4.) Simulations with ridge.

Let $\beta \in R^p$ and let x, y be random variables such that the entries of x are i.i.d. Rademacher random variables, and $y = \beta' x + \varepsilon$ where $\varepsilon \sim N(0,1)$

(a) Show for any function of form $f(u) = u'\alpha$ for all $u \in R^p$:

$$E_{x,y}[(f(x) - y)^2] = 1 + \|\alpha - \beta\|_2^2 = R(f) \text{ ... the risk of } f.$$

$$\begin{aligned}
\mathbb{E}[(f(x) - y)^2] &= \mathbb{E}[(f(x) + \mathbb{E}[y] - \mathbb{E}[y] - y)^2] \\
&= \mathbb{E}[f(x) - \mathbb{E}[y]]^2 + \mathbb{E}[y - \mathbb{E}[y]]^2 \\
&= (f(x) - \mathbb{E}[y])^2 + \mathbb{E}[y - \mathbb{E}[y]]^2 \\
&= \text{Bias}^2(y) + \text{Var}(y)
\end{aligned}$$

$\Rightarrow$ Variance of Rademacher R.V. is 1

$\Rightarrow$ Bias of $\alpha' x$ and $\beta' x$ is $\|\alpha - \beta\|_2$ ... the euclidean distance between α and β

because $x \in \pm 1$ $\quad = \|\beta - \alpha\|_2$

Thus ... $\mathbb{E}_{x,y}[(f(x) - y)^2] = 1 + \|\alpha - \beta\|_2^2$ ✓