McMahon Lab Post

Pete

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1 objectives

Given an initial configuration, we assign qubits for $v \in B_i \cap B_j$, for i even {resp odd} and j odd {resp even}, with $B^{\epsilon}_{x_i} = \sqcup_{\{i,j,k,l\} \in \mathcal{A}} B_{\mathcal{A}}$, and measure the time evolution of the initial state beginning at v. With additional samples of the Brownian bridge, a suitable equivalence towards assigning qubits of the time evolution of future states relies on enforcing parameter values for analyzing initial random data, dependent on the height h of the Brownian bridge sample. Specifically, this conversion of turbulent flow involves assigning a $|1>$ qubit to the bitstring if the sample satisfies $W(t_{i+1}) \geq h$, and otherwise, a $|0>$ qubit.

Depending on h, the emerging state space of qubits will either assign future states, from the Brownian samples, to the $|0>$ or $|1>$ state with overwhelming probability. To study the growth of the state space from a countable number of samples, we will compress over symmetries, to determine which projection reduces the state space the most substantially, whether polynomially or exponentially.

With a theoretically driven approach that will be numerically observed, a subinterval $[a_j, b_j]$ will be partitioned into finer subintervals on which Brownian samples will be collected. For a sufficiently high number of such samples, at each t_i during which the i th sample is collected, the Brownian bridge transitions to any neighboring vertex of v with fixed probability.

During successive increments following the state of the initial configuration, it is important to realiably measure the change that the Brownian bridge achieves some height h, by computing, conditional on the measure $\mathbf{P}^{h,v}(\cdot|\cdot)$ of the form

$$\mathbf{P}^{h,v}_{[t_i,t_{i+1}]}(A|B) = \frac{\exp(-\mathcal{H})}{Z^{v_i}_{\emptyset}(\mathcal{R})} ,$$

for Borel measurable A and B, $\mathcal{R} \subseteq \mathbf{R}^2$, where the normalization $Z^{v_i}_{\emptyset}$ denotes the partition function of the emerging initial state, with no boundary conditions placed on the initial state in $B^{\epsilon}_{x_i}$. For each additional increment of time, an additional state is appended to the summation over all configurations of the system, for an additional sample of $W(t)$. Recall that the Hamiltonian is of the form

$$\mathcal{H} = \sum_{i \sim j} \mathcal{J}_{ij} \bigg(\mathbf{1}_{|x_i - x_j| < 1} \exp(-i(t_i + t_j)) + \mathbf{1}_{|x_i - x_j| > 1} \exp(-it_i) \exp(-it'_j) \bigg) ,$$

in which the couplings satisfy, for arbitrary N and ϵ,

$$\mathcal{J}_{ij} = \begin{cases} N|t_i - t_j| \exp(\mathbf{1}_{W(t_j)=h}|x_i - x_j|) \text{ , for } |x_i - x_j| \leq \frac{\epsilon}{n} \\ \frac{\exp(\mathbf{1}_{W(t)\neq h}|x_j|)}{|x_i - x_j|} \text{ , for } |x_i - x_j| > \frac{\epsilon}{n} \end{cases} .$$

To enforce random initial data for studying the time evolution, admissible random initial data may be drawn from a specified distribution. With such a measure and knowledge of the Brownian height, sampled at different instances, symmetries within $B^{\epsilon}_{x_i}$, including

- First, $\frac{\pi}{2}$ rotational invariance : Any collection of Brownian samples, for configurations sampled at $v_i \neq v_j$ at t_i can be regarded, similar to duality arguments in horizontal and vertical crossing probabilities across rectangles in Russo-Seymour-Welsh Theory, as equivalent, and identical, configurations, because each configuration receives the same assignment from the the measures $\mathbf{P}^{h,v_i}_{[t_i,t_{i+1}]}(\cdot|B)$ and $\mathbf{P}^{h,v_j}_{[t_i,t_{i+1}]}(\cdot|B)$.

- Second, translation invariance : Introducing observations of qubits to the state space which are translates of each other, in which there exists an automorphism $\varphi : V \longrightarrow V : [v_i, v_{i+1}] \mapsto [v_{i+j}, v_{i+j+1}]$, which translates, over the plane, one collection of Brownian samples, from $t_1, \cdots, t_k$, to another collection of Brownian samples, from $t_{i+j}, \cdots, t_{k+j}$, both of which are equal in distribution.

Under the symmetries listed below, we will investigate whether the listed symmetry exponentially maps classical observation to qubits, so that the symmetry that one chooses to compress over prepares variables for quantum registers, while also having the symmetry not project onto too restrictive of a subspace of the original state space of all classically made observations. Explicitly, the conditions that will be required are centered around geometric constraints, enforced below:

- First, the Ricci curvature tensor : For a Riemannian manifold and ∇ the Levi-Civita connection, the Ricci curvature tensor, of the form $R(X,Y)Z = \nabla_X \nabla_Y Z - \nabla_Y \nabla_X Z - \nabla_{[X,Y]} Z$, measures the extent to which taking paths following vector fields X, and Y, commutes. The Ricci curvature itself provides a symmetry over which a solution $u(x,t)$ may be compressed over.

  To compute the probability that this random height model achieves a random samples of size h, one computes the Ricci curvature, and given the value and properties of the ambient metric assigned to the manifold, which in our case is the usual Euclidean distance enforced to measure the dispersion of random samples within a fixed interval of sampling, provide constraints over which the system may be compressed.

  The properties of the metric, which classify it as Riemannian, enforce a symmetry in the assignment of qubit states, due to the inherent symmetries of the Ricci curvature, all of which claim,

$$\begin{aligned} R(u,v) &= -R(v,u) \ , \\ < R(u,v)w, z > &= - < R(u,v)z, w > \ , \\ R(u,v)w + R(v,w)u + R(w,u)v &= 0 \ . \end{aligned}$$

- Second, for additional invariants : To optimally identify collections of Brownian samples that, based on the inherent randomness of independent sampling of distinct Brownian bridges, in which there is a nonzero probability of having distributions of samples, collected during finite lengths of time, for which samples in each collection are positively correlated to one another, in the sense that the collections of samples

$$\mathcal{A} = \{ T \in [t_1, \cdots, t_k] : W(T) \geq \beta \}$$

  from one Brownian bridge $W(t)$, for arbitrary β, in comparison to those from another bridge $W'(t)$, on the disjoint time interval given below,

$$\mathcal{B} = \{ T' \in [t'_1, \cdots, t'_k] : W'(T') \geq \beta \} \ ,$$

  with the same threshold can be put into correspondence with one another. Specifically, this can be achieved by making use of an algorithm to determines whether an injective mapping between one collection of samples and another collection can be established. Besides imposing the obvious condition that each collection of samples have equal cardinality, to determine whether nearly strict equality between two collections of Brownian samples can be achieved pairwise, define a suitable mapping $\Phi : \longrightarrow : \mapsto$. Otherwise, it is possible to run a modification of the algorithm by allowing sets to be paired with one another, namely that the intersection of state spaces $|\mathcal{S}(t_1, \cdots, t_k)| \cap |\mathcal{S}'(t'_1, \cdots, t'_k)| = \mathcal{E} = \cup_{i=1}^{n} e_i$, where e_i are singletons, with countable $\mathcal{E}$ and sufficient n.

With such a Φ, by virtue of the sample distributions converging to a fixed distribution for enough samples, one identifies the maximum random sample achieved, and searches through the other collection of random samples to determine whether a maximum value, given a tolerance of random sampling mismatch, ϵ^1. From the size of each collection of random samples, the process continues to search throughout the remaining collection of random samples, from which the process will determine whether the random sample, chosen uniformly at random from the collection, can be matched to another random sample from the other collection, given a tolerance ϵ^2. From the collection $\{\epsilon^k\}_{k \in \mathbf{N}}$ of k samples, each of which are independently drawn, the process ends after all of the points lying in the nonempty intersection of the collection $S(t_1), \cdots, S(t_k)$ and $S'(t'_1), \cdots S'(t'_k)$ have been explored.

One goal will be to simulate a large enough collection of random samples, on which the symmetry can be enforced. For convenience, the collections will first be drawn from disjoint, open intervals of $\mathbf{R}$, from which larger, more complicated samples of random data in the plane can be generated by gluing the intervals together, so as to form intersecting sides of each $B^{\epsilon}_{x_i}$, which partition $\mathcal{R}$.

2 code: plotting the random variation in the Ricci curvature, dependent on the collection of Brownian samples [Plots TBA for next post]

After establishing the height of one sample relative to a previous sample, independent of the coordinate representation, a random array on $\mathbf{Z}^2$ will be generated. To construct the short and long range terms for the Hamiltonian, random samples from a site, chosen uniformly at random with positive probability, will be sampled. Depending on the distribution of these random samples, with distance > 1 from the initial point of time evolution, fluctuations in the Ricci curvature, for each additional time step in a fixed number of samples, will be used to generate ansatzae that are incremented with each additional random sample that is drawn.

For future simulations, it would be interesting to impose boundary conditions, namely a nonzero initial excitation to the initial configuration that would serve as a starting point for future sampling on bridges. From qualities of Gaussian processes, we would expect that imposing the initial condition through random sampling on $\partial B^{v_i}_{\epsilon_i}$ will shift the magnitude of all future samples after the sample has been determined.

3 towards constructing an ambient manifold: defining collections of open sets whose interior, dependent on the magnitude of the random sample, enclose subsets of the plane [Plots TBA for next post]

Topologically, as a collection of open sets that forms a basis $\mathcal{U}_i = \bigg[(x_i) 2^{-j} \frac{W(t_i)}{\max_{j<i} W(t_j)}, (x_i) 2^{-j+1} \frac{W(t_{j+1})}{\max_{j+1<i} W(t_{j+1})} \bigg) \times \bigg[(y_i) 2^{-I} \frac{W(t_I)}{\max_{j<I} W(t_j)}, (y_i) 2^{-I+1} \frac{W(t_{I+1})}{\max_{I+1<I} W(t_{J+1})} \bigg)$, for each $j \in \mathbf{Z}$, representative of short term interactions due to the continuity of the Brownian bridge, such randomly generated open sets characterize intervals of existence of solutions with dependent on random initial conditions. If $j = i = 1$, the maximum quantity is set to 1.

After having generated such a collection of open sets at random, an MPS will be defined, iteratively, so that the collection of ansatzae will approximate the local solution from random samples, all of which are collected for times within the interval of existence, by making use of properties of Riemannian manifolds in the plane, to construct solutions with a fixed time interval of existence. As a measure again numerical instability of solutions, the interval of existence on which the ansatzae will be defined should qualify as paths, should they exist, respect to the conditions which not only include that the Hamiltonian for the trajectories do not exhibit infinite Ricci curvature, pairwise for any points of the path, but also that multiple paths in the manifold can be classified, by comparing the equality of samples, in distribution, for equal intervals of times for which random samples are generated.

4 repurposing the random initial data: correlating disjoint collections of random samples to each other that differ by fixed increments, over finite periods of time

As mentioned in the objectives, another possible symmetry besides enforcing only conditions from the the Ricci curvature also includes correlating collections of random samples, for distributions of samples that are nearly equal. That is, by prescribing arbitrary ϵ, there exists a δ so that collections of random variables, sampled at disjoint intervals of time, can be compressed over this symmetry. Specifically, the symmetry that we will enforce assigns trajectories of fluid motion, in correspondence with the magnitude from sampling.

From the correspondence, it is possible to generate MPS ansatzae by examining the correlation of one collection of random samples with that of another collection. Besides the approaches that have already been given, from the collection of open sets that we have generated from such sampling on the plane, ansatzae will be introduced depending on the behavior of the Ricci curvature on the manifold. Besides merely being representative of the curvature attributed to points in space, for the specified bond dimensions,

Ricci curvature sign	Ambient manifold structure	MPS
<0	hyperbolic	$\lvert\psi^{\mathrm{MPS}} >< \sum_{\mathrm{bonds}} W(t_i)$
>0	pseudo Riemannian	$\lvert\psi^{\mathrm{MPS}} >= \sum_{\mathrm{bonds}} W(t_i)$,
$=0$	Ricci flat	$\lvert\psi^{\mathrm{MPS}} >< \sum_{\mathrm{bonds}} W(t_i)$

with index sets as chosen from summations of the form

$$
\begin{aligned}
\lvert\psi^{\mathrm{MPS}}_{H,PR,RF} > &= \sum_{\text{i, j > 0 st for fixed } i,\ \{i\}_{J_i},\{j\}_{J_j},i_{J_i}\sim j_{J_j}>h\ \forall J_i,J_j} W(t_i) \\
&= \begin{cases} \sum_{i=1}^{k \text{ bonds}} W(t_i)\ , \\ \sum_{i=1}^{k' \text{ bonds}} W(t_i)\ , \\ \sum_{i=1}^{k'' \text{ bonds}} W(t_i)\ . \end{cases} \\
&\Rightarrow \begin{cases} < \sum_{K=k\text{bonds}} W(t_i)\ , \\ = \sum_{K=k'\text{bonds}} W(t_i)\ , \\ > \sum_{K<k''\text{bonds}} W(t_i)\ , \end{cases}
\end{aligned}
$$

An advantage behind introducing ansatzae in relation to the classical MPS ansatz given in Lubasch et al is that higher bond dimensions in the ansatz are introduced for subsets of the plane for which the Brownian samples randomly achieve a maximum sample, or subsets of the plane for which . By enumerating all such points over the plane for which the Brownian samples achieves such a maximum, or close to the maximum, the open set $\mathcal{U}$ (in the sequence of open sets $\mathcal{U}^j$ to be defined below) allows us to assign a coverage $\mathcal{C}$ of the initial configuration of the system before time evolution. From the strings $\lvert \prod_{i=1}^{n} \psi_i >$ of the bitspace, one can determine whether the following unitaries can be embedded in the quantum ansatz, by determining whether the assignment of qubits, dependent on h, serves as an approximate threshold for separating turbulence on shorter and longer scales.

Put differently, the bond dimensions of the ansatzae in which there are longer range interactions, as evidenced by collections of random samples corresponding to some initial configuration, require bond dimensions of higher degree, through multiple "sampling rounds" of the same interval.

5 limiting behavior of h, in relation to the coverage $\mathcal{C}$ of an initial configuration

To determine whether the height of the random sample can be accounted for, in which large deviations from the maximum random sample are penalized, we will introduce a dependence on open sets, which as open balls have a center surrounding the point of the plane at time t_i of sample. We can assert such a dependence by penalizing the probability of achieving such a random sample in our state space, through the extent to which the open set that corresponds to a random sample with larger deviation from the maximum sample, in comparison to another sample which has a smaller deviation from the maximum sample, has a smaller open ball circumscribed around the sampling point, which could prevent the open set surrounding this point from including the point of the initial time evolution.

With these drawbacks in mind, from the sample with maximum height we will assign, with

$$\varphi : V_i \longrightarrow \mathcal{H} : \{\forall v \in \partial B^{x_i}_\epsilon\} \mapsto \begin{cases} |0> & \text{w.p } \epsilon \\ |1> & \text{w.p } 1-\epsilon \end{cases} \quad ,$$

for arbitrary ϵ, V_i the vertex set of the subgraph G_i of G, and $\mathcal{H}$ a Hilbert space. With these quantities, an initial qubit assignment before the time evolution of the initial configuration will be determined. Next, to determine whether the measurable event, under $\mathbf{P}^{h,v_i}_{[t_i,t_{i+1}]}$ that the Brownian sample achieves height h occurs, the assignment that the measure attributes to such an event is influenced not only by the region of the plane in which the sample is drawn, but also on the random assignment of the initial qubit state. For each disjoint, random $\mathcal{U}_i \subset \mathbf{R}^2$, the empty set in the basis of the topology is identified as the points for which there does not exist an open set covering any of the nearest neighbors of the point. On the other hand, the maximum sample is the open set with the most coverage of the initial condition, which from a dyadic interval decomposition of each B^v_ϵ admits a construction of smooth surfaces.

From either short or long range interactions in $\mathcal{H}$, once qubits have been assigned, the energy at any point of the time evolution, if finite, is computed from the assignment of the Hamiltonian. From an initial configuration, an easily applicable topological characterization of the maximum sample and all of the samples below it can be studied in terms of open sets whose interior can, or cannot, contain the origin of the time evolution of the configuration. If an open set corresponding to a sample is sufficiently large so that it contains the starting points of multiple configurations sampled with some B^v_ϵ, this situation will be discussed in later posts.

Dependent on the time evolution from any $x \in B^v_\epsilon$, the sign of the Ricci curvature, which informs the choice of MPS ansatz, allows one to reduce the classical state space of observations, by studying cases in which a symmetry of interest is broken. In our case, the Ricci curvature as one enforceable symmetry that can be broken, is proportional to, for the event

$$\begin{aligned} \mathcal{E} = \{\exists (x,y),(z,w) \in \mathbf{R}^2 \Rightarrow R(x,y) \neq -R(x,y) \ , \\ < R(x,y)w, z > \neq - < R(x,y)z, w > \ \text{, and/or} \\ R(x,y)w + R(y,w)x + R(w,x)y \neq 0\} \ , \end{aligned}$$

assigns values continuously, satisfying the proportionality with constant,

$$\mathbf{P}_W(\mathcal{E}) \propto C\bigg(|\mathcal{A}|, |\mathcal{U}^{(j+1)} \cap \mathcal{U}^{(j)}| \bigg) \ ,$$

for the set

$$\mathcal{A} = \{\text{instances of } \mathcal{E}\} \ ,$$

which occurs with probability, for any events $\mathcal{S}$ depending on finitely many edges, under the law

$$\mathbf{P}_W(\mathcal{S}) = \frac{|(x,y) \in \mathbf{R}^2 \text{ for which symmetry breaking occurs, from the conditions of } \mathcal{E}|}{Z} \ ,$$

with an appropriate renormalizing of the form

$$Z = \sum_{\mathcal{A}=(x,y),(z,w)\subset \mathbf{R}^2} \mathbf{1}_{|(x,y)\in\mathbf{R}^2|} = \sum_{\mathcal{E}\text{ admissible }(x,y)} \mathbf{1}_{|(x,y)\in\mathbf{R}^2|} + \sum_{\mathbf{R}^2\backslash\mathcal{E}\text{ admissible }(x,y)} \mathbf{1}_{|(x,y)\in\mathbf{R}^2|} \ .$$

Heuristically, besides only observing that the partition Z is a probabilistic measure in the sense that is assigns the values between 0 and 1 for the chance of encountering admissible, or non admissible, coordinates in the plane which respect the Ricci curvature tensor symmetry, determining such collections of points can be realized through a planar search through all of the vertices that lie in open subsets of the randomly generated open sets, for constructing the ambient manifold.

Also, observe that the dependence of each ansatz on the bond dimension, because after assigning qubits from the bridge the ansatz with the highest bond dimension results from the higher number of samples, to ensure proper coverage of the behavior less predictable, well behaved, subsets of the plane. [1]

From the 3 possible cases of Ricci curvature, we also observe that the ansatzae each of which are separately generated depending on the curvature sign, shape the coverage $\mathcal{C}$ that will be defined now. By comparing against geodesic paths in 2 manifolds from the exponential map, which as a subset of the tangent space maps geodesics to geodesics, the extent to which there exists a finite subcover, from open sets that have been randomly generated in proportion to our sampling activity, is expressed as

$$\mathcal{C}^{h,v_i}_{t_1,\cdots,t_n} = \#\{ j \in \mathbf{N} : \mathcal{U}^{(j)} \subseteq \mathcal{U}_i^{t_{v_i}\to t_{v_{i+1}}} \Rightarrow E_1 \ , E_2, \text{ or } E_3 \text{ occur}\} \ ,$$

for $1 \leq i \leq n$, as the **coverage** of the initial configuration at v_i, of time step $t_{i+1} - t_i$. In particular, the following events, measurable against $\mathbf{P}_W$ because of the dependence on the diameter of each $\mathcal{U}_i$ on the magnitude of the random sample at t_i, for automorphisms γ of B^v_ϵ, realized as piecewise smooth curves, defined to be of the form

$$E_1 = \{\forall v_i \in B^v_{\epsilon,} k > j \ , \mathcal{U}^{(j)},\mathcal{U}^{(j+1)} \subset B^v_\epsilon \ , \mathcal{U}^{(j)} \cap \mathcal{U}_i^{t_{v_i}\to t_{v_{i+1}}} \neq \emptyset \Rightarrow \exists \ \gamma : B^v_\epsilon \to B^v_\epsilon$$
$$\text{satisfying, for each } t_* \in [t_1,\cdots,t_j] \ , \gamma(t_*) \in \partial\, \mathcal{U}^{(j)}\} \ ,$$

for points along γ, whose interval of existence is at the most the duration of samples from the initial configuration before time evolution, lie exactly on the boundary, a second event

$$E_2 = \{ \text{ same conditions } \Rightarrow \exists \ \gamma : B^v_\epsilon \to B^v_\epsilon \text{ satisfying, for each } t_* \in [t_1,\cdots,t_k] \ , \gamma(t_*) \in \mathcal{U}^{(j)}\} \ ,$$

for points lying in the interior of $\mathcal{U}^{(j)}$. Finally, for points lying in neither one of the open sets, the third event is of the form

$$E_3 = \{ \text{ same conditions } \Rightarrow \exists \ \gamma : B^v_\epsilon \to B^v_\epsilon \text{ satisfying, for each } t_* \in [t_1,\cdots,t_l] \ , \gamma(t_*) \notin \mathcal{U}^{(j)}\} \ .$$

Apriori we observe that the intervals of existence for such paths γ need not be equal depending on the duration and magnitude of samples, hence the different intervals in which t_* exists. From such events, it is evident that because $\mathcal{C}^{h,v_i}_{t_1,\cdots,t_n} \geq 0$ for any collection of samples between $t_1,\cdots,t_n$. As a result, identifying

[1] To avoid confusion, also observe that $\mathcal{H}'$, in comparison to $\mathcal{H}$ at the beginning, includes terms with respect to the Ricci curvature symmetry, which are being enforced differently rather than only long and short range considerations for calculating the energy of turbulence. Another interest of future study is to determine which terms from each Hamiltonian are more dominant depending on the samples, and moreover, whether any Hamiltonian $\mathcal{H}'' = \mathcal{H}' + \mathcal{H}$ can be expressed, so as to enforce multiple symmetries at once.

subsets of the plane for which $\mathcal{C}^{h,v_i}_{t_1,\cdots,t_n} > 0$, with N sufficiently large, by taking the limit as $\mathcal{U}$ exhausts $\mathcal{U}^{t_{v_i}\rightarrow t_{v_{i+1}}}$ entails that we introduce as many subsets $\mathcal{U}^j_i \subset \mathcal{U}^{(j)}$ satisfying $\mathcal{U}^{(j)} \subseteq \mathcal{U}^{(j)}_i \subseteq \mathcal{U}^{t_{v_i}\rightarrow t_{v_{i+1}}}_i$ for each i,[2] hence making the sequence $\{\mathcal{U}^{(j)}_i\}_{i\in\mathbf{N}}$, for j fixed, of the largest length possible, and therefore maximal.

Naturally, as a quantity for measuring the coverage of distinct $\mathcal{U}^{(j)}_i$ in the sequence $\{\mathcal{U}^{(j)}_i\}$, if pairwise for each $i=j$ and $i+1=j+1$ there does not exist $x\in\mathbf{Z}^2 \subset \mathcal{U}^{(j+1)}\cap\mathcal{U}^{(j)}$, which empirically can be checked for each open set at $j+1$ of the sequence, with the sequence having total coverage

$$\mathcal{C}^{\mathcal{U}^{(j)}} = \sum_{j=2}^{n} \mathcal{C}^{h,v}_{t_1,\cdots,t_j} \in [0,+\infty) \ ,$$

for $\mathcal{U}^{(j)}$ disjoint. Otherwise, another instance of the formula involves pairwise $\mathcal{U}^{(j+1)}\cap\mathcal{U}^{(j)} \neq \emptyset$, with $|\mathcal{U}^{(j)}_j| = n$, in which the total coverage of the sequence is of the form,

$$\begin{aligned}\mathcal{C}^{\mathcal{U}^{(j)}} &= \mathcal{C}^{\{\mathcal{U}^{(j)}\}_{j\in\mathbf{N}:\mathcal{U}^{(j+1)}\cap\mathcal{U}^{(j)}=\emptyset}} + \mathcal{C}^{\{\mathcal{U}^{(j)}\}_{j\in\mathbf{N}:\mathcal{U}^{(j+1)}\cap\mathcal{U}^{(j)}\neq\emptyset}} \ , \\ &= \sum_{\text{disjoint}j\in\mathbf{N},n'<n:\exists\text{ a subsequence }\{t_{j_1},\cdots,t_{j_n}\},\text{ with }|\mathcal{U}^{(j_i)}|\leq n'} \mathcal{C}^{h,v_{j_i}}_{t_{j_1},\cdots,t_{j_n}} \\ &+ \sum_{j'\in\mathbf{N}\backslash(\text{disjoint}j),n''<n:\exists\text{ a subsequence satisfying, for }\{t'_{j_1},\cdots,t'_{j_n}\},|\mathcal{U}^{j'_i}|\leq n''} \mathcal{C}^{h,v'_{j_i}}_{t'_{j_1},\cdots,t'_{j_n}} \ ,\end{aligned}$$

where if disjoint open coverings $\mathcal{U}_i$ of $\mathcal{R}$ are achieved, $n'+n''=n$. Combinatorially, the arrangement of open sets corresponding to the magnitude of a random sample dictates how many pairwise, disjoint $\mathcal{U}^{(j)}$ and $\mathcal{U}^{(j+1)}$ can be found in the sequence. In passing to a limit along subsequences of $\{\mathcal{U}^{(j)}\}$, it is possible for the probability of the existence of a piecewise smooth path between open sets $\mathcal{U}^{(j)}$ and $\mathcal{U}^{(j+1)}$ to exist. To accommodate for such possible events, namely events which have probability

$$\begin{aligned}\{\forall j,j',k'\neq k''\in\mathbf{N}\ \exists\text{ subsequences }\mathcal{U}^{(j)},\mathcal{U}^{(j')}\subset\mathcal{U}^{(j)},\ \text{with the property that} \\ \text{given suitable } j,j'\ ,\ |\mathcal{U}^{(j)}|<k',|\mathcal{U}^{(j')}|<k'',\text{ given }k'+k''=n\} \ ,\end{aligned}$$

from occurring, where n is the total number of subsets $\mathcal{U}^{(j)}\subset\mathcal{U}$ for which there exists piecewise paths γ. However, if there exists a similar event, of the form,

$$\begin{aligned}\{\forall\text{ sequences }\mathcal{U}^{(j)},\mathcal{U}^{(j')},i\neq i'\in\mathbf{N},\exists\text{ unique subsequences }\mathcal{U}^{(j)}_i\subset\mathcal{U}^j,\mathcal{U}^{(j)}_{i'}\subset\mathcal{U}^{(j')} \\ \Rightarrow |\mathcal{U}^{(j)}_i|<k'_1\leq k',|\mathcal{U}^{(j')}_{i'}|<k''_1\leq k''\} \ ,\end{aligned}$$

for which it is possible to obtain longer sequences of open sets, by deleting open sets in the sequence for open sets from which piecewise paths cannot be constructed. These events, of the form

$$\begin{aligned}\{\text{for sequences of open sets }\mathcal{U}^{(j)},\mathcal{U}^{(j')}\text{ and subsequences }\mathcal{U}^{(j)}_i,\mathcal{U}^{(j')}_{i'},j\text{ arbitrary,} \\ \exists\ \epsilon_1,\epsilon_2\in\mathbf{N},\ \text{open sets }\mathcal{V}_{ij}\subset\mathcal{U}^{(j)}_i,\mathcal{V}_{i'j'}\subset\mathcal{U}^{(j')}_{i'},\Rightarrow\ \text{either} \\ |\mathcal{U}^{(j)}_i\backslash\mathcal{V}_{ij}|<\epsilon_1\leq k',\ \text{and/or,}\ |\mathcal{U}^{(j')}_{i'}\backslash\mathcal{V}_{i'j'}|<\epsilon_2\leq k''\} \ ,\end{aligned}$$

[2]If at any point either the lower, or upper set equality is satisfied, the process of constructing such paths ends.

To combat such configurations from arising after random sampling, an exponential penalty of the form below is enforced

$$\mathbf{P}_W[\exists \mathcal{X} \sim \mathcal{N}(0, s-t), s > t \in \mathbf{N} : W(t_k) = \mathcal{X}] = 1 - \exp((t_i - t_j)^2) \text{ , for } t_i < t_j \text{ ,}$$

so as to more directly influence the types of configurations that are the most likely to be sampled. For different subsets of the plane over which samples are collected, a systematic search along the set of paths, generated by gluing edges of $(1,0)^T$ and $(0,1)^T$ together, given that the emerging path of edges $\{e_i\}$, specified by the starting point chosen uniformly at random, either for any $x \in \mathcal{U}^{(j+1)} \backslash \mathcal{U}^{(j)}$, or for any $x \in \mathcal{U}^{(j)}$ but $x \notin \mathcal{U}^{(j+1)}$. At the point for which we encounter such x, the sequence of open sets will have terminated. As $h \to 0^+$, the qubit assignment φ will fail to assign excited, turbulent states on larger scales, and will instead assign more turbulent configurations on shorter scales.

6 motivation for future work: leveraging the random topology towards computing the probability that random, chaotically driven paths in Riemannian manifolds are sensitive to perturbations in initial conditions

For the final goal, one clear motivation for studying such piecewise paths in the lattice arises from being able to characterize the construction of such paths based on turbulent flows. Depending on the value of h that is enforced as the random height, our construction, over large collections of samples collected in parallel that must be collected to evaluate $\mathcal{H}$ for each additional step in the time evolution, would have large coverage $\mathcal{C}$ in such nonempty subsets of the plane. For this randomly generated topology, studying discrepancies in the coverage, dependent on the position within the plane, can be of further use in optimal qubit assignment and ansatzae formation, as I want to further build on ansatzae that I have provided. [3]

7 brief thoughts, questions

- I am planning to further my programming, based on these ideas, to compute the quantities of interest that I have introduced
- It would be convenient to determine the number of samples, for which drawing samples past this threshold does not significantly impact one's ability to compress the sample space of classical observations when passing onto qubits, or of the ability to correlate random sequences of motion, which albeit collected at random, can be regarded are somewhat equivalent
- Run more simulations for modifications to $\mathcal{H}$, with the aim of maximizing coverage of the Hilbert space as mentioned in the paper that Thomas posted
- As of this very moment, this PDF already outlines several approaches that I will continue to implement for a demo for randomly generated fluid flow

[3] For a more thorough analysis, I am thinking about writing down some group action $\rho : G \times |\{t_1, \cdots, t_k\}| \to$, from which all integer multiples of difference in frequencies can be generated. From higher and lower differences in frequencies that can be continuously adjusted for, ρ, well defined for representative frequencies k_1 and k_2, maps the spatial variables to images $\rho(k_1)$ and $\rho(k_2)$, so that for particular choices of k_1 and k_2 the difference $k_1 - k_2$, if a multiple of an integer, can be used to analyze when different paths of fluid flow, when in different positions, can be nevertheless be temporally related by the direction and magnitude of their respective trajectories.