MATH 3043, Numerical Analysis I

Fall 2018

Lab 11

This lab will have you computing vector norms and finding eigenvalues and corresponding eigenvectors.

Solutions must be submitted on Canvas and are due **December 3** at the beginning of lab. Please submit a single script file `Lab11Lastname.m` and the corresponding published file `Lab11Lastname.pdf` (for example, my submitted files would be `Lab11Zumbrum.m` and `Lab11Zumbrum.pdf`). Each solution should

- be contained in a separate cell which includes the problem number and short problem description,
- run independent of other cells,
- be adequately commented.

1. Consider an r-dimensional column vector $\mathbf{x}$ whose entries are random real numbers between -5 and 5. For an integer r between 15 and 25 (inclusive),
   (a) compute $\|\mathbf{x}\|_\infty$ using a user-defined function `norminf`
   (b) compute $\|\mathbf{x}\|_1$ using a user-defined function `norm1`
   (c) compute $\|\mathbf{x}\|_2$ using a user-defined function `norm2`
   (d) compute $\|\mathbf{x}\|_p$ for $p = 3$ using a user-defined function `normp`

   Note: The ℓ_p norm is defined by

   $$\|\mathbf{x}\|_p = \left\{ \sum_{i=1}^{n} |x_i|^p \right\}^{1/p}, \ p \leq 1;$$

   this is consistent with the definitions for $\|\mathbf{x}\|_1$ and $\|\mathbf{x}\|_2$ we've already seen.

2. Redo Problem 1 (for the same vector $\mathbf{x}$) using the built-in function `norm`.

3. The following linear systems $A\mathbf{x} = \mathbf{b}$ have $\mathbf{x}$ as the actual solution and $\tilde{\mathbf{x}}$ as an approximate solution. Compute $\|\mathbf{x} - \tilde{\mathbf{x}}\|_\infty$ and $\|A\tilde{\mathbf{x}} - \mathbf{b}\|_\infty$.
   (a) $\mathbf{x} = \left(\frac{1}{7}, -\frac{1}{6}\right)^T$, $\tilde{\mathbf{x}} = (0.142, -0.166)^T$

   $$\begin{aligned} \frac{1}{2}x_1 &+ \frac{1}{3}x_2 = \frac{1}{63} \\ \frac{1}{3}x_1 &+ \frac{1}{4}x_2 = \frac{1}{168} \end{aligned}$$

   (b) $\mathbf{x} = (1.827586, 0.6551724, 1.965517)^T$, $\tilde{\mathbf{x}} = (1.8, 0.64, 1.9)^T$

   $$\begin{array}{rcrcrcr} 0.04x_1 & + & 0.01x_2 & - & 0.01x_3 & = & 0.06 \\ 0.2x_1 & + & 0.5x_2 & - & 0.2x_3 & = & 0.3 \\ x_1 & + & 2x_2 & + & 4x_3 & = & 11 \end{array}$$

4. Find the eigenvalues and corresponding eigenvectors for

$$\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$$

   (a) by hand

   (b) using the built-in function `eig`

5. Repeat Problem 4 for

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}.$$

   Note: What do you notice about the length of eigenvectors found using `eig`?