MATH 3043, Numerical Analysis I
Fall 2018

Lab 2

This lab will have you implementing fixed-point iteration and Newton's method to approximate solutions for several problems.

Solutions must be submitted on Canvas and are due **September 24** at the beginning of lab. Please submit a single script file `Lab2Lastname.m` and the corresponding published file `Lab2Lastname.pdf` (for example, my submitted files would be `Lab2Zumbrum.m` and `Lab2Zumbrum.pdf`). Each solution should

- be contained in a separate cell which includes the problem number and short problem description,
- run independent of other cells,
- be adequately commented.

As part of your solution for each problem, output the number of iterations required, the approximation, and the error tolerance formatted using the `fprintf` function similar to the sample output below:

```
n:  12   p12:  1.234567890   |error|:  0.000000012
```

For a solution accurate to within 10^{-k}, include at least k digits in the approximation output. Unless otherwise noted, use the stopping criteria

$$\left|\frac{p_n - p_{n-1}}{p_n}\right| < \epsilon.$$

1. The following four fixed-point iteration methods are proposed to compute $21^{1/3}$:

   a. $p_n = \dfrac{20p_{n-1} + 21/p_{n-1}^2}{21}$

   b. $p_n = p_{n-1} - \dfrac{p_{n-1}^3 - 21}{3p_{n-1}^2}$

   c. $p_n = p_{n-1} - \dfrac{p_{n-1}^4 - 21p_{n-1}}{p_{n-1}^2 - 21}$

   d. $p_n = \left(\dfrac{21}{p_{n-1}}\right)^{1/2}$

   For each method, compute approximations for the value using $p_0 = 1$ and $\epsilon = 10^{-10}$ for the stopping criteria $|p_n - p_{n-1}| < \epsilon$. Rank the methods in order based on the apparent speed of convergence.

2. Using Newton's method, find an approximation to $\sqrt[3]{25}$ correct to within $\epsilon = 10^{-10}$. Compare the number of iterations required for Newton's method and the number of iterations required for the Bisection method used in Lab 1 Problem 4 to solve the same problem.

3. An object falling vertically through the air is subjected to viscous resistance as well as to the force of gravity. Assume that an object with mass m is dropped from a height s_0 and that the height of the object after t seconds is

$$s(t) = s_0 - \frac{mg}{k}t + \frac{m^2 g}{k^2}\left(1 - e^{-kt/m}\right),$$

where $g = 32.17$ ft/s^2 and k represents the coefficient of air resistance in lb/s. Suppose $s_0 =$ 300 ft, $m = 0.25$ lb, and $k = 0.1$ lb/s. Use Newton's method to find the time, accurate to within 10^{-5} s, it takes this object to hit the ground.

4. A drug administered to a patient produces a concentration in the bloodstream given by $c(t) = Ate^{-t/3}$ milligrams per milliliter, t hours after A units have been injected. The maximum safe concentration is 1 mg/mL. What amount should be injected to reach this maximum safe concentration, and when does this maximum occur? An additional amount of this drug is to be administered to the patient after the concentration falls to 0.25 mg/mL. Determine, to the nearest minute, when this second injection should be given.

5. Approximate the zero of the function $f(x) = x^2 - 2e^{-x}x + e^{-2x}$ to within 10^{-8} using Newton's method with $p_0 = 1$.