Applied Mathematics 3911b
Winter 2020
Assignment # 2

1. Create a program that generates random numbers using the LCG genenator discussed in class for arbitraty a,c and m. Show that the choice used by IBM (find in book or Resources on OWL) creates the planes discussed in class by plotting (x_n, x_{n+1}, x_{n+2}) for a large number of random numbers in 3D. You may wish to write the numbers to a file and then use a separate program to do the plotting. I find gnuplot is a simple, free, plotting environment. See https://en.wikipedia.org/wiki/RANDU

2. The simplist test of randomness is uniformity. That means a random number sequence should contain numbers distributed in the unit interval, [0,1] with equal probability. To test this divide the interval into M equal size sub intervals or bins and place each random number into the appropriate bin. Do this for $N = 10^5$ random numbers using both the LCG randon number generator discussed in class (and in question 1) and the default random number generator of the programming language you use. (ie. Matlab, Python or C or C++) Plot a graph of N(i)/Z versus i where i is the bin number and N(i) is the number of random numbers in that bin and Z is the total number of random numbers generated. Comment on if there is a difference.

3. Short term correlations is another measure of randomness. For a sequence of random numbers the auto-correlation function is

$$C(k) = \frac{< x_{i+k} x_i > - < x_i >^2}{< x_i x_i > - < x_i >^2} \tag{1}$$

   where the $<>$ brackets means an ensemble average over the whole sequence of random numbers. Find $C(k)$ using $a = 106, c = 1283$ and $m = 6075$ for your RNG from question 1 and using the built in random number generator. In an infinite sequence of purely random numbers $C(k) = 0$. Try and do $k = 0$ to $k = 350$. Discuss your $C(k)$'s differences from this result.

4. Write your own program to simulate nuclear decay using a Monte Carlo simulation. You may use the program on pages 192-193 as a guide (but try not to copy it line for line). Your program should be designed to repeat the simulation multiple times ($ntrial > 1$) and average the results to find the ensemble average $< N(t) >$ as a function of time, t. Use the parameters $N_0 = 200, p = 0.01, tmax = 200$ and $ntrial = 20$. I would like you to hand in:
   (a) A copy of the program.
   (b) A plot of $N(t)$ versus t for one simulation.
   (c) A plot of $< N(t) >$ versus t from the average of all ntrial simulations.
   (d) A plot of the data in part c. plotted on a semi-log plot, along with a least-squares fit to the data using the exact solution . What value λ of do you calculate?

5. Graduate students: The probability P(n) that n nuclei decay in a time period is expected to be a Poisson distribution. Modify your program such that is outputs the probability that n nuclei decay in the first time step. Use $N_0 = 1000$,$p = 0.001$,$tmax = 1$ and $ntrial = 1000$. Plot P(n) versus n and compare your results to the Poisson distribution with your measured value of $< n >$.