

Background Study

The Brain1 and Brain3 datasets come from an fMRI (functional Magnetic Resonance Imaging) study where 20 participants learned a **finger-tapping task** over three days [1]. To track learning changes, we compared Day 1 (early learning) with Day 3 (after practice). The researchers divided each subject's brain into 112 anatomically defined regions (nodes). The edges between these nodes are undirected and denote significant **functional connections** (a high correlation in activity between two regions - in this case, there is a connection between any two regions whose low-frequency (0.06–0.12 Hz) coherence exceeded a **false discovery rate threshold**).

Visually, **Brain1 appears denser with more interconnections, while Brain3 is sparser**, reflecting a reduction in overall edges from Brain1 (2,500 edges) to Brain3 (1,540 edges). Early in training (Day 1), performance is effortful and recruits a broad network of brain areas, including motor, sensory, and higher-order control regions. By later sessions (Day 3), performance becomes faster and more automatic, the brain's motor system becomes more autonomous and less dependent on frontal “supervisory” regions.

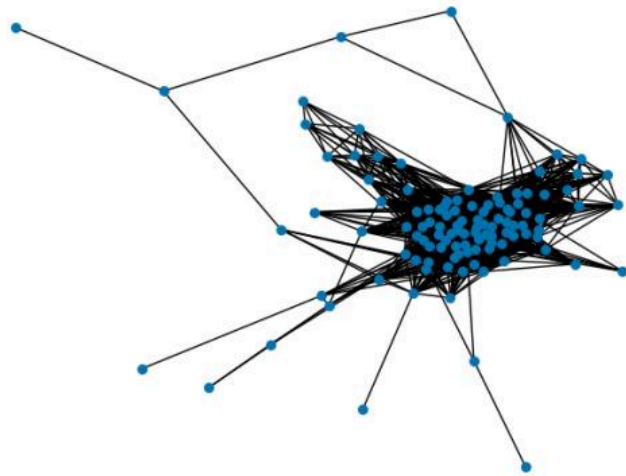


Figure 1: Brain Networks at day 1 (Brain1)

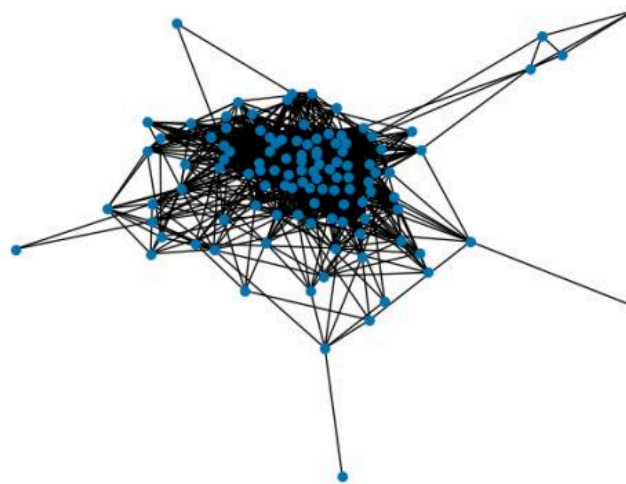


Figure 2: Brain Networks after day 3 (Brain3)

In network terms, we expect increased modularity (more distinct subnetworks) and a shift in hub roles from control regions to specialized sensorimotor regions as learning progresses. The **presence of an edge** indicates that the two regions have **significant interaction or communication** within the brain network model. In Brain1, nodes tend to have a **higher average degree** (more connections) than in Brain3. For instance, the highest-degree hub in Brain1 connects to approximately 60%–70% of all other regions, whereas Brain3's largest hub connects to less than half of the regions. **This suggests that the brain's connectivity is more globally integrated initially and becomes more focused on efficient, task-relevant pathways after learning.**

Community Structure via Spectral Partition

We divided each network into two communities using spectral partitioning to investigate the modular structure. Specifically, we used the eigenvector corresponding to the second-largest positive eigenvalue of the adjacency matrix and split the nodes based on the sign of their components.



The resulting bipartition for each graph yields two communities of roughly equal size: Brain1 has groups of 55 and 57 nodes, while Brain3 has 54 and 58 nodes. We then counted the number of edges within each community and between the two communities:

- **Brain1 (pre-learning):** Community 1 has 702 internal edges, Community 2 has 839 internal edges, and there are 959 edges between the two communities. About 61.6% of edges are within communities, and 38.4% are between communities.
- **Brain3 (post-learning):** Community 1 has 529 internal edges, Community 2 has 486 internal edges, and 525 edges between communities. Approximately 66.0% of edges are within communities, and 34.0% are between them.

These results indicate that both Brain1 and Brain3 exhibit modular structure, more connections occurring within communities than between them. However, **Brain3 shows a higher proportion of within-module connections compared to Brain1**, indicating its modules have become more distinct and segregated. **This aligns with the idea that as learning progresses, the brain's functional networks evolve from a more globally integrated state toward increased modularity, where specialized circuits become more focused.**

Modularity and Learning

A more modular (separated) network might mean the brain is refining its connections, with specialized regions handling specific tasks after learning. On the other hand, a less modular (more connected) network early on suggests that different brain areas communicate widely before things become more organized.

For example, if a brain region is connected to two others, there's a 75.6% chance those two are also directly linked in Brain1 (forming a triangle), compared to 56.7% in Brain3. This high level of local connections—combined with short paths between distant regions—shows the brain has a **"small-world"** structure, which is efficient for processing information.

The number of triangles (three fully connected nodes) drops sharply from 38,026 in Brain1 down to 11,348 in Brain3. Similarly, four-node loops (squares) plummet from 1,626,504 in Brain1 to just 303,982 in Brain3. **The abundance of 4-cycles in Brain1 suggests a highly redundant network.** In contrast, Brain3, with more reduced connections, shows substantially fewer 4-cycles. **It's as if early learning explores multiple possible connections, while mastery trims the excess, leaving behind more direct and efficient pathways.**

Comparison to Random Networks

To understand whether these results are meaningful, we compared the brain networks to randomly generated networks (Erdős–Rényi graphs) with the same number of nodes and connections. We created 60 random versions of each brain network and calculated their average clustering and motif counts.

For Brain1 (early learning, 112 nodes, 2,500 connections):

- The random networks had much lower clustering (~ 0.402) compared to Brain1's actual clustering (0.756)—meaning the real brain network is nearly twice as interconnected locally.
- They also had far fewer triangles ($\sim 14,815$ vs. Brain1's 38,026) and 4-cycles ($\sim 486,587$ vs. Brain1's 1,626,504).

For Brain3 (after learning, 112 nodes, 1,540 connections):

- The random networks were even less clustered (~ 0.248), though still below Brain3's real clustering (0.567).
- Their motif counts were also much lower ($\sim 3,462$ triangles vs. 11,348; $\sim 70,162$ 4-cycles vs. 303,982).

These comparisons confirm that the brain's organization isn't random—it has far more local connections and redundant pathways than chance would produce. The high clustering and abundance of motifs in Brain1 suggest a densely interconnected system early in learning, which later refines into a more efficient structure.

Centrality Analysis of Key Nodes

Degree Centrality (Number of Connections): The most connected brain regions - called "hubs" - changed significantly between early and late learning stages. In Brain1 (Day 1), the top hubs were regions **16**, **47**, **63**, **53**, and **43**, with region 16 standing out as particularly important - it connected to over 70 other areas. By Brain3 (Day 3), a different set of hubs emerged: regions **5**, **14**, **23**, **21**, and **27**. Noticeably, region **16** - once a major connection point - dropped out of the top five. This [suggests that as the task became more automatic, some previously critical hubs became less central, either losing connections](#) or seeing their importance shift to other regions. This hub reorganization shows how learning doesn't just change individual connections, but reshapes the brain's entire information network. The system appears to redistribute its key integration points as skills become more refined, potentially making processing more efficient.

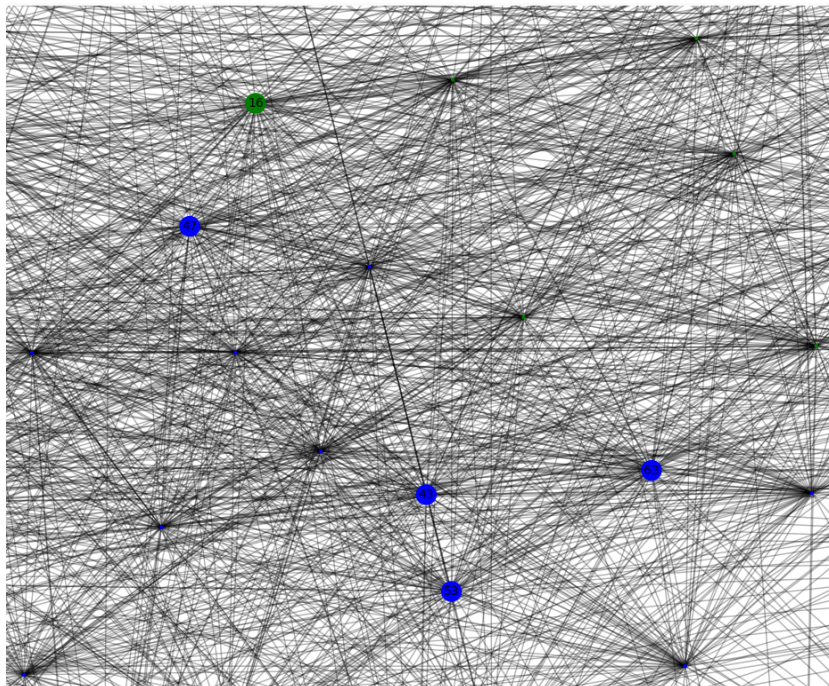


Figure 1: Degree centrality at day 1 (Brain1)

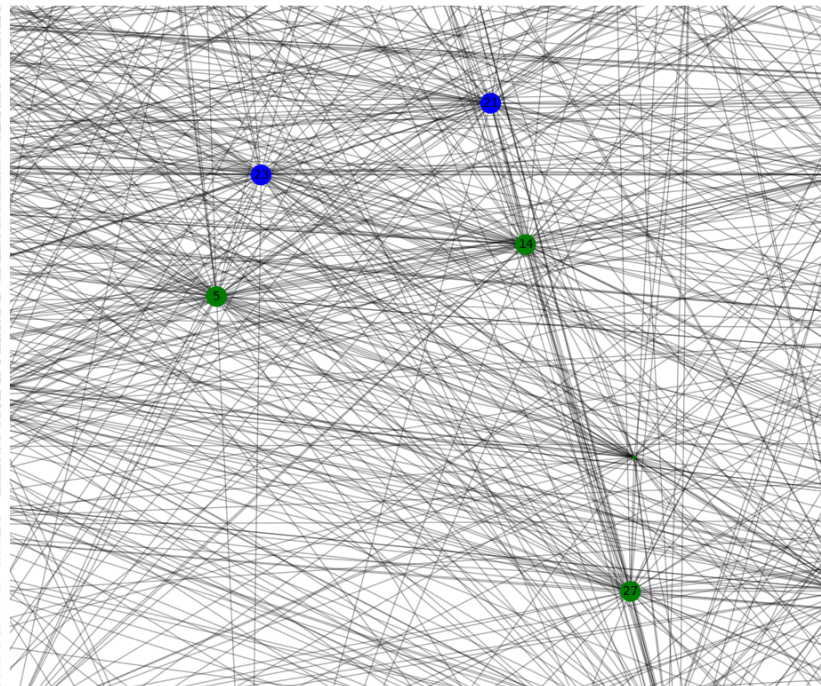


Figure 2: Degree centrality after day 3 (Brain3)

Closeness Centrality (Inverse of Average Distance to All Other Nodes): The brain's most connected regions (hubs) also tend to be the most centrally located in terms of communication efficiency. In Brain1 (Day 1), nodes **16**, **63**, **47**, and **53** ranked highest in both number of connections (degree) and ease of reaching other regions (closeness). Similarly, in Brain3 (Day 3), nodes **5**, **14**, **21**, and **23** topped both measures. This makes sense—regions with many direct links naturally serve as efficient communication hubs. However, node **30** presents an interesting exception: it appears among the top five most central nodes for closeness in both networks, despite not being highly connected (degree). This suggests it occupies a strategically important position—acting as a bridge that provides short pathways between other regions, even without numerous direct connections. Notably, **node 30 maintains this central role from Day 1 to Day 3, implying its consistent importance in facilitating efficient information flow throughout the learning process.** This stability contrasts with the broader reorganization of degree-based hubs, highlighting how different measures of centrality capture distinct aspects of the brain's evolving network architecture during skill acquisition.

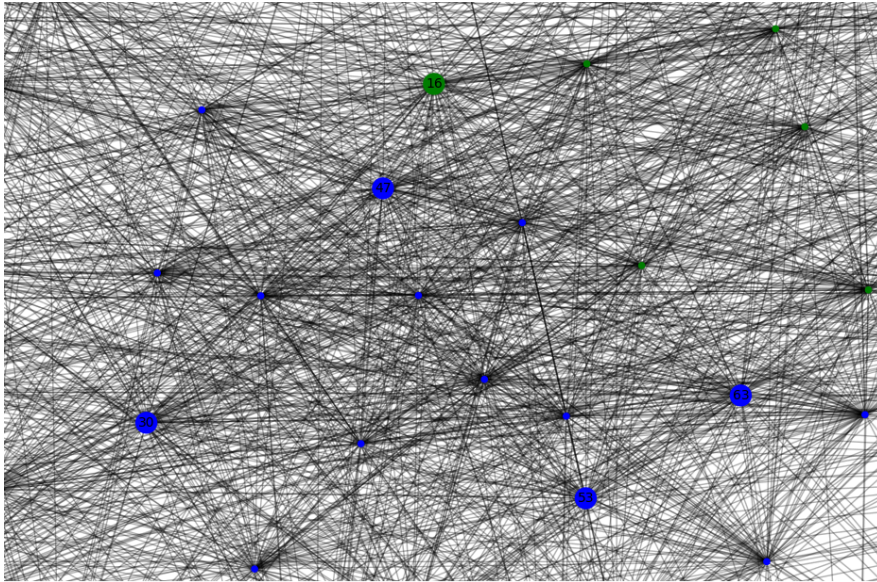


Figure 1: Closeness centrality at day 1 (Brain1)

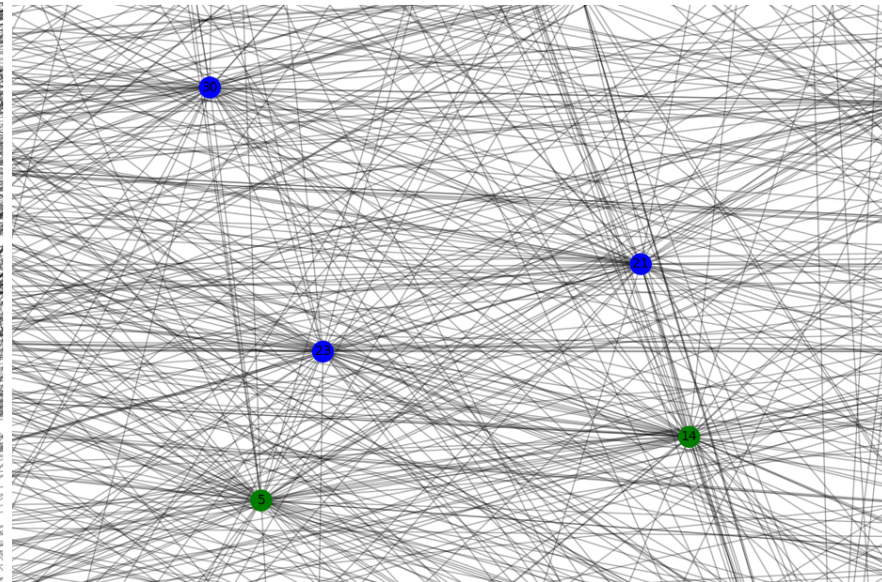


Figure 2: Closeness centrality after day 3 (Brain3)

Betweenness Centrality (Fraction of Shortest Paths Passing Through a Node): The brain uses different types of important nodes - not just highly connected hubs, but also critical "bridge" nodes that link separate regions. While degree and closeness centrality highlight the most connected and centrally located nodes, betweenness centrality reveals a different set of key players.

In Brain1 (early learning), nodes **2, 31, 76, 13, and 81** emerged as top betweenness nodes - even though they weren't especially highly connected. For example, **node 2 had only moderate connections but became crucial because it sat on the shortest paths between many distant brain regions**. These bridge nodes don't necessarily have many direct neighbors, but they strategically connect different neural communities. By Brain3 (after learning), the key bridge nodes had shifted to **0, 22, 5, 33, and 6**. This change suggests that as learning progresses, the brain not only reorganizes its hubs but also rewires how different neural communities connect through these vital bridges. This pattern shows an important network principle: some nodes specialize in connecting rather than collecting connections. While hubs integrate information within communities, bridges enable efficient communication between communities - both crucial for brain function. The reorganization of these bridge nodes during learning likely reflects optimization of information flow between specialized brain regions.

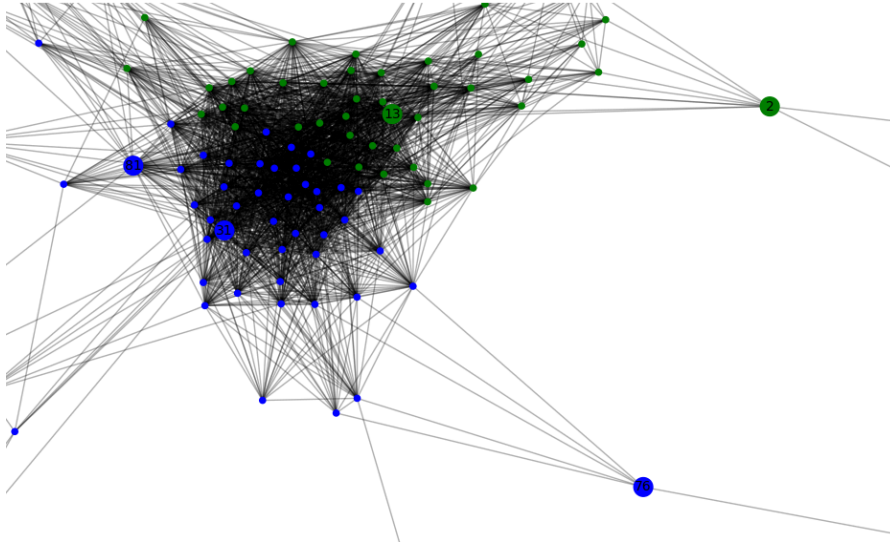


Figure 1: Betweenness centrality at day 1 (Brain1)

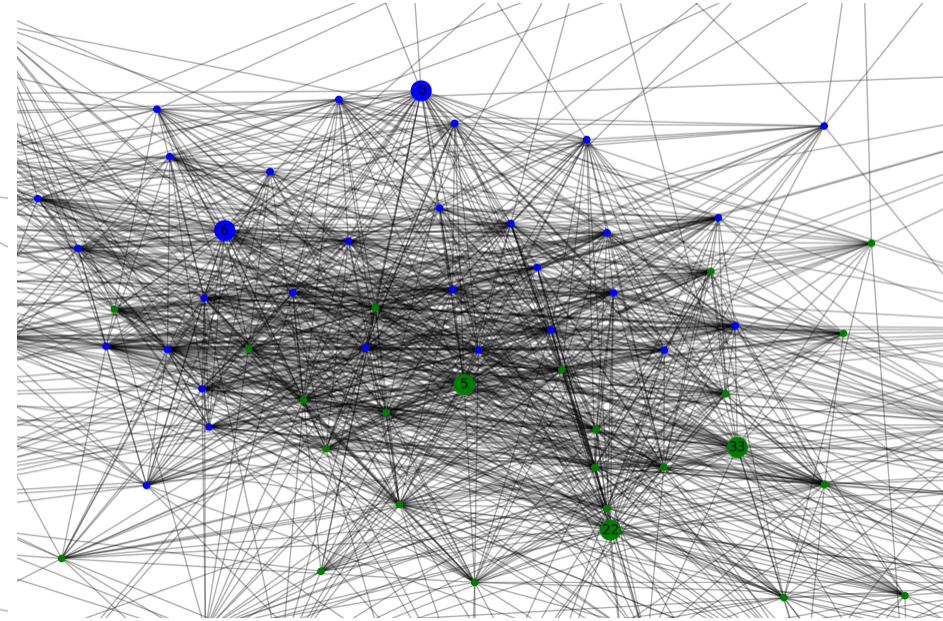


Figure 2: Betweenness centrality after day 3 (Brain3)

Comparison of Nodes and Differences: The comparison between Brain1 and Brain3 reveals a fascinating reorganization of critical brain regions. Node 5 stands out as particularly important in Brain3 - it appears among the top nodes in all three centrality measures (degree, closeness, and betweenness), suggesting it evolved into a "super hub" that both connects widely and facilitates efficient communication. Such versatile nodes likely play a special role in coordinating activity across the brain's specialized modules after learning.

The shift from Brain1 to Brain3 shows a clear pattern: initially prominent hubs (particularly in sensorimotor areas) lose their dominance, while other regions like node 5 gain importance. This reorganization aligns with findings from Mantzaris et al. (2013), who observed that while centrality generally decreased across all regions by Day 3, sensorimotor areas showed the most pronounced drops. Our single-network analysis suggests these changing nodes may reflect:

- Early-learning hubs in motor control regions (like primary motor cortex) that become less critical as movements automate
- Emerging connector regions that gain importance for integrating information in the learned state.

This dynamic reorganization supports the idea that learning doesn't just strengthen connections - it fundamentally reshapes how information flows through the brain's network. The system appears to shift from relying on motor execution hubs to depending more on integrative regions that coordinate automated performance.

References

[1] A. V. Mantzaris, D. S. Bassett, N. F. Wymbs, E. Estrada, M. A. Porter, P. J. Mucha, S. T. Grafton, and D. J. Higham. Dynamic network centrality summarizes learning in the human brain. *Journal of Complex Networks*, vol. 1, no. 1, pp. 83–92, 2013, doi:10.1093/comnet/cnt001.