MATH 3043, Numerical Analysis I
Fall 2018

Lab 3

This lab will have you implementing alterations to fixed-point iteration and Newton's method to approximate solutions for several problems.

Solutions must be submitted on Canvas and are due **October 1** at the beginning of lab. Please submit a single script file `Lab3Lastname.m` and the corresponding published file `Lab3Lastname.pdf` (for example, my submitted files would be `Lab3Zumbrum.m` and `Lab3Zumbrum.pdf`). Each solution should

- be contained in a separate cell which includes the problem number and short problem description,
- run independent of other cells,
- be adequately commented.

As part of your solution for each problem, output the number of iterations required, the approximation, and the error tolerance formatted using the `fprintf` function similar to the sample output below:

```
n:  12   p12:  1.234567890   |error|:  0.000000012
```

For a solution accurate to within 10^{-k}, include at least k digits in the approximation output. Unless otherwise noted, use the stopping criteria

$$\left|\frac{p_n - p_{n-1}}{p_n}\right| < \epsilon.$$

1. Plot the graph of $f(x) = x^2|\sin x| - 4$ for $x \in [0, 4]$. Use the Secant method to find the smallest positive zero accurate to within 10^{-6} using $p_0 = 3.6$ and $p_1 = 3.7$. [What happens if $p_0 = 2.8$ and $p_1 = 2.9$ are used?]
2. Use Newton's method to approximate the zero of the function $f(x) = x^2 - 2e^{-x}x + e^{-2x}$ accurate to within 10^{-8} using $p_0 = 1$. [What do you notice about the convergence of Newton's method?] Repeat the problem using the modified Newton's method and compare the number of iterations required for both methods.
3. Plot the graph of $f(x) = x^3 - 12.42x^2 + 50.444x - 66.552$ for $x \in [4, 6]$ and use Newton's method to find all zeros accurate to within 10^{-8} on this interval. [How are the convergence of the method, the multiplicity of the zeros, and the values of the derivative at the zeros related?]
4. Use fixed-point iteration to approximate the solution of $x = 5^{-x}$ accurate to within 10^{-8} using $p_0 = 0.5$. Repeat the problem using Steffenson's method and compare the number of iterations required for both methods.