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MATHS ASSIGNMENT.

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Ques. 1 $f(x) = \left(\frac{\pi - x}{2}\right)^2$ interval $0 \le x \le 2\pi$

$\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \ldots\ldots = \pi^2/6$

Soln: $f(x) = \left(\frac{\pi - x}{2}\right)^2 \Rightarrow a_0 = \frac{1}{\pi}\int_0^{2\pi}\left(\frac{\pi - x}{2}\right)^2 \cdot dx$

$\Rightarrow \frac{1}{4\pi}\int_0^{2\pi}(\pi^2 - 2\pi x + x^2)\,dx \Rightarrow \frac{1}{4\pi}\left[\pi^2 x - \frac{\cancel{2}\pi x^2}{\cancel{2}} + \frac{x^3}{3}\right]_0^{2\pi}$

$\Rightarrow \frac{1}{4\pi}\left[2\pi^3 + \frac{8\pi^3}{3} - 4\pi^3\right] \Rightarrow \frac{1}{{}_{2}\cancel{4\pi}}\left[\frac{\cancel{2}\pi^{\cancel{3}\,2}}{3}\right] = \frac{\pi^2}{6}$

$\Rightarrow a_0 = \frac{1}{\pi}\int_0^{2\pi}\left(\frac{\pi - x}{2}\right)^2 \cos nx \cdot dx$

$\downarrow$ u $\quad\quad$ $\downarrow$ v

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$$= \frac{1}{4\pi}\left[(\pi-x)^2\left(\frac{\sin nx}{n}\right) - \left(-2(\pi-x)\right)\left(\frac{-\cos nx}{n^2}\right) + 2\left(\frac{-\sin nx}{n}\right)\right]_0^{2\pi}$$

$$= 1/4\pi\left[-2(-\pi)(1/n^2) + 2(\pi)(1/n^2)\right]$$

$$= 1/4\pi\left[-2(-\pi)(1/n^2) + 2(\pi)(1/n^2)\right]$$

$$= 1/4\pi\left[\frac{2\pi}{n^2} + \frac{2\pi}{n^2}\right] = 1/4\pi\left[4\pi/n^2\right]$$

$$a_n = 1/n^2$$

$$\cancel{b_n = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx}$$

$$\cancel{f(x) = \left(\frac{\pi-x}{2}\right)^2} =$$

$$b_n = \frac{1}{\pi}\int_0^{2\pi}\left(\frac{\pi-x}{2}\right)^2 \cdot \sin nx \cdot dx$$

$$= 1/4\pi \int_0^{2\pi} \underset{u}{(\pi-x)^2} \cdot \underset{v}{\sin nx} \cdot dx$$

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$$= \frac{1}{4\pi}\left[(\pi-x)^2\left(\frac{-\cos nx}{n}\right) - (-2)(\pi-x)\left(\frac{-\sin nx}{n^2}\right) + 2\left(\frac{\cos nx}{n^2}\right)\right]_0^{2\pi}$$

$$= \frac{1}{4\pi}\left[\pi^2\left(\frac{1}{n}\right) + 2\left(\frac{1}{n^2}\right) - \left\{\pi^2\left(\frac{-1}{n}\right) + 2\left(\frac{1}{n^2}\right)\right\}\right]_0^{2\pi}$$

$$= \frac{1}{4\pi}\left[\cancel{-\pi^2/n} + \cancel{2/n^3} + \cancel{\pi^2/n} - \cancel{2/n^3}\right]_0^{2\pi}$$

$$b_n = 0$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx$$

$$f(x) = \left(\frac{\pi-x}{2}\right)^2 = \frac{\pi^2}{12} + \left(a_1 \cos x + a_2 \cos 2x + a_3 \cos 3x + \dots\right)$$

$$f(x) = \left(\frac{\pi-x}{2}\right)^2 = \frac{\pi^2}{12} + \left(\frac{1}{1^2}\overset{\cos x}{\uparrow} + \frac{1}{2^2}\cos 2x + \frac{1}{3^2}\cos 3x + \dots\right)$$

put $x = 0$

$$f(x) = \pi^2/4 = \pi^2/12 + \left[1/1^2 + 1/2^2 + 1/3^2 + \dots\right]$$

$$\pi^2/4 - \pi^2/12 = \left[1/1^2 + 1/2^2 + 1/3^2 + \dots\right]$$

$$\frac{2\pi^2}{12} = \left[1/1^2 + 1/2^2 + 1/3^2 + \dots\right]$$

$$\Rightarrow \pi^2/6 = \left[1/1^2 + 1/2^2 + 1/3^2 + \dots\right]$$

Ans

Ques. 2 $f(x) = x^2 \qquad -l \leq x < l$

$1/1^2 + 1/2^2 + 1/3^2 + \dots = ?$

Soln: $f(x) = x^2 \qquad f(-x) = f(x) =$ even funcn

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{l}\right)$$

$$a_0 = 1/l \int_{-l}^{l} x^2 dx = 1/l \left[x^3/3\right]_{-l}^{l}$$

$$a_0 = 1/l\left[2l^3/3\right] = \underline{\underline{2l^2/3}}$$

$$a_n = 1/l \int_{-l}^{l} x^2 \cdot \cos\left(\frac{n\pi x}{l}\right) \cdot dx$$

$$= 1/l \left[x^2 \frac{\sin\left(\frac{n\pi x}{l}\right)}{\left(\frac{n\pi}{l}\right)} + (2x) \frac{\cos\left(\frac{n\pi x}{l}\right)}{\left(\frac{n\pi}{l}\right)^2} + 2\frac{\left(-\sin\left(\frac{n\pi x}{l}\right)\right)}{\left(\frac{n\pi}{l}\right)^3} \right]_{-l}^{l}$$

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$$= \frac{1}{l}\left[\frac{2l^2}{n^2\pi^2} \times l(-1)^n + l(-1)^n\right]$$

$$= \frac{l}{n^2\pi^2}[4l](-1)^n \qquad = \frac{4l^2}{n^2\pi^3}(-1)^n.$$

$$f(x) = x^2 = \frac{l^2}{3} + \sum_{n=1}^{\infty}\left(\frac{4l^2}{n^2\pi^2}\right)\cos\left(\frac{n\pi x}{l}\right)(-1)^n.$$

$$x^2 = l^2/3 + 4l^2/\pi^2 \sum_{n=1}^{\infty} \cos\left(\frac{n\pi x}{l}\right) \times 1/n^2 \times (-1)^n$$

$$x^2 = l^2/3 + \frac{4l^2}{3}\left(\frac{-\frac{\cos \pi x}{l}}{1^2} + \frac{\frac{\cos 2\pi x}{l}}{2^{2}} - \frac{\frac{\cos 3\pi x}{l}}{3^2} + \dots\right)$$

$$\left(x^2 - \frac{l^2}{3}\right)\frac{\pi^2}{4l^2} = \left[\frac{-1}{1^2}\frac{\cos \pi x}{l} + \frac{1}{2^2}\frac{\cos 2\pi x}{l} - \dots\dots\right]$$

Put $x=0$

$$\frac{-\pi^2 l^2}{12 l^2} = \left[\frac{-1}{1^2} + \frac{1}{2^2} - \frac{1}{3^2} + \frac{1}{4^2} + \dots\dots\right]$$

$$-\pi^2/12 = -\left[1/1^2 - 1/2^2 + 1/3^2 - 1/4^2 + \dots\right]$$

$$\boxed{\pi^2/12 = \left[1/1^2 - 1/2^2 + 1/3^2 - 1/4^2 + \dots\right]}$$

Q.3. $f(x) = x(l-x)$ $(0, l)$

$1/1^3 - 1/3^3 + 1/5^3 + \ldots\ldots = ?$

Soln: $f(x) = x(l-x)$, $(0,l)$

$$b_n = 2/l \left(\int_0^l x(l-x) \cdot \sin\left(\frac{n\pi x}{l}\right) \cdot dx \right.$$

u v

$$b_n = 2/l \left[x(l-x)\left(\frac{-\cos n\pi x/l}{(n\pi/l)}\right) - (l-2x)\left(\frac{-\sin\left(\frac{n\pi x}{l}\right)}{\left(\frac{n\pi}{l}\right)^2}\right) + (-2)\left(\frac{\cos(n\pi x/l)}{(n\pi/l)^3}\right)\right]_0^l$$

$$b_n = 2/l \left[(-2)\left(\frac{l^3}{n^3\pi^3}\right)\left((-1)^n - 1\right)\right]$$

$$b_n = \frac{-4l^2}{n^2\pi^3}\left((-1)^n - 1\right)$$

$$f(x) = \sum_{n=1}^{\infty} \frac{-4l^3}{n^3\pi^3}\left((-1)^n - 1\right) \sin\left(\frac{n\pi x}{l}\right)$$

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$$x(l-x) = \frac{-4l^2}{n^3\pi^3} \sum_{n=1}^{\infty} \left((-1)^n - 1\right) \sin\left(\frac{n\pi x}{l}\right)$$

$$x(l-x) = \frac{-4l^2}{\pi^3}\left[\frac{-2}{1^3} \sin\frac{n\pi x}{l} + 0 - \frac{2}{3^3} \sin\left(\frac{3\pi x}{l}\right) + 0 \ldots\right]$$

$$x(l-x) = \frac{8l^2}{\pi^3}\left[\frac{1}{1^3} \sin\frac{\pi x}{l} + \frac{1}{3^3} \sin\frac{3\pi x}{l} + \ldots\right]$$

$$\cancel{\pi^3/8l^2 \,(x)(l-x) = 1/1^3 - 1/3^3 + 1/5^3 + \ldots}$$

~~by (Put $x = l/2$)~~

$$\cancel{\pi^3/8l^2 \,(l/2)(l/2) = 1/1^3 - 1/3^3 + 1/5^3 + \ldots}$$

$$\pi^3/8l^2 \,(x)(l-x) = \left[1/1^3 \sin\frac{\pi x}{l} + 1/3^3 \sin\frac{3\pi x}{l} + \ldots\right]$$

Put $x = l/2$

$$\pi^3/32 = 1/1^3 \sin \pi/2 + 1/3^3 \sin(\pi + \pi/2) + \ldots$$

$$\frac{\pi^3}{32} = 1/1^3 + 1/3^3 (\sin(-\pi/2)) + \ldots$$

$$\frac{\pi^3}{32} = 1/1^3 - 1/3^3 + 1/5^3 + \ldots$$

Ans

Ques.4. $f(x) = x \qquad (0, \pi)$

$$\frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \cdots\cdots = \frac{\pi^4}{96}$$

Soln: $f(x) = x$

$$a_0 = 2/\pi \int_0^{\pi} x.dx = 2/\pi \left[\frac{x^2}{2}\right]_0^{\pi} = \frac{2}{\pi}\left[\frac{\pi^2}{2}\right] = \underline{\underline{\pi}}$$

$$a_n = 2/\pi \int_0^{\pi} x.\cos nx.dx$$

$$a_n = 2/\pi \left[\frac{x\sin nx}{n} + \frac{\cos nx}{n^2}\right]_0^{\pi}$$

$$a_n = 2/\pi \left[\frac{(-1)^n - 1}{n^2}\right]$$

According to this, half range cosine series is :

$$\bar{y}^2 = \frac{a_0^2}{4} + 1/2 \sum_{n=1}^{\infty} a_n^2$$

$$\bar{y}^2 = \frac{1}{b-a} \int_a^b y^2 dx = 1/\pi \int_0^{\pi} x^2 dx$$

$$= 1/\pi \left[x^3/3\right]_0^{\pi} = \underline{\underline{\pi^2/3}}$$

Sub~~stituting~~ stituting $\bar{y}^2$, a_0 ; a_n, we get

$$\frac{\pi^2}{3} = \frac{\pi^2}{4} + \frac{1}{2}\sum_{n=1}^{\infty} \frac{4}{\pi^2}\left[\frac{((-1)^n - 1)}{n^2}\right]^2$$

$$\frac{\pi^2}{3} - \frac{\pi^2}{4} = \frac{1}{2}\sum_{n=1}^{\infty} \frac{4}{\pi^2}\left[\frac{((-1)^n - 1)}{n^2}\right]^2$$

$$\frac{\pi^2}{12} = \frac{4}{2\pi^2}\sum_{n=1} \frac{[(-1)^n - 1]^2}{n^4}$$

$$\frac{\pi^4}{24} = \sum_{n=1}^{\infty} \frac{[(-1)^n - 1]^2}{n^4}$$

$$\frac{\pi^4}{24} = \frac{4}{1^4} + 0 + \frac{4}{3^4} + 0 + \frac{4}{5^4} + \dots$$

$$\frac{\pi^4}{24 \times 4} = \frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \dots$$

$$\pi^4/96 = 1/1^4 + 1/3^4 + 1/5^4 + \dots$$

Ans

Ques.5. $y = f(x)$

x :	0	$\pi/6$	$\pi/3$	π	$2\pi/3$	$5\pi/6$	2π
$f(x)$:	10	12	15	20	17	12	10

Solⁿ: Since, the function is periodic with period 2π, exclude the last point $x = 2\pi$.

Let $f(x) = (a_0/2) + (a_1 \cos x + b_1 \sin x) + (a_2 \cos 2x + b_2 \sin 2x)$

x	$f(x)$	$\cos x$	$\sin x$	$\cos 2x$	$\sin 2x$	$\cos 3x$	$\sin 3x$
0	10	1	0	1	0	1	0
$\pi/6$	12	0.866	0.5	0.5	0.366	0	1
$\pi/3$	15	0.5	0.866	-0.5	0.366	-1	0
π	20	-1	0	1	0	-1	0
$2\pi/3$	17	-0.5	0.866	0.5	-0.866	1	0
$5\pi/6$	12	-0.866	0.5	0.5	-0.866	0	1

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f(x)cos x	f(x)sin x	f(x)cos 2x	f(x)sin 2x	f(x)cos 3x	f(x)sin 3x
10	0	10	0	10	0
10.392	6	6	10.392	0	12
7.5	12.99	-7.5	12.99	-15	0
-20	0	20	0	-20	0
-8.5	14.782	-8.5	-14.722	17	0
-10.392	6	6	-10.392	0	12

Now, $a_n = \dfrac{2\sum f(x)}{6} = \dfrac{2(10+12+15+20+17+12)}{6} = \dfrac{172}{6} = +28.66$

$\dfrac{a_0}{2} = \dfrac{28.66}{2} = 14.33$

$a_1 = \dfrac{2\sum f(x)\cos x}{6} = \dfrac{2\times -11}{6} = 3.66$

$a_2 = \dfrac{2\sum f(x)\sin x}{6} = \dfrac{2\times 26}{6} = 8.66$

$a_3 = \dfrac{2\sum f(x)\cos 2x}{6} = 0$

$b_1 = \dfrac{2\times 39.71}{6} = 13.23$

$b_2 = \dfrac{2\times 0}{6} = 0$

$b_3 = \dfrac{2\times 2}{6} = 0.66$

$f(x) = \dfrac{a_0}{2} + (a_1\cos x + b_1\sin x) + (a_2\cos 2x + b_2\sin 2x) + (a_3\cos 3x + b_3\sin 3x)$

$f(x) = 14.33 - 3.66\cos x + 13.23\sin x + 8.66\cos 3x + 0.66\sin 3x$

Ans