Pricing American style options using binomial tree models

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American options

Definition

An American option is a contract which gives the right to buy or sell a risky asset at for the strike price K at **any point** before the expiration date T.

The difference between European and American style options is that Europen options can only be exercised right at the expiration date, whereas American options can be exerciesed before time T as well. This aspect renders the standard pricing approaches (via Black-Scholes or other stochastic volatility models) unable to provide a closed formula for American option pricing.

Binomial tree models

To remedy this, we are going to use the **binomial tree models**, which model the option prices by, simply speaking, working out all possible outcomes which arise from possible price movement patterns, which itself is modeled via a Bernoulli random variable with probability $p \in (0, 1)$.

- Cox-Ross-Rubinstein model – essentially discretizes the Black-Scholes model by setting the up-down movements to $u, d = e^{\pm\sigma\sqrt{\Delta t}}$ with the probability $p = \frac{e^{\Delta t} - d}{u - d}$.
- Jarrow-Rudd model – similar to CRR model but p is forced to be $1/2$, with u and d adjusted accordingly.
- Tian model – uses higher moments to more accurately "tilt" the binomial tree.

Results and conclusions

We have implemented all three models in Python, and tested their performance for on some stocks. There are the graphs we get for AAPL:

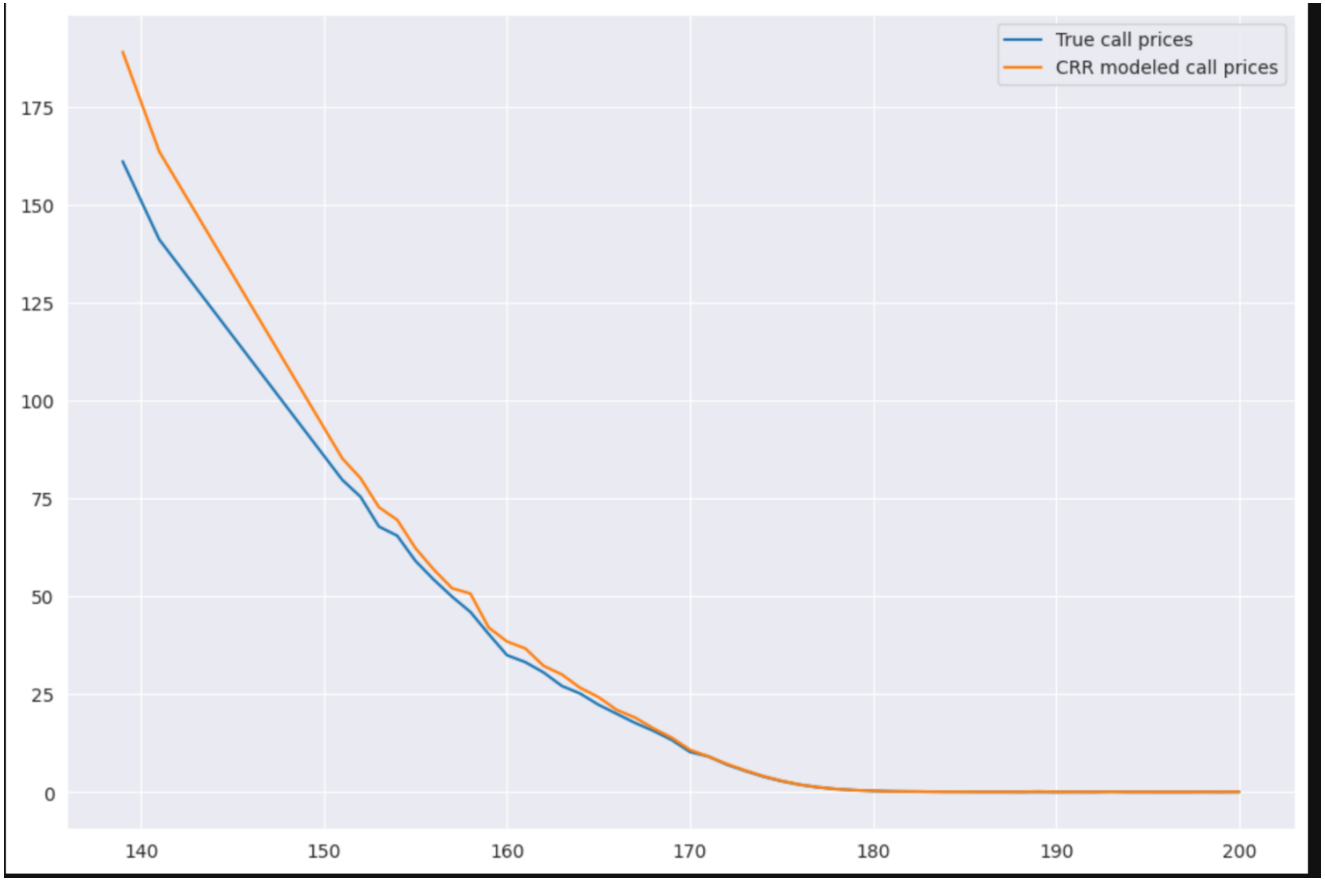

Figure: CRR model for AAPL prices with Dec 19 exp date

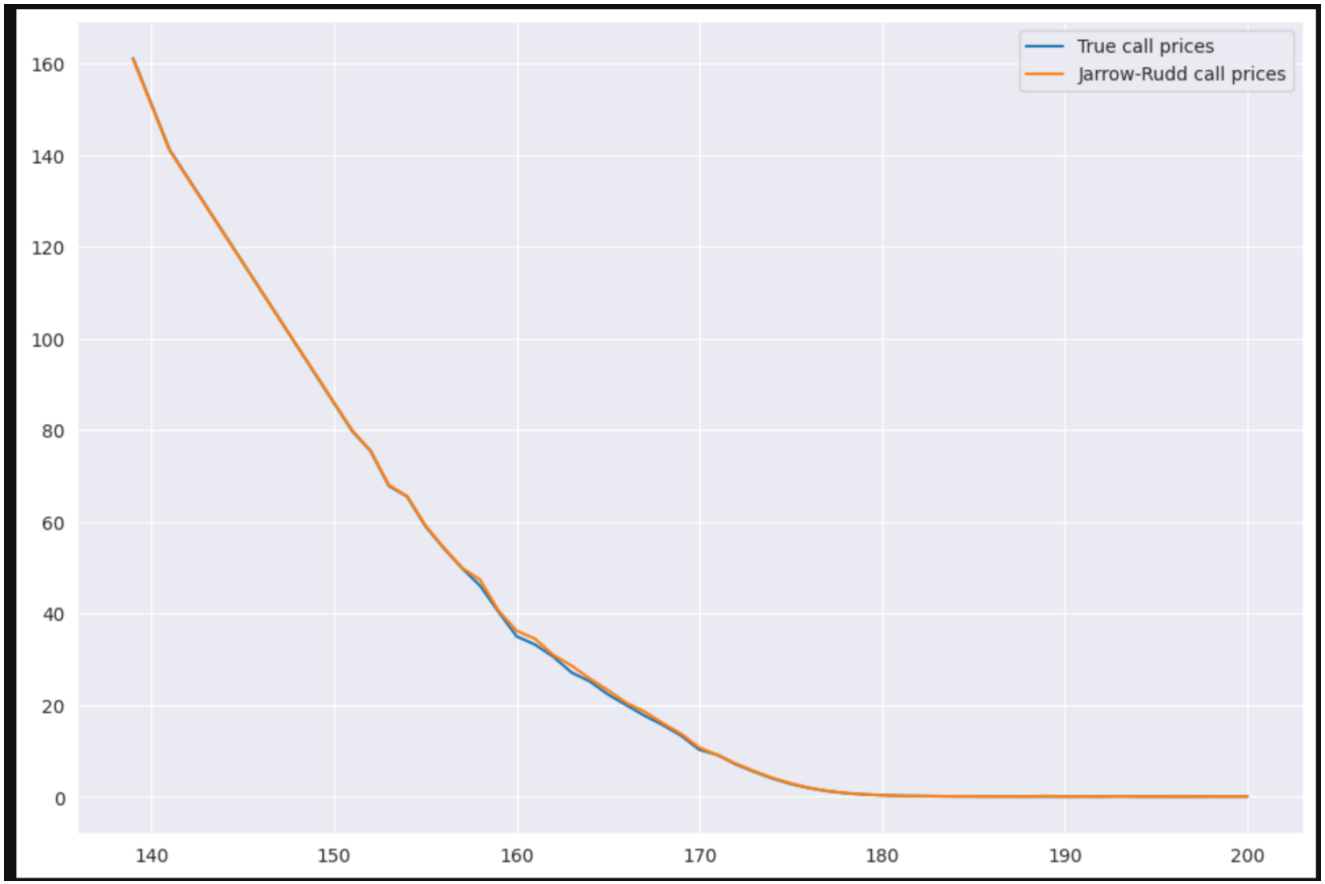

Figure: JR model for AAPL prices with Dec 19 exp date

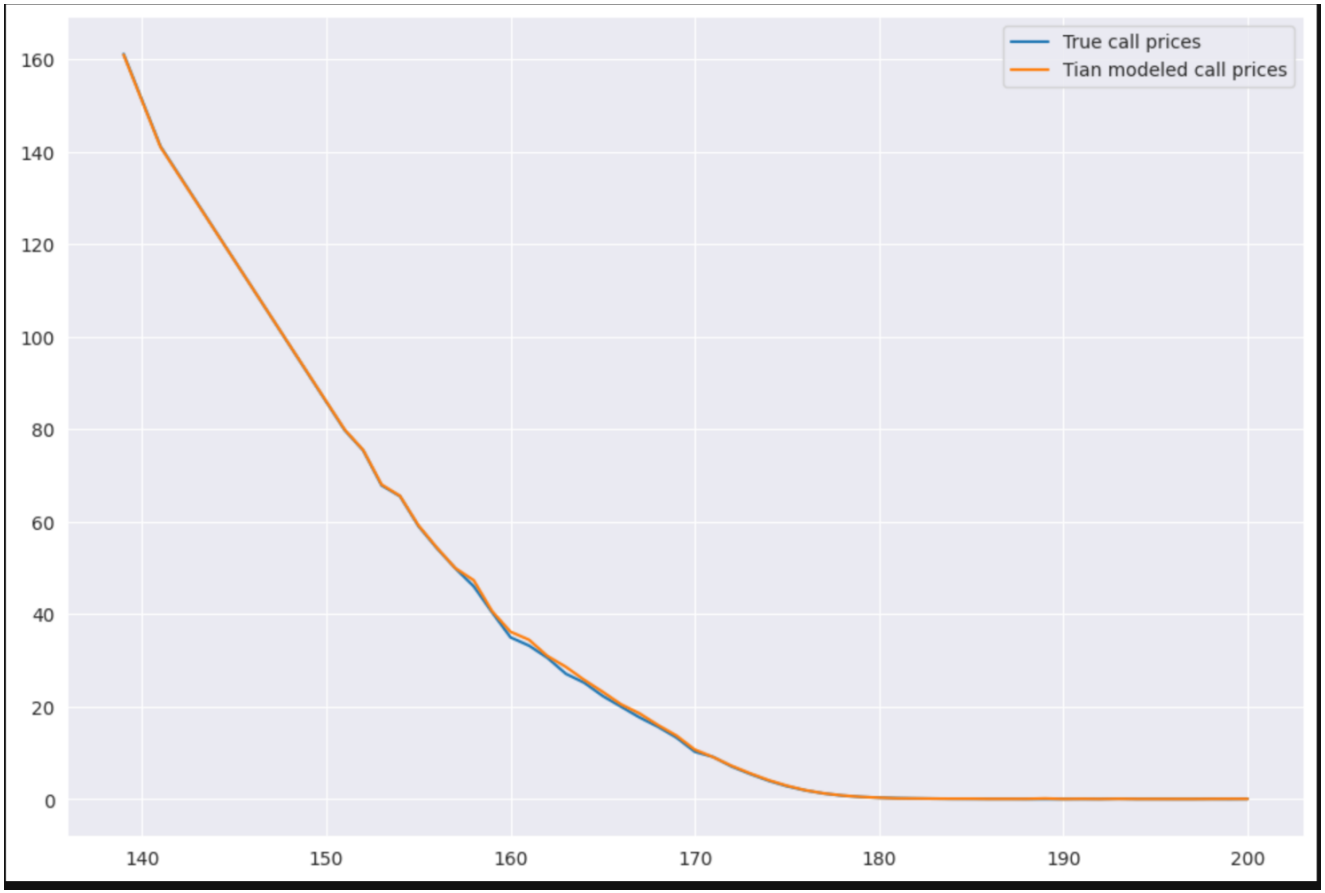

Figure: Tian model for AAPL prices with Dec 19 exp date

References

[1] Cox, J. C., Ross, S. A., Rubinstein, M., "Option Pricing: A Simplified Approach", Journal of Financial Economics (1979)

[2] Jarrow R., Rudd A., Approximate option valuation for arbitrary stochastic processes, Journal of Financial Economics, Volume 10, Issue 3, 1982, p. 347-369,

[3] Tian, Y.". (1999), A flexible binomial option pricing model. J. Fut. Mark., 19: 817-843.