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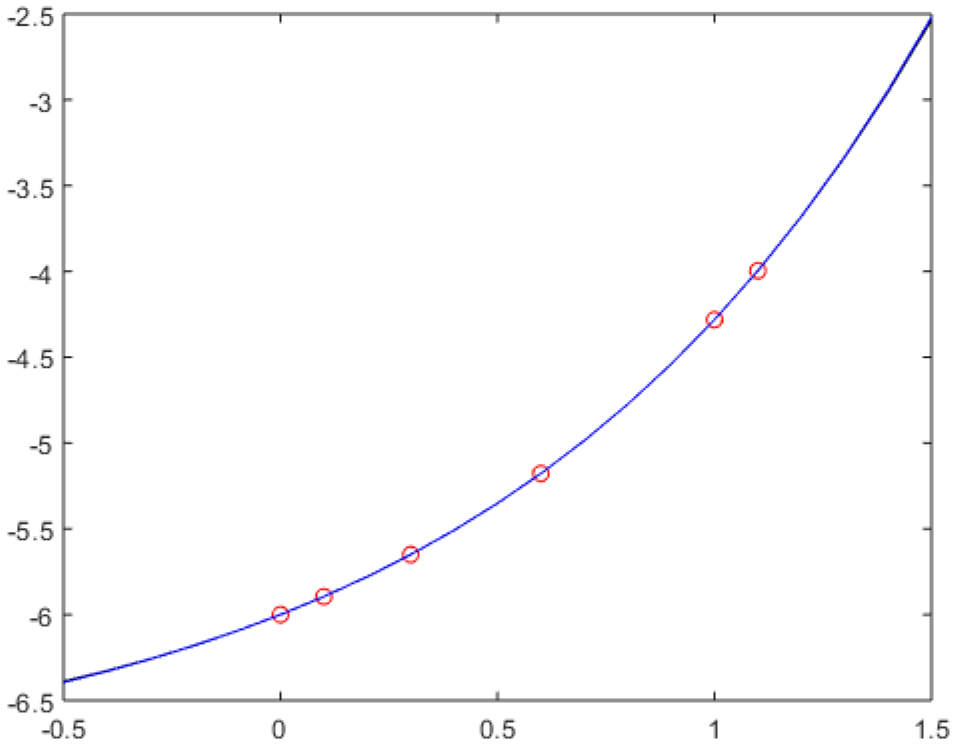
Problem 1

Use divided differences to construct the interpolating polynomial of degree four for the following data:

```
%       x   | f(x)
%       ---------------
%       0.0 | -6.00000
%       0.1 | -5.89483
%       0.3 | -5.65014
%       0.6 | -5.17788
%       1.0 | -4.28172

fprintf('Problem 1\n')

% create input and output data vectors
inData = [0.0, 0.1, 0.3, 0.6, 1.0];
outData = [-6.00000, -5.89483, -5.65014, -5.17788, -4.28172];

%Intialized divided differences matrix
inFij = InitializeFij(outData);

% Generate divided differences matrix
startRow = 2;
inFij = GenerateFij(inData, inFij, startRow);

% Create set of xVals in -0.5..1.5
xVals = [-0.5:0.1:1.5];
% Evaluate the interpolating polynomial at each xVals
yVals1a = CalcInterPoly(inFij, inData, xVals);

% Plot original data points and the interpolating polynomial on same
 graph
figure(1)
plot(inData, outData, 'ro')
hold on
plot(xVals, yVals1a, 'k-')
hold off

% 1b) Calculate f(0.9) using interpolating polynomial
```

```
inVec = [0.9];
fof0point9 = CalcInterPoly(inFij, inData, inVec);
fprintf('Using the interpolating polynomial, f(0.9) is approximately
 %.6f\n', fof0point9)

% Problem 1c

% Add f(1.1) = -3.99583 to data set
% Define vector for additional input data
addInData = [1.1];
% Update inData vector with additional input data vector
inData = [inData, addInData];
% Define vector for additional output data
addOutData = [-3.99583];
% Update inData vector with additional output data vector
outData = [outData, addOutData];

% get size of additional output data vector
sizeOfAddOutData = length(addOutData);

% get size of Fij
[row, col] = size(inFij);

% append the additional output data to the end of Fij
inFij(row+1:1:row + sizeOfAddOutData,1) = [addOutData]';

%Fill in Fij for additional Data
startRow = row + 1;
inFij = GenerateFij(inData, inFij, startRow);

% Calculate new yVals based on new interpolating polynomial
yVals1c = CalcInterPoly(inFij, inData, xVals);

% Plot origdata points and both interpolating polynomials on same
 graph
figure(2)
plot(inData, outData, 'ro')
hold on
plot(xVals, yVals1a, 'k-')
hold on
plot(xVals, yVals1c, 'b-')
hold off
```

Problem 1



Problem 2

The fastest time ever recorded in the Kentucky Derby was by a horse named Secretariat in 1973. He covered the 1 1\4 mile track in 1:59.4 (one minute and 59.4 seconds). Times at the quarter-mile, half-mile, and mile poles were 0:25.2, 0:49.2, and 1:36.4. Use interpolation to predict the time at the three-quarter mile pole and compare this to the actual time of 1:13.

```
%   distance (mile) | time (s)
%   ------------------------------
%       0.00        |     0.0
%       0.25        |    25.2
%       0.50        |    49.2
%       1.00        |    96.4
%       1.25        |   119.4

fprintf('Problem 2\n')

% Define vectors for distance and time
distance = [0, 0.25, 0.5, 1, 1.25];
time = [0, 25.3, 49.2, 96.4, 119.4];

% Order of data based on proximity to 0.75 miles
indexArray = [3, 4, 2, 5, 1];

% Sort arrays based on index
distanceSorted = distance(indexArray);
```

```
timeSorted = time(indexArray);

% Initialize Fij using timeSorted array
inFij = InitializeFij(timeSorted);

% Generate divided differences matrix using distanceSorted array
startRow = 2;
Fij = GenerateFij(distanceSorted, inFij, startRow);

% Approximate time(0.75 miles)
inVec = 0.75;
approxTime = CalcInterPoly(Fij, distanceSorted, inVec);

fprintf('The time at 0.75 miles is approximately %.1f\n', approxTime)

% The actual time at 0.75 miles was 73 seconds according to the
 homework
actualTime = 73;

% Relative error between actual time compared to approximate time
relError = (abs(actualTime - approxTime) / actualTime) * 100;

fprintf('The relative error between %.1f seconds and %.1f seconds is
 %.2f%%\n', ...
    actualTime, approxTime, relError)
```

Problem 2

Problem 3

The following function data represents points on a polynomial (see below comment block). Use divided differences to determine the degree of the polynomial.

```
%   x   |   f(x)
% ------------------
%   0   |   0
%   1   |   -2
%   2   |   -8
%   3   |   0
%   4   |   64
%   5   |   250
%   6   |   648
%   7   |   1372

fprintf('Problem 3\n')

% Create vectors for the inputs and outputs of function f
xVec = [0:1:7];
fofx = [0, -2, -8, 0, 64, 250, 648, 1372];

% Initialize divided differences matrix
inFij = InitializeFij(fofx);
```

```
% Generate divided differences matrix
startRow = 2;
Fij = GenerateFij(xVec, inFij, startRow);

% Use real in conjunction with sym to creat a symbolic variable x that
% represents a real number. Otherwise, the dot function in
 CalcInterPoly
% returns conj(x) instead. Since I don't know if this divided
 differences
% works the same way with complex numbers, I'm doing this to be safe.
x = real(sym('x'));

% Use expand to multiply out all of the products in each summand and
 add
% them together to get a single polynomial. Use matlabFunction to make
 this
% expression a function of x.
interPoly = matlabFunction(expand(CalcInterPoly(Fij, xVec, x)))

fprintf('According to the output, this is a 4th degree polynomial\n')

% Check that the symbolic interpolating polynomial interpolates data
interPolyResults = interPoly(xVec);
checkMatrix = [fofx; interPolyResults] % visual check by leaving off ;
```

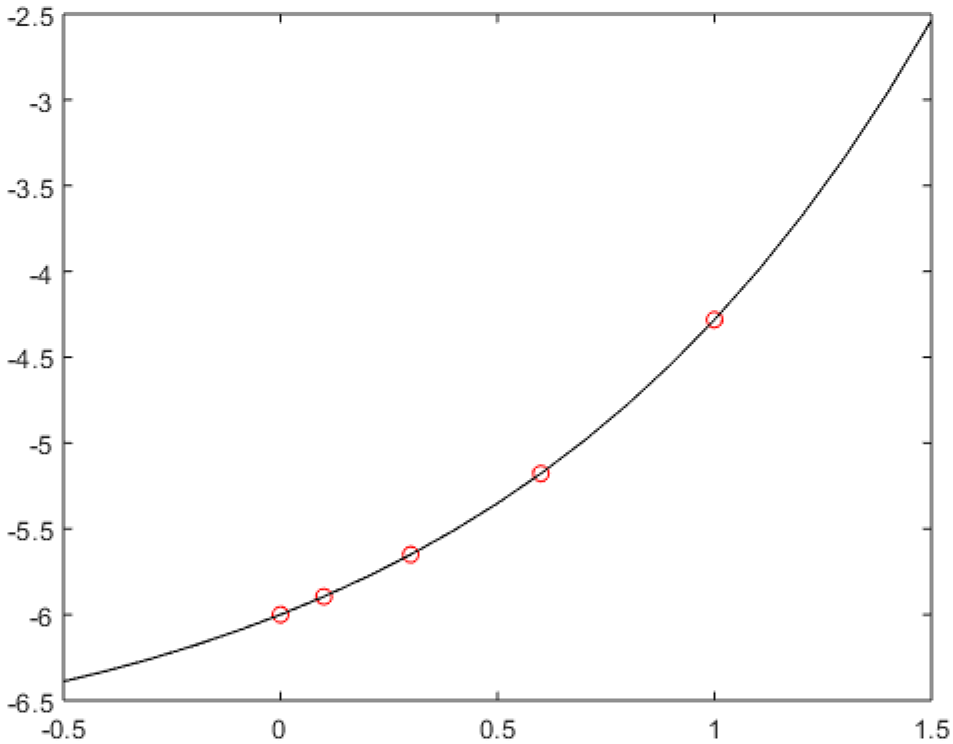
Problem 3

Local Helper Functions

```
++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
```

Make Coefficient Vector

```
function coeffVec = MakeCoeffVec(inputMatrix)
% This function take a square matrix as its input (in this case, the
%   divided differences matrix) and returns a row vector containing
 all of
%   the entries along the princple diagonal
%
%    INPUT: inputMatrix -  square matrix
%   OUTPUT: coeffVec - row vector containing entries allong principle
%           diagonal of square
[numRowFij numColFij] = size(inputMatrix);

n = numRowFij;

%   make coefficient vector
coeffVec = [];
for i = 1: n
    coeffVec(i) = inputMatrix(i,i);
end
end
```




```
Using the interpolating polynomial, f(0.9) is approximately -4.540223

The time at 0.75 miles is approximately 72.8
The relative error between 73.0 seconds and 72.8 seconds is 0.26%


interPoly =

  function_handle with value:

    @(x)real(x).^3.*-3.0+real(x).^4

According to the output, this is a 4th degree polynomial

checkMatrix =

  Columns 1 through 6

           0          -2          -8           0          64
 250
           0          -2          -8           0          64
 250

  Columns 7 through 8

         648        1372
         648        1372
```

Calculate output of Divided Difference Interpolating Polynomial

```
function approxVec = CalcInterPoly(Fij, inData, inVec)
% This function computes the approximated output of the interpolating
%   polynomial given a divided difference matrix (Fij), the set of
 input
%   data used to in generating Fij, and the input at which the user
 would
%   like to evaluate the interpolating polynomial
%
%    INPUT - Fij: divided differences matrix
%            inData: input values used in the generation of Fij
%            inVec: vector of inputs at which to evaluate the
 interpolating
%            polynomiat
%   OUTPUT - approximation: vector containing approximations for inVec

% Make Coefficient Vector
coeffVec = MakeCoeffVec(Fij);

approxVec = [];
for i = 1: length(inVec)
    % row vec [1,(x-x0),(x-x0)(x-x1),..,(x-x0)(x-x1)..(x - xnMinus1)]
    prodVec = [1];
    for m = 2: length(inData)
        prodVec = [prodVec, (inVec(i) - inData(m - 1)) * prodVec(m
 -1)];
    end

    approximation = dot(prodVec, coeffVec);
    approxVec = [approxVec, approximation];
end
end
```

Initialize Divided Differences Matrix F(n,n)

```
function initFij = InitializeFij(outVals)
% This function creates a zero matrix and then replaces the first
 column
% with the data in the output vector.
%    INPUT - outVals: the output values from the data set
%   OUTPUT - initFij: initialized divided difference matrix with first
%            column being the output data set and the rest being zeros

i = length(outVals);

initFij = zeros(i,i);
```

```
for m = 1: i
    initFij(m,1) = outVals(m);
end
end
```

Generate Divided Differences Matrix F(n,n)

```
function Fij = GenerateFij(inVals, inFij, startRow)
% This function polpulates the divided difference matrix using the
 input
% values and some initial divided difference matrix. The initial
 divided
% difference matrix does not have to be the one generated by
 InitializeFij.
% The algorithm will start on the specified row.
%    INPUT - inVals: input values from data set
%            inFij: initial divided differences matrix
%            startRow: tells the algorithm where to start
%   OUTPUT - Fij: completed divided differences matrix

i = length(inVals);

Fij = inFij;

for i = startRow: i
    xi = inVals(i);
    for j = 1: i-1
        xiMinusj = inVals(i-j);
        Fij(i, j+1) = (Fij(i,j) - Fij(i-1, j)) / ( xi - xiMinusj );
    end
end

end
```

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