = | Uninstalled Avionics Weight [pounds] |
| $W_{fl}$ | = | Total Fuel Weight [pounds] |

Finally, the user should build the 1x13 coordinates vector. This vector takes the form of

**coords = [Wing, Vertical Tail, Horizontal Tail, Main Landing Gear, Nose Landing Gear, Fuselage, Boom, Furnishings, Fuel System, Flight Control, Engine, Avionics, Electrical System]**

where each of the entries are the x-coordinate of the centroid of the listed component group.

With all three of these vectors complete, the user should call the mass properties function with the line

**[weights, cg] = mass_props(inputs, config, coords)**

The output vector 'weights' will have weights in pounds of each of the component groups in the same order as the 'coords' vector, with the 14th entry being the total weight of the aircraft. The CG output is the x-location in feet of the center of gravity, measured from the same datum as the coordinates given to the 'coords' vector.

As a result of the large number of inputs for this function, group b believes the most efficient way to use it is by organizing the inputs in an excel spreadsheet and then importing the columns into Matlab. This allows for quickly copying and pasting columns and just changing a few inputs in order to compare configurations.

# VII. Appendix

## A. Source Code

```
% MASS PROPERTIES
function [weights, CG] = mass_props(inputs, config, coords)
% INPUTS:
%   inputs - vector of all necessary inputs (in order):
%   1   Sw - wing area                                                  [ft^2]
%   2   Wdg - design gross weight                                       [lbs]
%   3   Nz - ultimate load factor                                       [g]
%   4   A - Aspect ratio
%   5   lambda14 - 1/4 chord sweep                                      [deg]
%   6   Vh - design cruise EAS                                          [kts]
%   7   lambda - taper ratio
%   8   tc - root t/c
%   9   Wfw - fuel weight in wings                                      [lb]
%   10  Kb - point load position/half span
%   11  Kp - point load fraction of Wb/Wdg
%   12  Kd - distributed fuel weight as fraction of Wdg
%   13  Svt - V Tail area                                               [ft^2]
%   14  lambda14_vt - Vstab 1/4 chord sweep                             [deg]
%   15  lambda_vt - Vstab taper ratio
%   16  Sht - H Tail area                                               [ft^2]
%   17  lambda14_ht - Htail 1/4 chord sweep                             [deg]
%   18  lambda_ht - Htail taper ratio
%   19  Nl - landing load factor                                        [g]
%   20  Wl - design max landing weight                                  [lb]
%   21  Lm - main gear length                                           [ft]
%   22  Ln - nose gear length                                           [ft]
%   23  Sf - Fuselage Wetted Area                                       [ft^2]
%   24  Lt - wing quarter-MAC to tail quarter-MAC                       [ft]
%   25  L_D - fuselage length to structural depth ratio
%   26  deltaP - max pressure differential (0 for unpressurized)    [psi]
%   27  Sb - Boom Wetted Area                                       [ft^2]
%   28  L_Db - boom length to structural depth ratio
%   29  Vt - fuel tank volume                                           [gal]
%   30  Vi_Vt - fraction of fuel tanks that are integral
%   31  Nt - number of tanks
%   32  Nen - number of engines
%   33  Bw - wingspan                                                   [ft]
%   34  L - fuselage length                                             [ft]
%   35  Wen - uninstalled weight of one engine                          [lb]
%   36  Wuav - uninstalled avionics weight                              [lb]
%   37  Wfl - total fuel weight                                         [lb]

%   Config vector:
%       config(1) - 0 = conventional horizontal tail or canard
%                   1 = T-tail configuration
%       config(2) - 1 = conventional single fuselage configuration
%                   2 = twin fuselage configuration
%                   3 = three fuselage/two boom configuration
%       config(3) - Number of vertical tails
%       config(4) - Number of horizontal tails

%   coords vector (all in ft)):
```

```
%   1   Wx - wing centroid x-coord
%   2   Vtx - vertical stab x-coord
%   3   Htx - horizantal stab x-coord
%   4   Mlgx - main landing gear x-coord
%   5   Nlgx - nose landing gear x-coord
%   6   Fusex - fuselage centroid x-coord
%   7   Bmx - boom centroid x-coord
%   8   Furnx - furnishings centroid x-coord
%   9   FSx - fuel systems centroid x-coord
%   10  FCx - flight control system centroid x-coord
%   11  Engx - engine cg x-coord
%   12  Avx - avionics centroid x-coord
%   13  Elcx - electrical systems centroid x-coord


% Wing Group:
Sw = inputs(1);
Wdg = inputs(2);
Nz = inputs(3);
A = inputs(4);
lambda14 = inputs(5);
Vh = inputs(6);
lambda = inputs(7);
tc = inputs(8);
Wfw = inputs(9);
Kb = inputs(10);
Kp = inputs(11);
Kd = inputs(12);

fuelDistCorr = (1-1.2*(Kb^.42-0.7*Kp^.3*Kb)*Kp)*(1-0.8*Kd);

rhoSL = .002377; % [slug/ft^3]
q = .5*rhoSL*(Vh*1.68781)^2; % dynamic pressure calculation

W_wing = (0.036*Sw^0.758*Wfw^0.0035*(A/cosd(lambda14)^2)^0.6*q^0.006*...
    lambda^0.04*(100*tc/(cosd(lambda14)))^-0.3*(Nz*Wdg)^0.49)*fuelDistCorr;


% Vertical Tail
Svt = inputs(13);
lambda14_vt = inputs(14);
lambda_vt = inputs(15);

Ht_Hv = config(1);
N_Vt = config(3);

W_vt = N_Vt*(0.073*(1 + 0.2*(Ht_Hv))*(Nz*Wdg)^0.376*q^0.122*Svt^0.873*...
    (100*tc/cosd(lambda14_vt))^-0.49*(A/
(cosd(lambda14_vt)^2))^0.357*lambda_vt^0.039);


% Horizantal Tail
Sht = inputs(16);
lambda14_ht = inputs(17);
```

```
lambda_ht = inputs(18);
N_Ht = config(4);

W_ht = N_Ht*(0.016*(Nz*Wdg)^0.414*q^0.168*Sht^0.896*(100*tc/
cosd(lambda14))^-0.12...
    *(A/(cosd(lambda14_ht)^2))^0.043*lambda_ht^-0.02);


% Landing Gear
Nl = inputs(19);
Wl = inputs(20);
Lm = inputs(21);
Ln = inputs(22);

W_main_LG = 0.095*(Nl*Wl)^0.768*(Lm)*0.409;

W_nose_LG = 0.125*(Nl*Wl)^0.566*(Ln)^0.845;


% Fuselage
Sf = inputs(23);
Lt = inputs(24);
L_D = inputs(25);
deltaP = inputs(26);

if deltaP == 0 % fuselage not pressurized
    W_press = 0;
else % fuselage pressurized
    W_press = 11.9 + (Vpr*deltaP)^0.271;
end

if config(2) == 1 || config(2) == 3 % single fuselage or triple config
    W_fuse = 0.052*Sf^1.086*(Nz*Wdg)^0.177*Lt^-0.051*(L_D)^-0.072*q^0.241 +
 W_press;
elseif config(2) == 2 % twin fuselage configuration
    W_fuse = 2*(0.052*Sf^1.086*(Nz*Wdg)^0.177*Lt^-0.051*(L_D)^-0.072*q^0.241 +
 W_press);
end


% Booms
Sb = inputs(27);
L_Db = inputs(28);

if config(2) == 3 % fuselage + two booms config
    W_booms =
 2*(0.052*Sb^1.086*(Nz*Wdg)^0.177*Ltb^-0.051*(L_Db)^-0.072*q^0.241 + W_press);
end


% Furnishings
correction = W_fuse/Wdg/0.154; % correct based on fuselage weight

W_furnishing = (0.0582*Wdg - 65)*correction;
```

```
% Fuel System
Vt = inputs(29);
Vi_Vt = inputs(30);
Nt = inputs(31);
Nen = inputs(32);
Wfl = inputs(37);

Fref = 0.8898;
K_fs = (1-.9*(Wfl/Wdg)^.8)/Fref;

W_fuelSys = K_fs*2.49*Vt^0.726*(1/(1 + Vi_Vt))^0.363*Nt^0.242*Nen^0.157;


% Flight Control System
Bw = inputs(33);
L = inputs(34);

W_flightCon = 0.053*L^1.536*Bw^0.371*(Nz*Wdg*10^-4)^0.8;


% Engine
Wen = inputs(35);

W_engine = 2.575*Wen^0.922*Nen;


% Avionics
Wuav = inputs(36);

W_avionics = 2.117*Wuav^0.933;


% Electrical System

W_elec = 12.57*(W_fuelSys + W_avionics)^0.51;


% Output Vector
weights = [W_wing, W_vt, W_ht, W_main_LG, W_nose_LG, W_fuse, W_booms, ...
    W_furnishing, W_fuelSys, W_flightCon, W_engine, W_avionics, W_elec];
W_total = sum(weights);
weights(14) = W_total;



% Center of Gravity Calculation
Wx = coords(1);
Vtx = coords(2);
Htx = coords(3);
Mlgx = coords(4);
Nlgx = coords(5);
Fusex = coords(6);
```

```
Bmx = coords(7);
Furnx = coords(8);
FSx = coords(9);
FCx = coords(10);
Engx = coords(11);
Avx = coords(12);
Elcx = coords(13);
CG = (W_wing*Wx + W_vt*Vtx + W_ht*Htx + W_main_LG*Mlgx + W_nose_LG*Nlgx +...
    W_fuse*Fusex + W_booms*Bmx + W_furnishing*Furnx + W_fuelSys*FSx +...
    W_flightCon*FCx + W_engine*Engx + W_avionics*Avx + W_elec*Elcx)/W_total;

end
```

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