

**MATH 3043, Numerical Analysis I**  
Fall 2018

**Lab 10**

This lab will have you working with matrix inverses and matrix factorizations.

Solutions must be submitted on Canvas and are due **November 26** at the beginning of lab. Please submit a single script file `Lab10Lastname.m` and the corresponding published file `Lab10Lastname.pdf` (for example, my submitted files would be `Lab10Zumbrum.m` and `Lab10Zumbrum.pdf`). Each solution should

- be contained in a separate cell which includes the problem number and short problem description,
  - run independent of other cells,
  - be adequately commented.
1. Consider an  $n \times n$  matrix  $A$  where  $n$  is a random integer between 4 and 6 (inclusive) and entries of  $A$  are random numbers between 0 and 1 (exclusive). Determine if  $A$  is nonsingular; if it is, find the inverse of  $A$  (by either directly solving a system or by extracting entries from an augmented matrix in reduced row echelon form). If it is singular, print a message stating this.

**Hints:**

- The determinant of a matrix can be calculated using the built-in function `det`
  - The reduced row echelon form of a matrix can be found using the built-in function `rref`
  - The solution of this problem can be found in less than 10 lines of code (and without the use of a loop)
2. Find the Doolittle factorization for  $A$  (or  $PA$  if necessary) and use it to solve the system  $A\mathbf{x} = \mathbf{b}$  where

$$A = \begin{bmatrix} 3 & -6 & 9 & 3 \\ 2 & 1 & 4 & 1 \\ 1 & -2 & 2 & -1 \\ 1 & -2 & 3 & 0 \end{bmatrix} \quad \text{and} \quad \mathbf{b} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}.$$

**Hints:**

- The Doolittle factorization (and permutation matrix  $P$ ) for the matrix  $PA$  can be found in MATLAB using `[L, U, P] = lu(A)`
  - What do you notice about  $P$  in this problem?
3. Repeat Problem 2 for the following coefficient matrix:

$$A = \begin{bmatrix} 1 & 1 & -1 & 0 \\ 1 & 1 & 4 & 3 \\ 2 & -1 & 2 & 4 \\ 2 & -1 & 2 & 3 \end{bmatrix}$$

**Hint:** What do you notice about  $P$  in this problem? Be sure to solve the correct system!

4. [Section 12.1 Example 1] Determine the steady-state heat distribution in a thin square metal plate with dimensions 0.5 m by 0.5 m using  $n = 4$  subintervals. Two adjacent boundaries are held at  $0^\circ\text{C}$ , and the heat on the other boundaries increases linearly from  $0^\circ\text{C}$  at one corner to  $100^\circ\text{C}$  where the sides meet.

To find a numerical solution for this problem, solve the linear system

$$\begin{bmatrix} 4 & -1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 4 & -1 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 4 & 0 & 0 & -1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 4 & -1 & 0 & -1 & 0 & 0 \\ 0 & -1 & 0 & -1 & 4 & -1 & 0 & -1 & 0 \\ 0 & 0 & -1 & 0 & -1 & 4 & 0 & 0 & -1 \\ 0 & 0 & 0 & -1 & 0 & 0 & 4 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & -1 & 4 & -1 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & -1 & 4 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \\ w_5 \\ w_6 \\ w_7 \\ w_8 \\ w_9 \end{bmatrix} = \begin{bmatrix} 25 \\ 50 \\ 150 \\ 0 \\ 0 \\ 50 \\ 0 \\ 0 \\ 25 \end{bmatrix}$$

for temperatures  $w_i$  at interior grid points (at the corresponding points labeled in Figure 12.7). The code below constructs the coefficient matrix and right-hand side vector for a system with a general number of subintervals  $n$ .

```
%specifies the number of subintervals in each direction
n = 4;

%creates the coefficient matrix
A = toeplitz([4 -1 zeros(1, n-3) -1 zeros(1, (n-2)*(n-1)-1)]);

%updates specific coefficient matrix entries in the identity blocks
for i = 1:n-2
    A(i*(n-1), i*(n-1)+1) = 0;
    A(i*(n-1)+1, i*(n-1)) = 0;
end

%creates the right-hand side vector
b = zeros((n-1)*(n-1), 1);

%updates entries based on the boundary conditions
b(1:n-1) = b(1:n-1) + 100/n*[1:n-1]';
b(n-1:n-1:end) = b(n-1:n-1:end) + 100/n*[n-1:-1:1]';
```

- (a) What do you notice about the coefficient matrix? Use the  $LU$  decomposition of the coefficient matrix to solve the system and save the temperatures in a vector  $w$ .

With the numerical approximations for the temperature at the interior grid points stored in the column vector  $w$ , the code below plots the resulting numerical solution for the temperature distribution on the entire metal plate.

```
%creates a matrix for temperatures at all grid points
W = zeros(n+1, n+1);

%inserts temperatures at interior grid points
W(2:end-1, 2:end-1) = reshape(w, n-1, n-1)';

%inserts temperatures on the boundaries
W(1, :) = 100/n*[0:n];
W(:, end) = 100/n*[n:-1:0];

%creates vectors for plotting
x = 0.5/n*[0:n];
y = 0.5/n*[n:-1:0];

[xx, yy] = meshgrid(x,y);

surf(xx,yy,W)
```

- (b) Produce the plot for the numerical solution of the temperature distribution with  $n = 4$  intervals.
- (c) Redo parts (a) and (b) for  $n = 32$  intervals.
- (d) Update the code to use the temperature distributions  $f(x) = 1600x^4$  and  $f(y) = 1600y^4$  at the non-zero boundaries and use  $n = 32$  intervals. **Hint:** Only update the right-hand side vector and solve the new system for the approximations of the temperature at the interior grid points; use the  $LU$  decomposition you found in part (c)!

**Note:** If this problem is interesting to you, register for MATH 3044, Numerical Analysis II!