Deep Reinforcement Learning Agents learn Predatory Pricing

relAI
Konrad Zuse School of Excellence in Reliable AI

Motivation

- Complex economic models have been infesible to analyse until recently
- **Reinforcement Learning** (RL) can solve complex decision making problems

$\Longrightarrow$ What if RL controls economic agents?

Dynamic Oligopoly Model

Agents are companies competing in an oligopoly over multiple rounds:

Require:
Set of agents $\mathcal{N} = \{1, \dots, N\}$
Number of rounds T
For each agent i: initial demand D_1^i, unit production cost c_i, policy π_i, observation function Φ_i
for $t = 1, 2, \dots, T$ **do**
  for $i \in \mathcal{N}$ **do**
    i observes $o_t^i = \Phi_i(s_t) = \Phi_i(t, D_t^1, \dots, D_t^N)$
    i selects a price $p_t^i \sim \pi_i(o_t^i)$
    i sells quantity $D_t^i - p_t^i$
    i receives reward $r_t^i = (p_t^i - c_i)(D_t^i - p_t^i)$
  end for
  Compute the average price as $\bar{p}_t = \frac{1}{N}\sum_{j \in \mathcal{N}} p_t^j$
  for $i \in \mathcal{N}$ **do**
    Compute the price difference $\Delta p_t^i = p_t^i - \bar{p}_t$
    Transition demand to $D_{t+1}^i = D_t^i - \Delta p_t^i$
    Optionally, drop out i if $D_{t+1}^i < c_i$ (see Eq. (2))
  end for
end for
Reward each agent i with $U_i = \sum_{t=1}^{T} r_t^i$

The dropout mechanism removes unprofitable companies from the game

$\Longrightarrow$ **Discontinuous game dynamics**

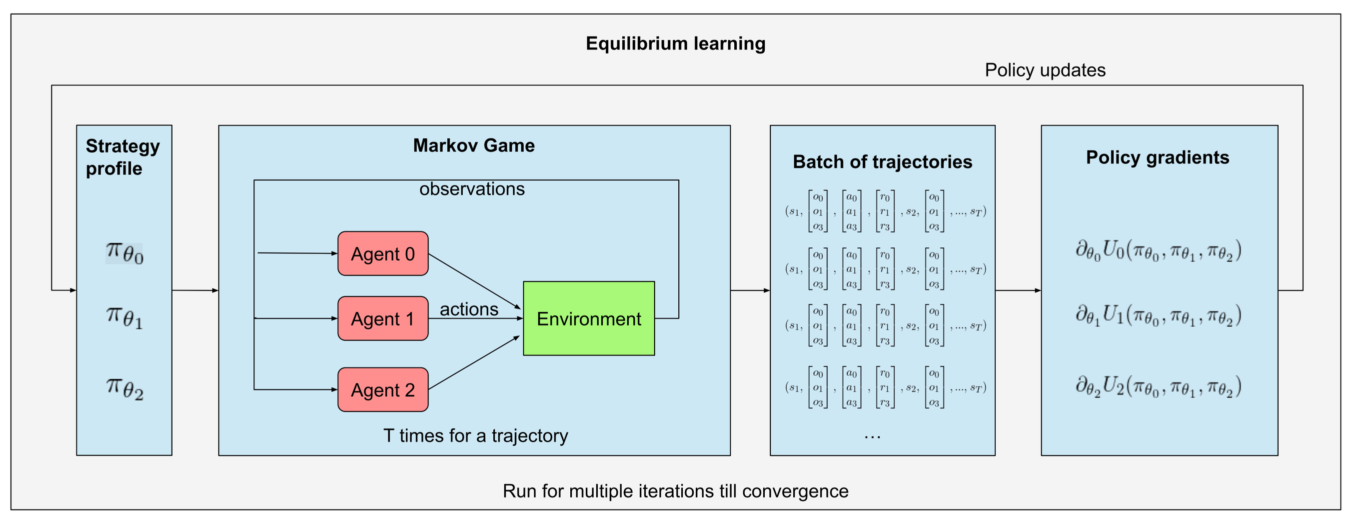
Analytical solutions unknown with dropouts

Equilibrium Learning

We search **Nash equilibria** as fixed points of the game where no player gains by deviating:

$$\sup_{\pi_i \in \Sigma_i} U_i(\pi_i, \pi_{-i}^*) - U_i(\pi_{\mathcal{N}}^*) \leq \varepsilon \quad \forall i \in \mathcal{N}$$

Each company is controlled by an RL agent aiming to maximize its profit:



We use REINFORCE (or its variants) to update all agents' policies simultaneously:

$$\theta_i \leftarrow \theta_i + \alpha \nabla_{\theta_i} U_i(\pi_{\theta_i}, \{\pi_{\theta_j}\}_{j \in \mathcal{N} \setminus \{i\}})$$

We hope this procedure converges to a Nash equilibrium, as measured by:

$$\mathcal{L}_{\text{bf}} = \sum_{i \in \mathcal{N}} \sup_{\pi_i \in \Sigma_i^K} U_i(\pi_i, \pi_{-i}) - U_i(\pi_i, \pi_{-i})$$

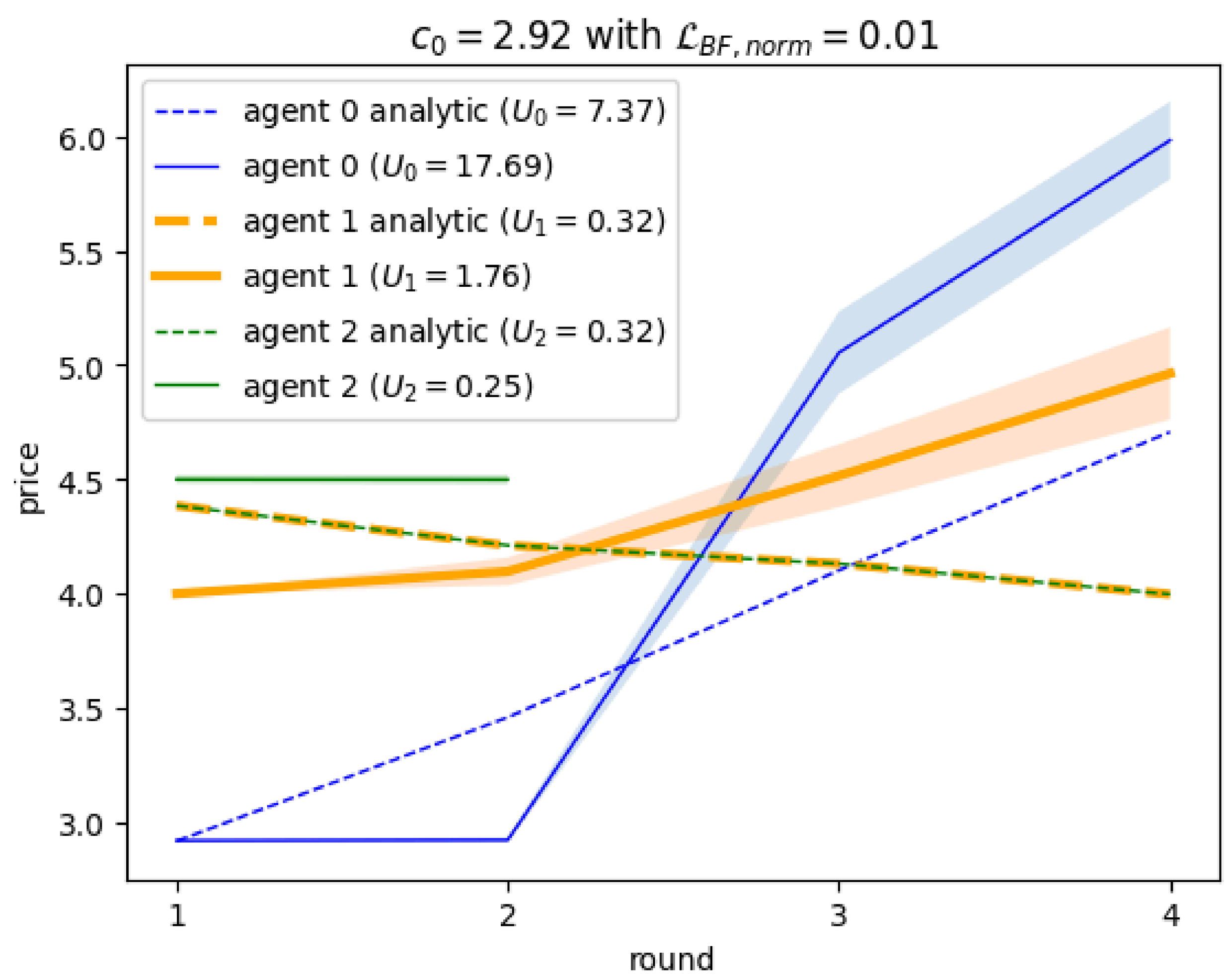
Results

Setup:

- $N = 3$ companies, $T = 4$ rounds
- Company 0 can produce at a lower cost

Findings:

- Firm 0 learns **predatory pricing**
- The result is a verified Nash equilibrium



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