

An Introduction to Bond Graph Modelling

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Goals of this talk

My first aim is to convince you that:

- ▶ Bond graphs are a ‘natural’ way to model energy distribution across networks
- ▶ Bond graphs are very useful for modelling multi-domain systems.

Secondly, I will introduce some basic bond graph components, particular those for

- ▶ distribution,
- ▶ storage, and
- ▶ dissipation.

The Fundamental Law: Conservation of Energy

There is a fact, or if you wish, a law, governing all natural phenomena that are known to date. There is no known exception to this law—it is exact so far as we know. The law is called the conservation of energy.

- Richard Feynman, 1961 [1].

Energy Networks

Bond Graphs model systems that:

- ▶ use energy as the core currency.
- ▶ can be decomposed into functionally discrete subsystems.
- ▶ transfer of energy between subsystems.

Hence Bond Graphs have a network structure!

Implied Network Structure

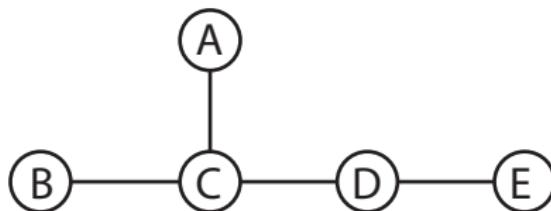
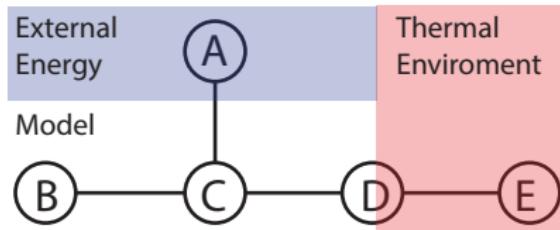


Figure: An Energy Network

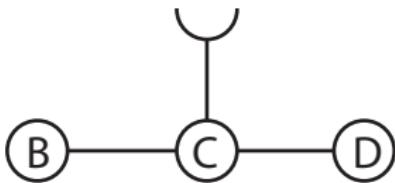
Nodes represent energetic subsystems.

Edges represent transfer of energy between subsystems.

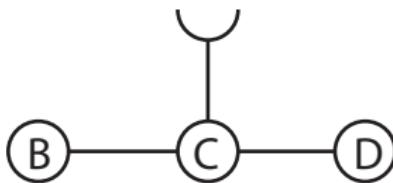
Open Systems



is modelled as



Power



Power is exchanged between subsystems B, C, D .

Power is represented in terms of *effort* e and *flow* f such that

$$\text{Power} = \text{effort} \times \text{flow} \quad \iff \quad P = ef$$

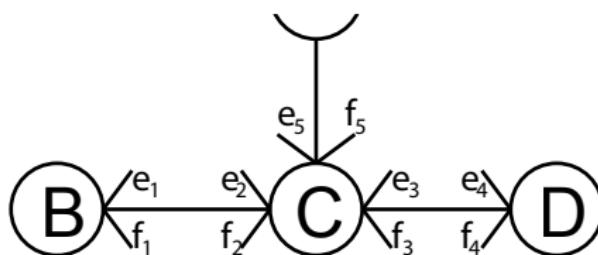
Domain Specific Power

Domain	q	$f = \dot{q}$	p	$e = \dot{p}$
Translational Mechanics	position	velocity	momentum	force
Rotational Mechanics	angle	angular velocity	angular momentum	torque
Electronics	charge	current	flux linkage	voltage
Hydraulics	volume	flow	pressure momentum	pressure
Thermo-dynamics	entropy	entropy flow	temperature momentum	temperature
Chemistry	moles	molar flow	chemical momentum	chemical potential

Figure: State space (q, p) and phase space (e, f) variables [2, 3].

Network Structure with 2-variable Edges

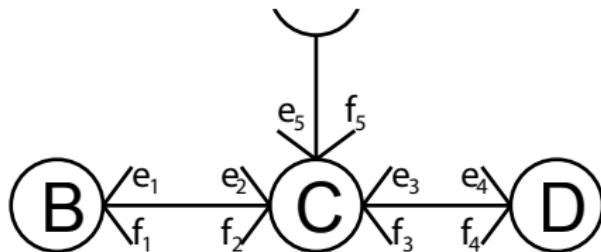
The power transferred into B is given by $e_1 \times f_1$.



Hence the energetic behaviour of the B subsystem can be specified by $\Phi_B(e_1, f_1) = 0$. Similarly for the C and D subsystems.

Φ_B describes some kind of physical process like energy storage or dissipation.

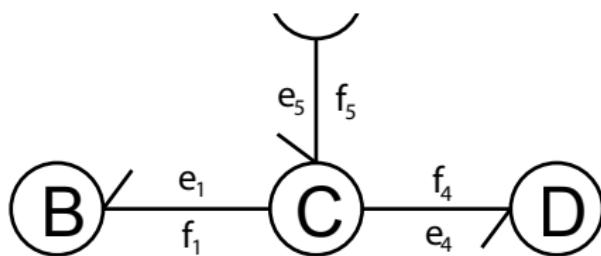
Directed Network Structure



Conservation of Energy must hold across edges, hence

$$e_1 f_1 + e_2 f_2 = 0, \quad e_3 f_3 + e_4 f_4 = 0.$$

Directed Network Structure

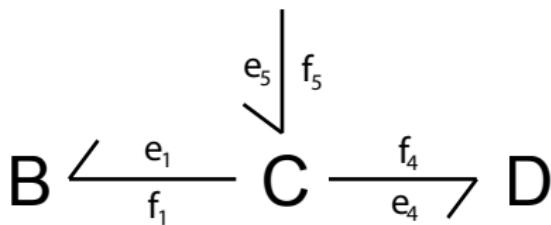


We choose a sign, and denote the direction of positive f by a half-arrow. Hence

$$e_2 = e_1, \quad f_2 = -f_1, \quad e_3 = e_4, \quad f_3 = -f_4,$$

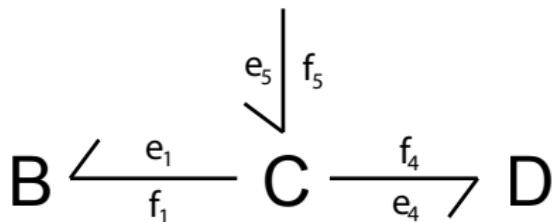
which are used to update the co-ordinates of Φ_B , Φ_C and Φ_D .

Directed Network Structure



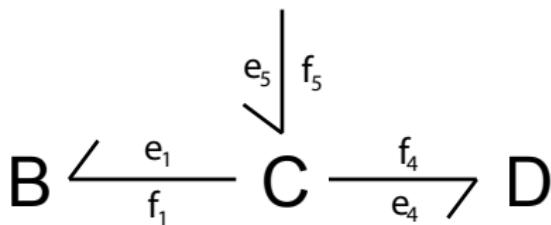
Discard the extra circles, we get an acasual bond graph.

Key Features of a Bond Graph



- ▶ Subsystem defined by constitutive relations $\Phi_B(e_1, f_1) = 0$ etc.
- ▶ Bonds show *shared variables* through which power is balanced.
- ▶ Arrows indicate positive f (sign convention).

Strengths and Weaknesses

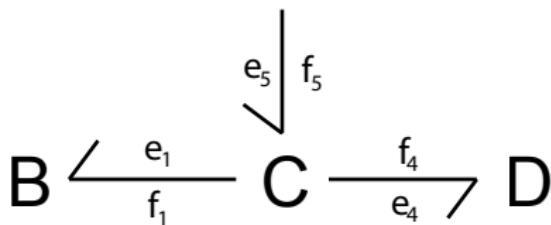


Conservation laws plus networked structure implies that:

A bond graph is physical iff it's subsystems are physical.

Caveat: Modellers must take care when constructing Φ .

Strengths and Weaknesses



No explicitly defined physical domain implies that:

Bond graphs are useful for cross-domain modelling.

Caveat: No domain specific encoding.

Part II: Subsystem Relations

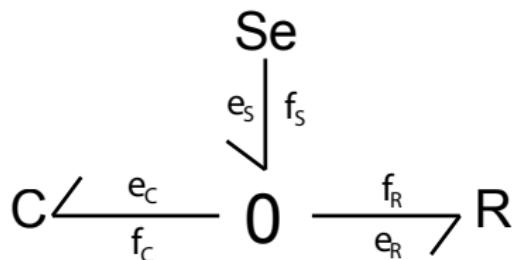


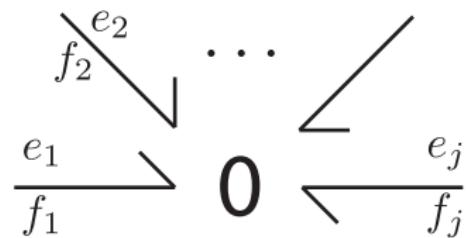
Figure: A simple bond graph

We wish to use this bond graph of as a simple example of how to generate model equations.

We need the constitutive relations for the components.

0-Junction

The 0-Junction, or common effort junction, conservatively balances power across many bonds, with equal effort.



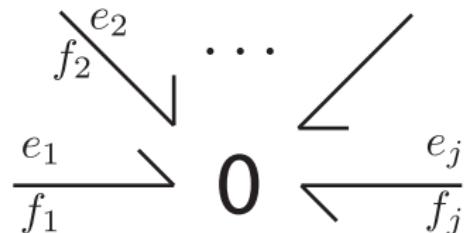
Conservation of Energy implies $\sum_{i=1}^j e_i f_i = 0$.

Equal effort implies $e_1 = e_2 = \dots = e_j$, hence $\sum_{i=1}^j f_i = 0$.

This is Kirchoff's Current Law!

0-Junction

The 0-Junction, or common effort junction, conservatively balances power across many bonds, with equal effort.



$$\Phi_0(e_1, f_1, e_2, f_2, \dots, e_j, f_j) = \begin{pmatrix} e_1 - e_2 \\ \cdots \\ e_{j-1} - e_j \\ f_1 + f_2 + \dots + f_j \end{pmatrix} = 0$$

R component

The R component dissipates power proportionally to f^2 (or e^2).

$$\frac{e}{f} \searrow R$$

The constitutive relation is given by

$$\Phi_R(e, f) = e - Rf = 0.$$

This is both Ohms law and Newtonian friction.

C component

The C component stores potential energy.

$$\frac{e}{f} \searrow \mathbf{C}$$

$$\Phi_C(e, f) = Ce - \int f dt = 0$$

This is Hooke's law and the ideal capacitor relation.

Se component

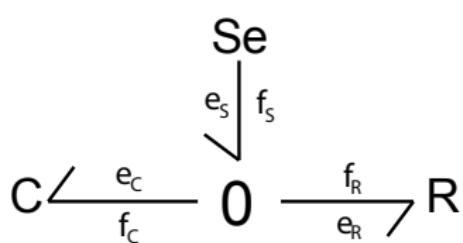
The ideal effort source provides power into the system.

$$\frac{e}{f} \searrow \text{Se}$$

Here f is allowed to vary freely and effort is specified by a control variable u

$$\Phi_{\text{Se}}(e, f) = e - u.$$

Putting it Together



$$\Phi_C = Ce_C - \int f_C dt \quad (1)$$

$$\Phi_R = e_R - Rf_R \quad (2)$$

$$\Phi_{Se} = e_S - u \quad (3)$$

$$\Phi_0 = \begin{pmatrix} e_S - e_C \\ e_S - e_R \\ f_S + (-f_R) + (-f_R) \end{pmatrix} \quad (4)$$

Goal: Find f_S .

Notice that (3) and (4) give $u = e_S = e_R = e_C$ which when combined with (1) and (2) gives $f_C = C^{-1}\dot{u}$ and $f_R = R^{-1}u$ respectively.

The last equation of (4) then gives our result:

$$f_S = \left(\frac{1}{R} + \frac{1}{C} \frac{d}{dt} \right) u.$$

In Review

Hopefully I've convinced you that bond graphs :

- ▶ 'naturally' model energy distribution across networks
- ▶ are very useful for modelling multi-domain systems.
- ▶ conserve energy when connecting network subsystems.

Network subsystems can be:

- ▶ derived from first principals,
- ▶ defined phenomenologically,
- ▶ constructed hierarchically.

Thanks

Thanks to

- ▶ Prof. Edmund Crampin and the Systems Biology Lab at UniMelb.
- ▶ Andre, the CellML workshop organisers, and the Auckland Bioengineering Institute.
- ▶ Others?

Resources

These slides, and more, can be found at
<https://github.com/peter-cudmore/>

References

-  Feynman, Richard. *The Feynman Lectures on Physics; Volume 1*. U.S.A: Addison Wesley, 1964. ISBN 0-201-02115-3.
-  Karnopp, Dean C. and Margolis Donald L. and Rosenberg, Ronald C. *System Dynamics: a Unified Approach*, Wiley, New York, 1990.
-  Baez, John. *This Weeks Finds in Mathematical Physics (Week 289)*. 2010.
<http://math.ucr.edu/home/baez/week289.html>