主題: Eulerian Circuit

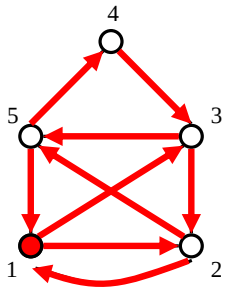
- 基礎
- 應用
- 作業與自我挑戰

基礎

- Eulerian circuit
- Extend to directed graphs

Eulerian circuit

- 給一個 undirected graph (allowing multiple edges)，一筆劃走完該 graph 所有 edges 的走法稱為 Eulerian path
- 一筆劃走完而且回到出發點的走法稱為 Eulerian circuit



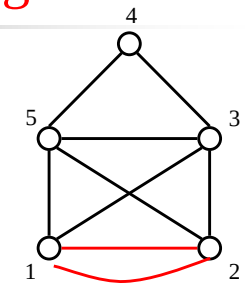
1325435121 is an Eulerian circuit

Property

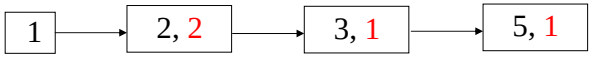
- 假設 graph 是 connected
- 一個一筆劃走完的路徑，除了起點與終點之外，所有點的 degree 必為偶數
- 若沒有 odd degree 的 vertex，則由任一點 x 出發可以造一條 Eulerian circuit 回到 x
- 若有 2 個 odd degree 的 vertices x 和 y，則可以造一條由 x 到 y 的 Eulerian path

Handling multiple edges

- Method 1. use adjacency-matrix
  - A[1, 2] = 2



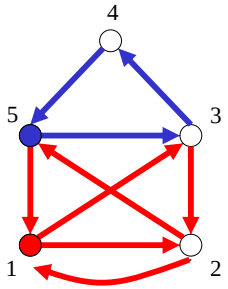
- Method 2. use adjacency-lists

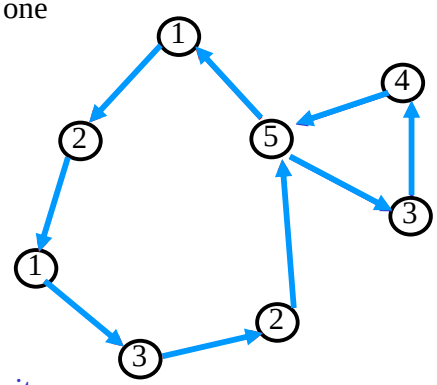


Circuit Construction

Step 1. Find small cycles one by one

Step 2. Merge small cycles into one

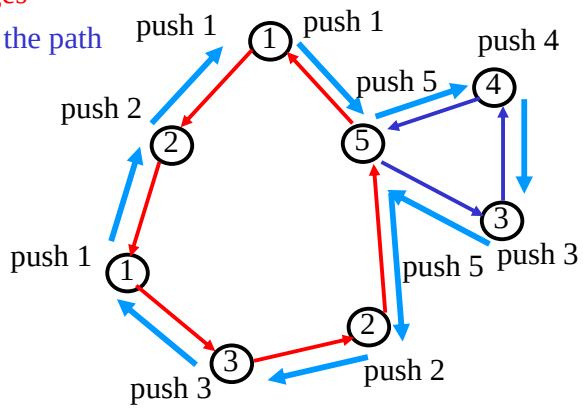




1213253451 is an Eulerian circuit

How to do merging efficiently?

- Perform DFS on "edges"
- Use a stack S to keep the path



Pseudo code

```
void find_circuit(int i)
{  int j;
   if (degree[i] == 0) { push(S, i); return; }

   for (j = 0; j < n; j++) //檢查所有的 neighbors,
   {  if (A[i][j] > 0) {  // 至少存在一條 edge (i, j)
            A[i][j]--, A[j][i]--;  // 拿掉一根 (i, j) (j, i) edge
            degree[i]--, degree[j]--;  // degree 減 1
            find_circuit(j);
         }
   }
   push(S, i);
}
```

Solution

- Assume that an adjacency matrix is used.
- Step 1. 判斷 graph 是否相連: $O(n^2)$
  - DFS
- Step 2. 為每個點計算 degree 數: $O(n^2)$
- Step 3. find_circuit: O(mn)
- Time: O(mn)
  - can be improved into O(m) by using adjacency-lists

Extend to directed graphs

- 存在 Eulerian path
  - 只有一個點的 out-degree 比 in-degree 大 1 (出發點)
  - 且只有一個點的 in-degree 比 out-degree 大 1 (終點)
- 存在 Eulerian circuit
  - 所有點的 in-degree 與 out-degree 皆相同
  - 由任一點 x 出發可以造一條 Eulerian circuit 回到 x
- modification of find-circuit
  - while going from i to j, only an edge (i, j) is removed

應用

- 應用一: Smallest Eulerian circuit
- 應用二: A.10054 The necklace

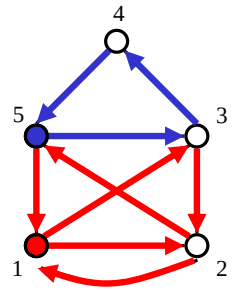
應用一: Smallest Eulerian circuit

- 給一個 undirected graph
- 輸出 lexicographical order 最小的一個 Eulerian circuit
  - 如: 1213254351 < 1325435121
- $|V| \leq 44$, $|E| \leq 1995$

Solution

- Problem: How to find the smallest one?
  - 由 i 向外走時，id 最小的優先
    - adjacency matrix
      - for j = 0, 1, ..., n-1
    - adjacency-lists
      - list 中的 nodes 按 id 大小順序排列
  - Start from vertex 1

Example



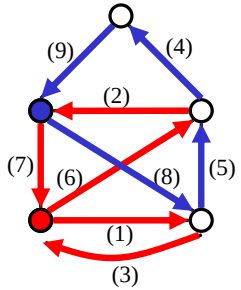
1213253451 is the smallest Eulerian circuit

類題: A.302 John's trip

- 給一個 undirected graph
  - 指定一個起點
  - 每條 edge 有編號
  - 以 edge sequence 表示 Eulerian circuit
    - 如: 136285497 表示 $e_1e_3e_6e_2e_8e_5e_4e_9e_7$
- 輸出 lexicographical order 最小的一個 Eulerian circuit
- |V| ≤ 44, |E| ≤ 1995

Solution

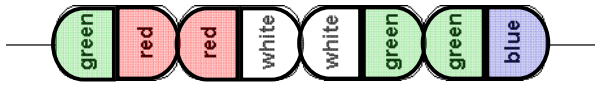
- 由某個點向外走時，編號小的 edge 優先



136285497 is the smallest Eulerian circuit

應用二: A.10054 The necklace

- 有 n 個長橢圓珠，每顆珠子的兩半端各有一個顏色

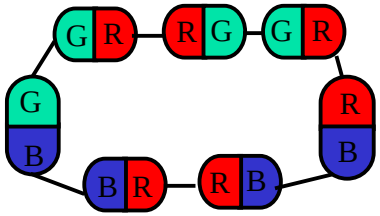


- 請問，是否可以把這些珠子串成一條項鍊，這條項鍊須滿足「相鄰兩棵珠子接觸的兩端顏色相同」
- 如果可以，找出一個串法

- $5 \le n \le 1000, 1 \le color \le 50$

Example

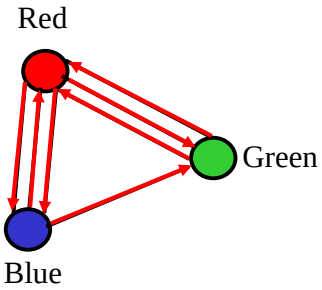
(Red, Green) × 3
(Red, Blue) × 3
(Blue, Green) × 1



Solution

Example: **(R, G)×3, (R, B)×3, (B, G)×1**

an undirected graph G



A Eulerian circuit:

(B, G)(G, R)(R, G)(G, R)(R, B)(B, R)(R, B)

作業與自我挑戰

- 作業
  - 練習題
    - A.302 John's Trip http://uva.onlinejudge.org/external/3/302.html
  - 挑戰題
    - A.10248 The Integer All-time Champ
      http://uva.onlinejudge.org/external/102/10248.html
- 其它有趣的題目
  - A.10506 Ouroboros
    http://uva.onlinejudge.org/external/105/10506.html
  - A.10129 Play on Words
    - http://uva.onlinejudge.org/external/101/10129.html
  - A.10596 Morning Walk
    - http://uva.onlinejudge.org/external/105/10596.html