MATH 3043, Numerical Analysis I
Fall 2018

Lab 8

This lab will have you approximating the solution of an integral equation by implementing composite numerical integration methods and solving linear systems.

Solutions must be submitted on Canvas and are due **November 5** at the beginning of lab. Please submit a single script file `Lab8Lastname.m` and the corresponding published file `Lab8Lastname.pdf` (for example, my submitted files would be `Lab8Zumbrum.m` and `Lab8Zumbrum.pdf`). Each solution should

- be contained in a separate cell which includes the problem number and short problem description,
- run independent of other cells,
- be adequately commented.

1. A Fredholm integral equation of the second kind is an equation of the form

$$u(x) = f(x) + \int_a^b K(x,t)u(t)\,dt,$$

where a and b and the functions f and K are given. To approximate the function u on the interval $[a,b]$, a partition $x_0 = a < x_1 < \cdots < x_{m-1} < x_m = b$ is selected, and the equations

$$u(x_i) = f(x_i) + \int_a^b K(x_i,t)u(t)\,dt, \quad \text{for } i = 0,\dots,m,$$

are solved for $u(x_0), u(x_1), \dots, u(x_m)$. The integrals are approximated using quadrature formulas based on the nodes $x_0, \dots, x_m$. In our problem, $a = 0, b = 1, f(x) = x^2$, and $K(x,t) = e^{|x-t|}$.

(a) Set up and solve the linear system that results when the Composite Trapezoidal rule is used with $n = 4$.

(b) Repeat part (a) using the Composite Simpson's rule.

Note: Please make your code general enough that simply updating the value of n will produce approximations at the updated number of nodes.

Hint: Use the built-in function \ to solve linear systems (`x = A\b` can be used to solve the system $A\mathbf{x} = \mathbf{b}$); the built-in function `meshgrid` may be helpful for the evaluations of $K(x,t)$.