Applied Mathematics 3911
Winter 2020
Assignment # 3

Due: Monday March 9th, 2020 (by 5pm, in the my office. Rm 285 MSC)

1. Write a program to use Monte Carlo simulation to simulate the two-dimensional Ising model on a square lattice. The Ising model is a spin model of magnetism (See the book for help) with a Hamiltonian or energy of a configuration given by

$$E = -J \sum_{<i,j>} S_i S_j, \tag{1}$$

where J is a constant, $< i, j >$ means a sum over nearest neighbour pairs and S_i represents the spin at site i. Use $J = 1$ and assume the spins can be in one of two states given by $S_i = +1$ or $S_i = -1$. For Matlab use a lattice size of 30 by 30 with periodic boundaries. Use a larger system if time (cpu time) will allow. Use importance sampling via a Markov process with $k_B = 1$ and transition probability T given by

$$T(S_i \to -S_i) = \frac{e^{\frac{-\Delta E}{k_B T}}}{1 + e^{\frac{-\Delta E}{k_B T}}} \tag{2}$$

Hand in a copy of your code (printed nicely in a PDF document)

2. Using the code of part 1 start with a random system. After 2000 initial Monte Carlo steps calculate the average energy by sampling every 10th Monte Carlo (MC) step in a simulation of at least 5000 MC steps at temperatures of $T = 1$, $T = 2$,$T = 3$ and $T = 4$. Plot $< E >$ vs T and $< |M| >$ vs T. Note E is energy of the system and $|M|$ is the absolute value of the magnetization of the system. Do more temperatures in this range if you have time.

3. Find (on-line) the exact solution to the 2D Ising model and compare the exact solution to your results in part 2 by plotting both on the same graphs.

4. Graduate Students only. Calculate the $< E^2 >$ and $< M^2 >$ and use these to calculate the specific heat and magnetic susceptibility using your Ising Model code. How to calculate these quantities is discussed in the text.