STSCI 5080 Homework 5

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November 27, 2020

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1 Rice 8.10.7 modified

1.1 Part A

To determine the MLE, we first

2 Rice 8.10.16 modified

2.1 Part A

3 Problem 5: Fischer information for location family

3.1 Part A

To show that the Fischer information of the given location family is of the indicated form, we make use of a change of variables, in which

$$\begin{aligned} I(\mu) = \int_{-\infty}^{\infty} \left(\frac{\partial \log(f(u-\theta))}{\partial \theta} \right)^2 f(u-\theta) \mathrm{d}x \\ = \int_{-\infty}^{\infty} \frac{f'(u-\theta)}{f(u-\theta)} \ , \end{aligned}$$

from which substituting $x = u - \theta$ gives,

$$\int_{-\infty}^{\infty} \frac{(f'(x))^2}{f(x)} \mathrm{d}x \ .$$

3.2 Part B

To determine the Fischer information of the logistic family, we begin by observing that

$$f(x) = \frac{e^{-x}}{(1+e^{-x})^2} = \frac{1}{1+e^{-x}} - \frac{1}{(1+e^{-x})^2} \ .$$

Differentiating $f(x)$ gives

$$
\begin{aligned}
f'(x) &= \frac{e^{-x}}{(1+e^{-x})^2} - \frac{2e^{-x}}{(1+e^{-x})^3} \\
&= f(x)\left(1 - \frac{2}{1+e^{-x}}\right) \\
&= \left(\frac{e^{-x}-1}{e^{-x}+1}\right) f(x)\ .
\end{aligned}
$$

Finally,

$$
I(\mu) = \int_{-\infty}^{\infty} \frac{f'(x)^2}{f(x)} \mathrm{d}x = \int_{-\infty}^{\infty} (\frac{e^{-x}-1}{e^{-x}+1})^2 \frac{e^{-x}}{1+e^{-x}} \mathrm{d}x\ .
$$