Modeling exponential decay in maximum capacitance across specified flight patterns in small aircraft

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background

Hogge et al, **2015** describes

- experimental data gathered for a specific aircraft,
- predictions for future flights depending on whether a state of charge (SOC) threshold is satisfied,
- ingredients for a prognostics algorithm which generates robust predictions for the change in battery discharge across a sequence of flights

joint work with C. Kulkarni to further develop upon methods in the Hogge et al paper

motivation

to study the optimal schedule of maintenance for different aircraft, we employ strategies with the goal of determining

- distinct flight patterns which are dependent on the duration of each stage of the flight in addition to the magnitude of current exerted,
- polynomial approximations to sinusoidal, nonconstant current versus time flight data through computational routines to continuously approximate readings,
- how to generate flight predictions from given data by computing the MSE of Fourier cosine and sine coefficients, introduced through approximation methods that will be applied to previous recordings

defining the exponentially decaying, temporally weighed factor for computations of the decay in $\mathcal{C}_{\max}$

to accurately reflect upon the rate at which the maximum capacitance in an engine decays across different points in a specified sequence of flights, or even within a particular flight, we introduce

- a mapping Φ whose images satisfy $\Phi(\beta_i) < \Phi(\beta_j)$, for $\beta_i < \beta_j \Leftrightarrow |t_i - t_j| < |t_i' - t_j'|$,
- an exponential factor, of the form

$$\mathcal{C}_{\max_{i+1}} = e^{-\sum_i \beta_i |t^{(i)}_{\mathrm{final}} - t^{(i)}_{\mathrm{initial}}|} \mathcal{C}_{\max_i} \ ,$$

to keep track of the rate at which the maximum capacitance of the battery in the engine is negatively impacted with further use, from flight cycle i to flight $i+1$

Table 1

Case	Parameters	$C_{\max}$ on flight 2
1	$\beta_1 = \frac{1}{500}, \beta_2 = \frac{1}{5000}$	$\approx 0.73184C_1$
2	$\beta_1 = \frac{1}{100}, \beta_2 = \frac{1}{5000}$	$\approx 0.43509C_1$
3	$\beta_1 = \frac{1}{2500} = \beta_2$	$\approx 0.67686C_1$
4	$\beta_1 = \frac{1}{25000} = \beta_2$	$\approx 0.99610C_1$

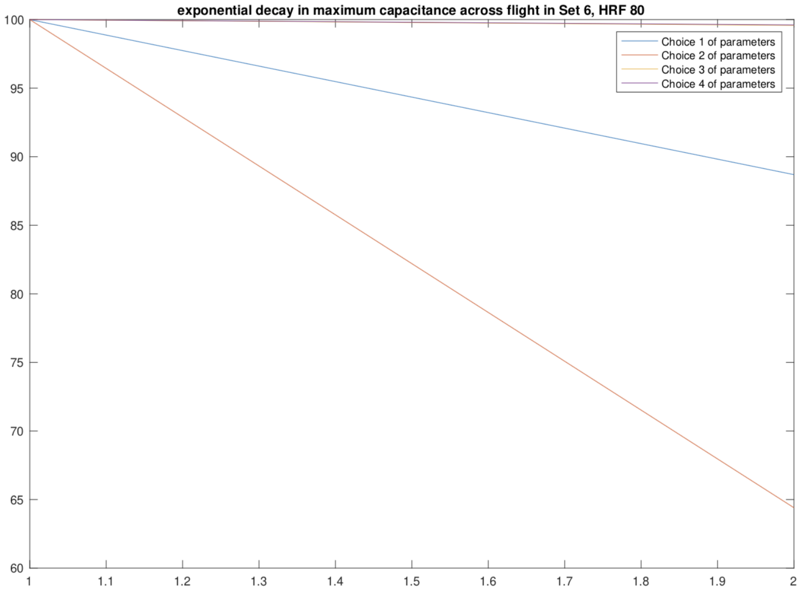
decay of $\mathcal{C}_{\max}$

stage number	duration of stage, in seconds
1	≈ 80
2	≈ 200
3	≈ 380

β value stages 1,2	% change in $\mathcal{C}_{\max}$
$\frac{1}{1800}, \frac{1}{1500}, \frac{1}{400}$	≈ 0.320683
$\frac{1}{2000}, \frac{1}{1800}, \frac{1}{600}$	≈ 0.4563732

generation of combinatorial space of flight outcomes

flight plan type	parasitic load classification	t_{init}, β
1	exp	$\approx 270, \frac{1}{5000}$
2	exp	$\approx 530, \frac{1}{8000}$
3	exp	$\approx 670, \frac{1}{3500}$



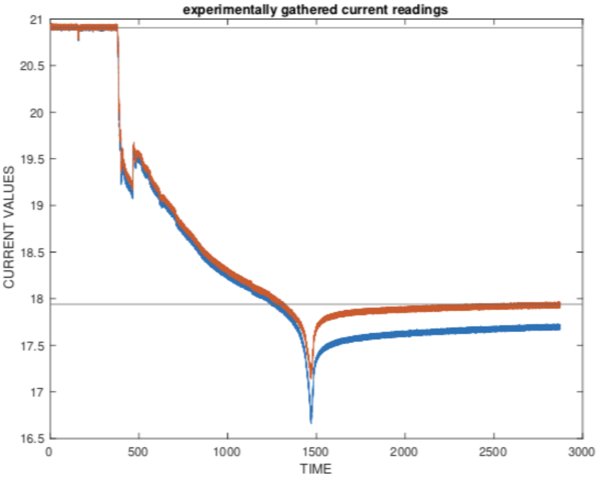
exponential decay in maximum capacitance across flight in Set 6, HRF 80
Choice 1 of parameters
Choice 2 of parameters
Choice 3 of parameters
Choice 4 of parameters
100
95
90
85
80
75
70
65
60
1
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
1.9
2

auxiliary algorithm

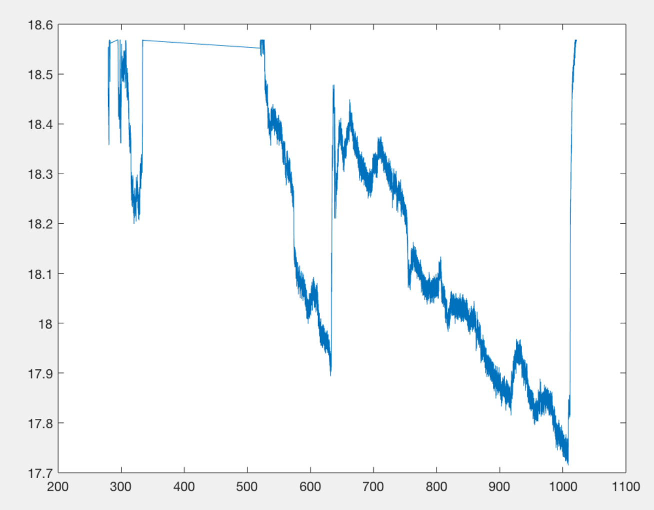
to more easily obtain the approximate time intervals during each stage of a given flight, we run an auxiliary algorithm, denoted for flights of multiple types, where the type classification is inclusive of the stage of the flight at which failure can occur, by:

- determining the value at which the current versus time readings "levels off" at in a given flight pattern,
- introducing a threshold parameter from which linear inspection of the current versus time readings can determine all admissible currents, and accompanying times, which meet the criterion that is uniquely determined from the asymptotic current in each flight,
- substituting in approximate time durations for each stage of the flight into the power of the exponential, to qualitatively measure an updated $\mathcal{C}_{\max}$ value

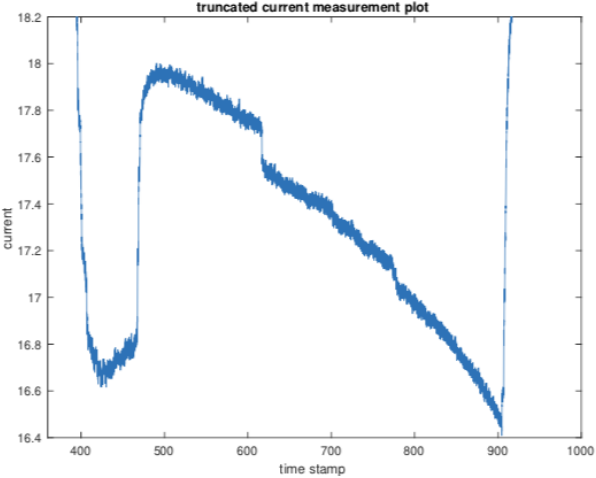
asymptotic current cutoff



Type 1 flight



isolating portions of a flight with the auxiliary algorithm



500 *time stamp flight segment*

obtaining smooth polynomial approximations to flight data

once estimates for the decay of $\mathcal{C}_{\max}$ have been obtained with suitable β_i, a polynomial root finding method returns smooth approximations to flight data by

- shifting the current versus time readings towards the t axis so that a polynomial is closely representative of flight data by subtracting all current measurements by real, fixed α,
- constructing a product of linear factors of the form $\prod_{i=1}^{n} k(x - r_i)$, where $k, r_i \in \mathbf{R}$,
- translating the approximation upwards past the t axis by the same amount that it was shifted below the axis

$$\boxed{\prod_{1}^{n} k(x - r_i) + \alpha}$$

Fourier series representation

with the polynomial approximation f,

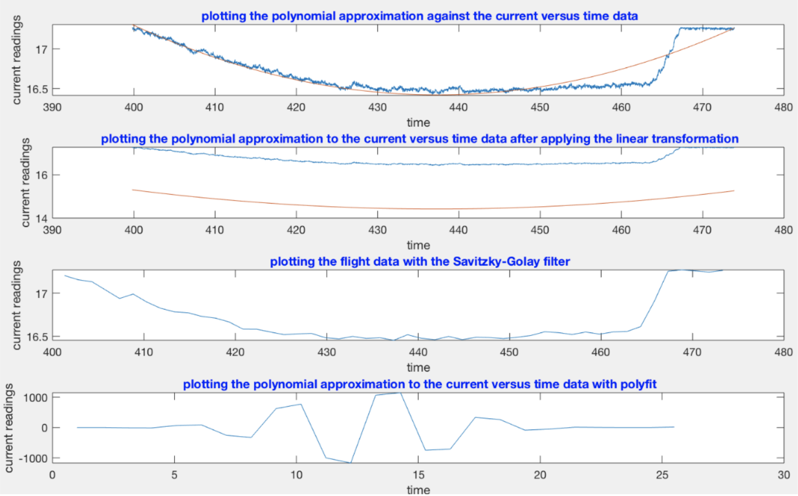
- Fourier sine and cosine coefficients of a series expansion can be determined,

$$\frac{1}{\sum_i t_i}\int_{\mathcal{T}_i} f(t)\cos\left(\frac{n\omega_0 t}{\mathcal{T}_i}\right)\mathrm{d}t\ ,\frac{1}{\sum_i t_i}\int_{\mathcal{T}_i} f(t)\sin\left(\frac{n\omega_0 t}{\mathcal{T}_i}\right)\mathrm{d}t\ ,$$

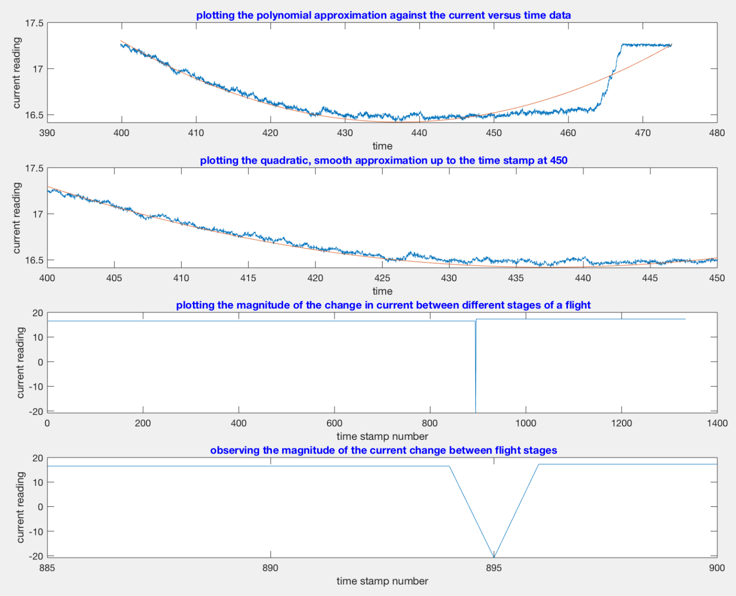
  with the exception that each coefficient is normalized by the duration of the stage of the flight, $\mathcal{T}_i$ is the specified stage of the flight, $\mathcal{M}$ is the number of Fourier modes in the series, $n, t \in \mathbf{N}$, $\omega_0 = 2\pi$

- MSE can also be computed, to assess reliability of the capacitance decay, and capacity, for future trials

approximation approaches



polynomial approximation



dependence of the MSE on the number of Fourier modes

to generate different quantitative behaviors in the MSE, we

- determine whether Fourier cosine, sine, or combination of both the sine and cosine coefficients that will be included in the Fourier series representation of the flight profile,
- analyze the numerical behavior of the Fourier coefficients over different ω_0, for 200, $2,000$ and $10,000$ modes,

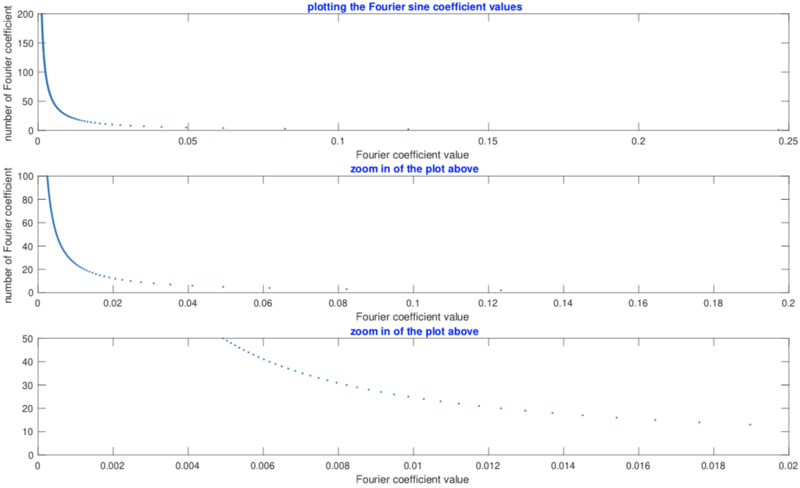
Fourier cosine coefficient behavior

Fourier modes	Max.	Min.
200	≈ 0.3139	$\approx 7.928 \times 10^{-6}$
$2,000$	≈ 0.3139	$\approx 7.857 \times 10^{-8}$

Fourier modes	MSE
200	$\approx 0.192276077317043$

incorporating only the cosine coefficients in the profile approximation numerically demonstrates that the magnitude of these coefficients monotonically decreases to 0 *in the limit of infinite Fourier modes*

asymptotics about the origin in the infinite mode limit



tuning steepness of maximum capacitance decay

- from conditions placed on the terms to which we exponentiate, we produce capacitance decay that is proportional with respect to observables that can be easily gathered experimentally,
- while simultaneously being able to reduce dependence on lab measurements of capacitance degradation, exhibiting a range of parameters over which
    - the curvature of capacitance degradation curves can be tuned in accordance to the magnitude of current exerted over an isolated stage of the flight,
    - the critical quantity $\mathcal{F}_{\mathrm{crit}}$ beyond which a decrease of the MSE within an arbitrary tolerance $\epsilon > 0$ is observed,
    - room for additional work exists through generalizations of our approach to other aircraft

computations of state of charge for polynomial and exponential parasitic loads

$$\mathrm{SOC}_1 = 1 - \frac{q_{\max} - q_b}{e^{\sum_i -\beta_i |t^{(i)}_{\text{initial}} - t^{(i)}_{\text{final}}|} C_{\max}} \ ,$$

$$\mathrm{SOC}_2 = 1 - \frac{q_{\max} - q_b}{\left(e^{\sum_i -\beta_i |t^{(i)}_{\text{initial}} - t^{(i)}_{\text{final}}|} \times e^{\beta^{\mathrm{PL}, \mathcal{T}^{(i)}}_{\mathcal{T}^{(i)}_{\mathrm{PL}}} |{t^{PL}}_{\text{final}} - {t^{PL}}_{\text{initial}}|} \right) C_{\max}} \ ,$$

$$\mathrm{SOC}_3 = 1 - \frac{q_{\max} - q_b}{\left(1 + (\beta^{\mathrm{PL}, \mathcal{T}^{(i)}}_{\mathcal{T}^{(i)}_{\mathrm{PL}}} |{t^{PL}}_{\text{final}} - {t^{PL}}_{\text{initial}}) {t^{PL}}_{\text{final}} + (\beta^{\mathrm{PL}, \mathcal{T}^{(i)}}_{\mathcal{T}^{(i)}_{\mathrm{PL}}} |{t^{PL}}_{\text{final}} - {t^{PL}}_{\text{initial}}) \frac{({t^{PL}}_{\text{final}})^2}{2} + \cdots \right) C_{\max}} \ ,$$

future goals

in the future, we are interested in

- applying machine learning models from available prognostics data sets,
- generalizing portions of the code for more robust prediction generation