MATH 3043, Numerical Analysis I
Fall 2018

Lab 12

This lab will have you implementing the Jacobi and Gauss-Seidel methods to approximate solutions of linear systems.

Solutions must be submitted on Canvas and are due **December 10** at the beginning of lab. Please submit a single script file `Lab12Lastname.m` (for example, my submitted file would be `Lab12Zumbrum.m`); each solution should

- be contained in a separate cell which includes the problem number and short problem description,
- run independent of other cells,
- be adequately commented.

Write separate MATLAB functions to implement the Jacobi and Gauss-Seidel methods (these may be included at the end of your script as local functions). Each function should accept five inputs (the matrix and right-hand side vector for the linear system, the maximum number of iterations, the stopping criteria tolerance, and the column vector with the initial approximation) and return two outputs (the number of iterations required and the approximation of the solution). Output the number of iterations required and the approximation for the solution for each problem (displaying at least k digits for solutions accurate to within 10^{-k}). Use the stopping criteria

$$\frac{\|\mathbf{x}^{(k)} - \mathbf{x}^{(k-1)}\|_\infty}{\|\mathbf{x}^{(k)}\|_\infty} < \text{tolerance.}$$

1. For the linear system

$$\begin{array}{rcrcrcrcr} 10x_1 & - & x_2 & + & 2x_3 & & & = & 6 \\ -x_1 & + & 11x_2 & - & x_3 & + & 3x_4 & = & 25 \\ 2x_1 & - & x_2 & + & 10x_3 & - & x_4 & = & -11 \\ & & 3x_2 & - & x_3 & + & 8x_4 & = & 15, \end{array}$$

   (a) use the Jacobi method to solve the linear system with $\mathbf{x}^{(0)} = \mathbf{0}$ and tolerance 10^{-3}
   Hint: This problem is Section 7.1 Example 1.

   (b) repeat part (a) using the Gauss-Seidel method
   Hint: This problem is Section 7.1 Example 3.

2. For the linear system

$$\begin{array}{rcrcrcrcrcrcr}
4x_1 & - & x_2 & & & & & & & & & = & 0 \\
-x_1 & + & 4x_2 & - & x_3 & & & & & & & = & 5 \\
 & - & x_2 & + & 4x_3 & & & & & & & = & 0 \\
 & & & & & + & 4x_4 & - & x_5 & & & = & 6 \\
 & & & & & - & x_4 & + & 4x_5 & - & x_6 & = & -2 \\
 & & & & & & & - & x_5 & + & 4x_6 & = & 6,
\end{array}$$

   (a) use the Jacobi method to solve the linear system with $\mathbf{x}^{(0)} = \mathbf{0}$ and tolerance 10^{-4}

   (b) repeat part (a) using the Gauss-Seidel method

3. For the linear system $A\mathbf{x} = \mathbf{b}$, where the entries of A are

$$a_{ij} = \begin{cases} 2i, & \text{when } j = i \text{ and } i = 1, 2, \ldots, 80, \\ 0.5i, & \text{when } \begin{cases} j = i+2 \text{ and } i = 1, 2, \ldots, 78, \\ j = i-2 \text{ and } i = 3, 4, \ldots, 80, \end{cases} \\ 0.25i, & \text{when } \begin{cases} j = i+4 \text{ and } i = 1, 2, \ldots, 76, \\ j = i-4 \text{ and } i = 5, 6, \ldots, 80, \end{cases} \\ 0, & \text{otherwise,} \end{cases}$$

   and the entries of $\mathbf{b}$ are $b_i = \pi$ for $i = 1, 2, \ldots, 80$,

   (a) use the Jacobi method to solve the linear system with $\mathbf{x}^{(0)} = \mathbf{0}$ and tolerance 10^{-5}

   (b) repeat part (a) using the Gauss-Seidel method

4. For the linear system

$$\begin{array}{rcrcrcr}
2x_1 & - & x_2 & + & x_3 & = & -1 \\
2x_1 & + & 2x_2 & + & 2x_3 & = & 4 \\
-x_1 & - & x_2 & + & 2x_3 & = & -5
\end{array}$$

   with exact solution $\mathbf{x} = (1, 2, -1)^T$,

   (a) compute $\rho(T_j)$ and $\rho(T_g)$

   (b) use your answers from part (a) to choose one method to approximate the solution of the linear system to within 10^{-5} using $\mathbf{x}^{(0)} = \mathbf{0}$

5. For the linear system

$$\begin{array}{rcrcrcr}
x_1 & + & 2x_2 & - & 2x_3 & = & 7 \\
x_1 & + & x_2 & + & x_3 & = & 2 \\
2x_1 & + & 2x_2 & + & x_3 & = & 5
\end{array}$$

   with exact solution $\mathbf{x} = (1, 2, -1)^T$,

   (a) compute $\rho(T_j)$ and $\rho(T_g)$

   (b) use your answers from part (a) to choose one method to approximate the solution of the linear system to within 10^{-5} using $\mathbf{x}^{(0)} = \mathbf{0}$